

Evaluation of the Effectiveness of No-Fines Encapsulated Embankment Solution in Enhancing Flood Resilience of Railways

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ABSTRACT: Flooding events in Australia have increasingly caused substantial damage in recent years, with notable impact on the rail and road systems. Railway embankments have been recognised as highly susceptible to damage when tracks become submerged or overtopped. This vulnerability stems from the fact that, unlike constructed levees, water containment is not a primary design consideration during the development and construction of rail embankments, especially for older structures in Australian floodplains. As the frequency and intensity of floods escalate, rail authorities urgently require effective, economical, and easily deployable solutions to promptly rehabilitate flood-damaged rail embankments and restore the network to full functionality. Furthermore, enhancing embankment resilience at susceptible railway locations before devastating floods occur is essential for maintaining optimal network performance and minimising potential damage. This paper evaluates the effectiveness of a no-fines encapsulated embankment solution (NFEES) in mitigating flood vulnerability for rail embankments within Australian floodplains, through a real case study of rail embankment failure. The objective is to establish design criteria and specifications for NFEES, facilitating water flow through a permeable top section of the embankment, (also allowing for overtopping and limiting the loss of fines material) while accommodating rail dynamic loads throughout its design lifespan. The paper begins by outlining the background and methods before describing the standard rail embankment design approach and failure mechanisms due to flooding. The paper reviews existing flood mitigation strategies for transport embankments, emphasising the need for simpler, more easily implemented solutions. It then introduces the development of the no-fines encapsulated embankment solution. The paper incorporates numerical analysis to create and evaluate the feasibility and engineering specifics of the proposed solution, as well as determine its resilience against floodwater exposure.

KEYWORDS: flood resilience, rail embankment, no-fines, piping, overtopping.

1 INTRODUCTION

In recent years, floods in Australia have caused extensive damage, including severe disruption to the transport network (Figure 1). Railway embankments have been identified as particularly vulnerable, exhibiting significant damage when tracks are inundated or overtopped which can often result in significant washouts, especially adjacent drainage systems or under bridges. The fundamental reason for this vulnerability is that, unlike in constructed levees, water impoundment is not a primary design focus during rail embankment design and construction, particularly for aged embankments in Australian floodplains (ARA, 2023).



Figure 1. Flood damage to the Trans-Australian railway line near Tarcoola, South Australia, Jan 2022.

Due to the rise in the frequency and severity of flooding events, rail authorities need efficient, cost-effective, and easily implementable solutions that can promptly restore flood-damaged rail embankments and the network to full operation, often at times when the local road network has been compromised by the same flood events. Additionally, addressing embankment resilience at vulnerable locations of the railway prior to catastrophic flooding events is crucial to ensure optimal network performance and mitigate potential damages.

2 RAIL EMBANKMENT DESIGN AND THE NFEES SOLUTION

2.1 Traditional rail embankments

(Selig and Waters, 1994) provided practical guidance for the design, construction, maintenance, and renewal of railway track from a geotechnical perspective. Key functions of various track components are examined, along with the properties of the ballast and subgrade materials. A single-track railway embankment is a typical example of a railway in rural, and in many cases flood-prone, areas. For modern railway design and construction, as shown in Figure 2, the total rail formation typically comprised the following layers from top as described in (Selig and Waters, 1994) and in (ARTC, 2019):

- Ballast layer under sleeper;
- Capping layer;
- Structural fill layer below capping;
- General fill embankment;
- Prepared foundation at natural ground.

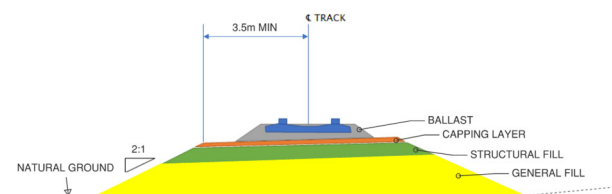


Figure 2. Traditional embankment layers.

In general, relevant authorities require rail embankment to be designed and constructed to, for example (ARTC, 2022):

- be resistant to global and local failure during the required Design Life to a minimum factor of safety of 1.5, or for short term cases such as storm rapid drawdown events or during an earthquake, 1.25;

- have a minimum bearing capacity factor of safety of 3 under static loading;
- Track formations must comply with the total vertical movement of 200mm and differential movement limit of 1 in 500 to 800;
- Be able to accommodate for transient vertical deflection of rail under dynamic loading between a minimum of 3.3mm and a maximum of 6.4mm for all trains;
- Shall provide mitigation for flood impacts.

Design assessments can be carried out using commercially available geotechnical software. For example slope stability analyses of the embankments have normally been undertaken using SLOPE/W computer program (Seequent, 2022). Settlement of the embankment under the rail loading can be assessed using PLAXIS (Brinkgreve et al., 2021). The protection of the earthworks from scour and erosion has been highlighted in modern railway design guidelines however no design criteria are provided.

2.2 Failure mode of traditional embankment due to flooding

Observations from individual instrumented earthwork sites and from the analyses of network-level data show that the differing material composition and construction method of embankments influence both the dominant failure mechanisms and the triggers to failure (Briggs et al., 2017). During flooding events, rail embankment failure has been observed to occur predominantly at low points where drainage culverts are present. These low points are typically areas where water can accumulate and create excess hydrostatic pressure, leading to instability of the embankment. The presence of a drainage culvert can exacerbate this situation by allowing water to flow under the embankment and potentially erode the soil, further reducing stability. It is therefore imperative that any measures taken to mitigate embankment vulnerability during flooding events account for these factors, and specifically address the risk posed by low points and drainage culverts.

The heights of railway embankments are generally small compared to dams and riverbanks. In extreme flooding events, seepage within embankments can generate two kinds of processes from a geotechnical point of view as shown in Figure 3 (USACE, 2014):

- External erosion along the embankment slopes - overtopping;
- internal erosion inside the embankment – piping.

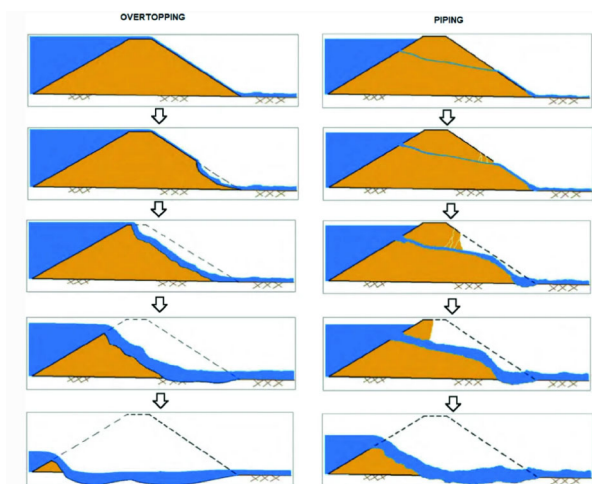


Figure 3. Failure modes of embankment due to flooding (USACE, 2014).

Both the processes can induce, under particular conditions, the general failure of the embankment. In particular, the external erosion failure of the embankment slopes derives from the reduced suctions and the development of pore water pressures, which are responsible for the reduction of the effective soil shear strength.

On the other hand, under specific seepage conditions, water flow can gradually induce internal erosion processes (piping), which may occur under different modes such as that has been described by (Fell and Fry, 2007):

- backward erosion: if seepage exit gradient along the downstream area of the embankment is sufficiently high to mobilize soil particles and can evolve upstream if the hydraulic gradient remains high throughout the embankment. Various researchers e.g. (Terzaghi, 1943) or (Richards and Reddy, 2007) presented method in assessing the critical hydraulic gradient
- suffusion: occurs when fine particles migrate through coarser particles under seepage forces, typically in internally unstable soils such as broadly or gap-graded soils. Various researchers have proposed methods to assess this risk potential e.g. (Kenney and Lau, 1985) and (Tomlinson and Vaid, 2000).
- concentrated leakage erosion: that can initiate through cracks or voids from settlement, desiccation, or poor compaction when flow-induced shear stress exceeds the soil's critical shear stress, and progresses as seepage erodes the sides of the flow path. This typically occurs adjacent to drainage structures.

The specific nature of railway embankments failure during and post flooding poses a challenge for design and implementation of rehabilitation measures. The normally adopted measures, such as replacement with a rail under bridge / culverts or embankment reconstruction after a washout can all be costly and time consuming. It should also be noted that the existing ageing railway embankments are prone to failure during flooding events and do not offer comparable levels of capability and resilience to modern engineered slopes, and this mainly owes to the known stressors such as the more prominent climate impacts and increased operational dynamic axle loading.

2.3 Proposed NFEES Solution

Figure 4 shows the proposed NFEES solution for two cases: (1) high embankments, where NFEES is applied to the top 2–3 m, and (2) low embankments under 3 m high, where NFEES is applied to the full height. The no-fines encapsulated embankment is intended to serve four primary purposes:

- as a porous structure, and when used adjacent existing drainage systems increases the cross-sectional area,
- as a spillway to enable safe floodwater passage, improving the embankment's ability to withstand hydraulic and slope stability challenges,
- to reduce the amount of fines or capping loss in the event of overtopping, reducing return to operations timeframes; and,
- mitigating erosion and scour.

The NFEES comprises several components, as illustrated in Figure 4. These include:

- redesigned edge baskets or proprietary systems which serve as retention structures to contain the rock fill;

- the rock fill, which is present both within and between the end baskets,
- Geogrid reinforcement is required to bind the end baskets and encapsulate the rock fill to prevent lateral spreading;
- Geofabric separation is also required at the bottom of the ballast to prevent infiltration or contamination of the rockfill;
- Further, to safeguard against scouring of the existing embankment, which may occur as water overtops the bottom part of the embankment, scour protection is needed, which may include the use of proprietary systems such as the reno mattress or concrete mats (https://www.concretemats.com.au);
- Finally the NFEES can be integrated with high and low flow drainage pipes, culverts and adjacent rail under bridges as required.

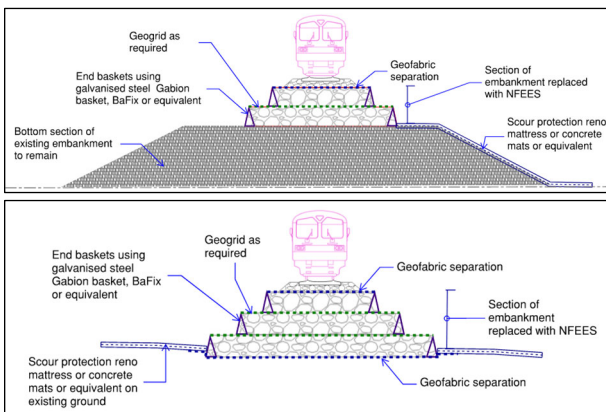


Figure 4. Research cases (Up: high embankment; below: low embankment).

3 VALIDATION OF THE STABILITY EFFECTIVENESS OF THE NFEES SOLUTION

3.1 The real case

The case of Main North Line embankment failure at Quarry Creek Tocal in Hunter Valley after April 2015 flood (shown in Figure 6) is adopted for the current investigation. Tocal is located in the Hunter Valley region of New South Wales, Australia. The Hunter Valley is part of the larger Sydney Basin, a sedimentary basin that extends from the coast to the Great Dividing Range. The geological context of Tocal and the surrounding region is characterised by sedimentary rocks, including sandstones, shales, and coal seams. It is expected the construction of the Main North Line has used locally won material derived from the above rock formations.

In April 2015, severe storms and heavy rainfall caused significant flooding in various parts of New South Wales, including Tocal. The Paterson River, which runs through the Tocal area, experienced its highest flood level in 60 years. According to the Bureau of Meteorology (BOM), the Paterson River at Paterson (which is upstream of Tocal) reached a peak flood level of 14.92 meters on April 21, 2015. This was the highest flood level recorded at this location since 1955. The floodwaters caused significant damage to roads and bridges in the area, including a major embankment washout along the Main North Line which is shown in Figure 6. The failure embankment had a height of 5.4m.

A detailed back-analysis of this study case has been performed, and this case has been used to assess the effectiveness of the NFEES. A diagrammatic cross-section of the embankment being investigated can be seen in Figure 5, accompanied by a photograph capturing the washout at the base of the embankment shown in Figure 6.

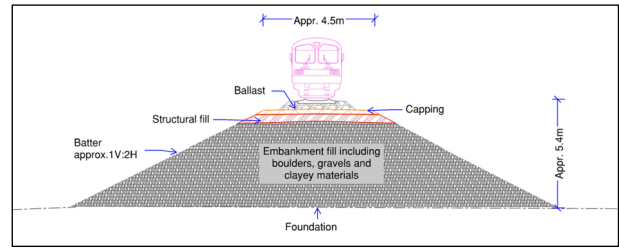


Figure 5. Schematic cross-section of the Tocal embankment



Figure 6. Photograph of the Tocal rail embankment washout taken at the bottom of embankment.

3.2 Back-analysis of the existing embankment

3.2.1 Seepage analysis

A finite element study was conducted to analyse the transient seepage process occurring within the embankment while considering unsaturated soil conditions. This was achieved using GEOSTUDIO software (Geo-Slope, 2022). The purpose of the study was to investigate how the flow regime changes over time as a result of the provisional impoundment on the upstream side. The study results were utilised to assess the hydraulic conditions that can trigger the onset of the piping process, as well as to back-calculate the sliding processes observed along the down-stream embankment slopes. The seepage analysis mesh utilised for the analysis is illustrated in Figure 7.

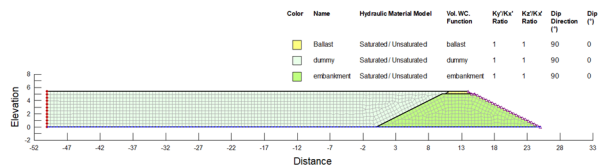


Figure 7. mesh and boundary conditions adopted for the seepage analysis of existing embankment.

A water head boundary condition variable with time in accordance with the impoundment level data recorded by BOM has been imposed on the upstream, 50m away from the upstream embankment face.

A free-draining boundary condition has been prescribed along the downstream face, whereas an impermeable boundary condition has been imposed along the bottom of the embankment. A saturation line at the bottom of the embankment has been assumed as initial condition (t =0).

Embankment materials have been assumed to have isotropic hydraulic properties. Since no field test could be performed on the material that had been placed in-situ, the value of the coefficient of permeability of the saturated soil forming the embankment has been assumed to be 1×10^{-5} m/s based on an examination of the material on the photo shown in Figure 6. For the rail ballast material, (Biabani and Indraratna, 2015) suggested a ballast permeability of 0.37m/s which is adopted here.

Based on the corresponding grain size distributions, the soil-water characteristic curve of all materials has been derived by means of the (Van Genuchten, 1980) relationship.

The results of the seepage analysis show that the saturation line reaches the maximum level during the raising of the impoundment level, shown in Figure 8 below. It can be seen that the embankment is almost fully saturated during the peak impoundment.

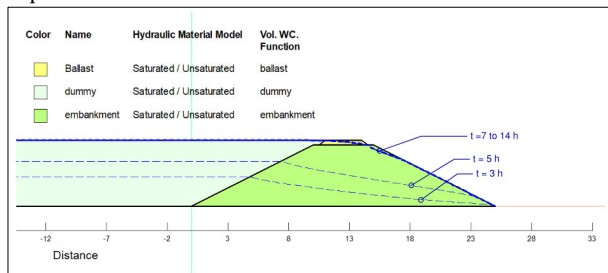


Figure 8. Saturation level in the embankment at different times.

The finite element analysis results indicate that at the downstream slope face, the assessed hydraulic gradient is 0.46 to 0.50 at the time when the embankment is fully flooded ($t = 13.4$ hour). The critical hydraulic gradient is assessed to be 0.4 based on the methods developed by (Kenney and Lau, 1985) and (Skempton and Brogan, 1994). Hence, an internal erosion failure within the embankment may have occurred.

3.2.2 Stability analysis

Based on the pore pressure distributions calculated by means of the transient seepage analysis presented above, limit equilibrium back-analyses have been carried out using the Morgenstern-Price method (Morgenstern and Price, 1965) with respect to sliding surfaces along the downstream embankment slope. In this back-analysis, the strength parameters of the embankment material have been assigned to be $c' = 5$ kPa and $\phi' = 25^\circ$. These parameters can be assumed as representative of a lower boundary of the shear strength of the examined embankment material, however, are considered to be not unexpected from site won residual material from sandstones, siltstones, and shales. Figure 9 illustrates the slope stability that was back analysed. At the time of peak impoundment, the downstream slope was unable to maintain its stability as the assessed factor of safety was below 1.0. This can be attributed to the extremely unfavourable flow conditions that occurred within the embankment during the flooding event.

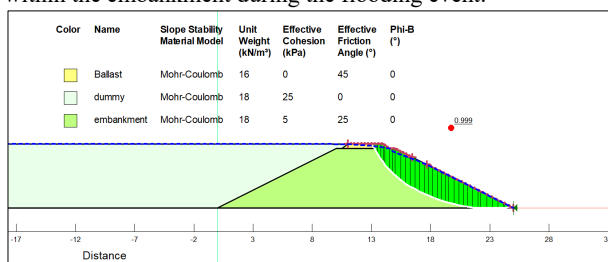


Figure 9. Back-analysis of stability of the saturated embankment

Nonetheless, if the embankment had not been subjected to such unfavourable pore water conditions, the evaluated factor of safety would have satisfied the stability requirements, as illustrated in Figure 10 below for the purpose of a direct comparison.

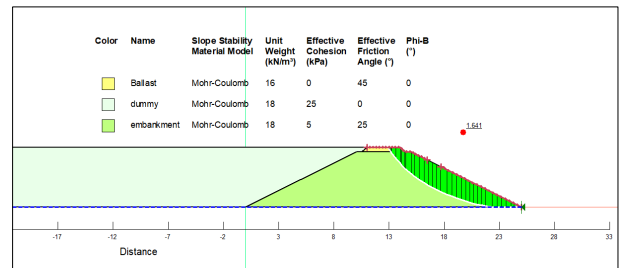


Figure 10. Stability of the unsaturated embankment

The above back analysis identified the hydraulic conditions that could trigger the onset of the piping process and sliding processes on the downstream slope. At the time of peak impoundment, the embankment was unable to maintain stability, largely due to unfavourable pore water conditions.

3.3 Evaluation of effectiveness of the NFEES solution

An evaluation of the efficacy of the NFEES solution entails conducting a range of seepage and stability assessments, along with an evaluation of internal erosion, to compare with the results of analysis carried out for the traditional embankment discussed in Section 3.2. Additionally, an assessment of the deformation characteristics of the NFEES embankment under the influence of dynamic loading on the rail is also performed, which is presented separately in Section 4.

3.3.1 Seepage Analysis

The seepage model and mesh adopted for the assessment is similar to that discussed in Section 3.2.1, however with a merely 2m high no-fines encapsulated embankment section incorporated at the top of the embankment.

Flowing through rockfill is inherently turbulent and therefore not amenable to a classical seepage analysis on the basis of Darcy's law. A non-Darcy flow model has been adopted for the rock fill based on (Joseph et al., 1982), the appropriate form of the momentum equation for a non-Darcian or turbulent flow model can be written as follows:

$$\nabla h = -\frac{q_w}{k_w} - C_F \sqrt{\frac{\rho_w}{k_w g \mu_w}} |q_w| q_w \quad (1)$$

where h is the total head, q_w is the superficial flux vector, and k_w is the hydraulic conductivity, $|q_w|$ is the Euclidean norm of the superficial flux vector, C_F is the dimensionless form-drag constant, ρ_w is the fluid density, g is the gravity vector, and μ_w is the dynamic viscosity of the fluid.

The saturation level within the embankment is shown in Figure 11 below.

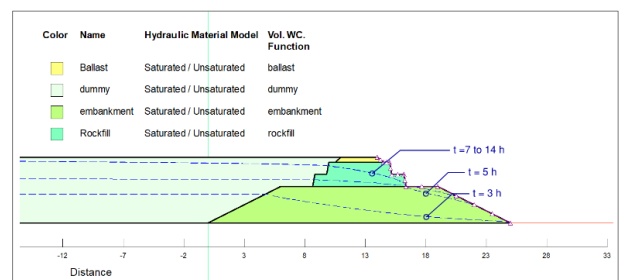


Figure 11. Saturation level in the embankment at different times

The introduction of the NFEES system has overall resulted in a significant reduction of the hydraulic gradient along the downstream slope with an assessed hydraulic gradient less than 0.4.

3.3.2 Stability analysis

An analogous stability assessment was conducted for the NFEES embankment, taking into account the pore pressure distributions derived from the transient seepage analysis.

The outcome of this evaluation is visually represented in Figure 12, which depicts the examined slope stability for the NFEES embankment. The factor of safety was calculated in this case as 1.38, above 1.25, normally accepted by Rail Authorities for this type of scenario.

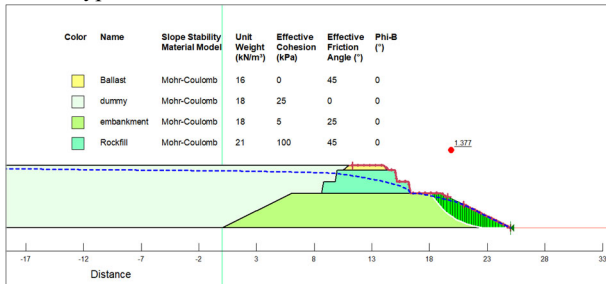


Figure 12. Analysis of the slope stability of NFEES embankment at time 7h to 14h

4 VALIDATION OF RAIL PERFORMANCE OF NFEES

4.1 Basic structural requirements of NFEES

The behaviour of a NFEES embankment under rail dynamic loading is strongly dependent on the material properties of the rock fill, the stiffness of the geogrid and the type of edge baskets.

Rail formation design is normally based on the functional design criteria nominated by the relevant rail authorities. As discussed in Section 2.1, the embankment system shall accommodate for transient vertical deflection of rail under dynamic loading between a minimum of 3.3mm and a maximum of 6.4mm.

Traditional solutions for the rail formation design, shown in Figure 2, include a high stiffness capping layer, which shall have a minimum California Bearing Ratio (CBR) of 50%, and a structural fill layer with a minimum CBR of 8%. The layers consisting of structural fill and capping materials are known as sub-ballast layers (Selig and Waters, 1994) which are located below the ballast and above the subgrade. The sub-ballast layers, with a stiffness of 80 to 300MPa (Butler & Deng, 2012), play a crucial role in maintaining the structural integrity and overall performance of the railway system in the rail formation design. Sub-ballast layers help to distribute the load exerted by the train over a wider area, reducing the stress applied to the subgrade and providing additional stiffness and strength to the track system. By doing so, they minimise deformation and settlement of the underlying subgrade and prevent uneven track settlement.

The NFEES embankment solution replaces the sub-ballast layers and a section of the subgrade below with a no-fines encapsulated rock fill system. Rock fill generally has much lower stiffness typically. In order to maintain the same functions as the sub-ballast layers the following considerations are given:

- **Rock fill:** Choose appropriate rock fill materials that are capable of providing adequate load distribution and drainage. The rock fill should have sufficient strength and durability to withstand the dynamic loads and stresses associated with train operations, and also should be

permeable, allowing for effective drainage while preventing the migration of fines. Rock fill is recommended to be 100mm to 250mm in particle size with a minimum point load strength of 1MPa; For the current assessment, an initial modulus of 20MPa and a reloading modulus of 60MPa has been adopted based on (Hunter and Fell, 2003) and calibrated against test data provided by (Jiang and Wang, 2011).

- **Edge baskets:** Install edge baskets along the perimeter of the no-fines encapsulated rock fill layer. These baskets provide confinement to the whole system, enhancing the load-bearing capacity and overall stability of the rock fill. The basket mesh is recommended to have a minimum steel wire diameter of 3.4mm for a nominal mesh size of 80 mm x 100mm. Wire shall be zinc galvanised to provide long term protection for steel wire against oxidation. The zinc galvanised wire is coated with special PVC (Polyvinylchloride) of 0.4~0.6 mm thick to give full protection against the corrosion from heavily polluted environment.
- **Horizontal reinforcement:** Horizontal reinforcement is to provide additional structural support, confinement and stability to the no-fines encapsulated rock fill system. Place the horizontal reinforcement at a recommended interval of 800mm to 1000mm within the rock fill layer to improve its load-bearing capacity and resistance to deformation. Horizontal reinforcement may be either steel or geogrid and is recommended to have a minimum stiffness (EA) of 600kN/m at a low strain of 0.5%;
- **Geotextile layer:** Introduce geotextile layers between the no-fines encapsulated rock fill system and the subgrade, and between the no-fines encapsulated rock fill system and the ballast. This layer will provide separation between the materials, preventing the intermixing of subgrade soil particles or ballast and rock fill. Additionally, geotextile will help with filtration and drainage, further ensuring the longevity and stability of the track structure.
- **Vertical reinforcement:** Vertical reinforcement may be required to increase the stiffness of the NFEES block. This may be implemented with prefabricated gabion baskets below the rail sleeper horizon;
- In addition, the width of the NFEES may be increased to allow the spreading of the dynamic rail load so as to reduce the rail deflection under transient loading conditions. A minimum width of 7m is recommended,

By integrating geogrid reinforcement along with the careful selection of materials, designing the layer thickness, introducing a geotextile layer, ensuring proper compaction, and implementing a robust drainage system, the no-fines encapsulated rock fill system can effectively replace the sub-ballast layer while meeting the same functions in a rail formation design.

4.2 Validation of rail performance of NFEES

4.2.1 Geometry Model

This study focuses on analysing the load-displacement behaviour of a no-fines encapsulated rail embankment using PLAXIS 2D geotechnical software and evaluating the role of geogrid reinforcement in meeting the specified corridor system design requirements. A typical rail embankment with slopes as 2H : 1V and the top 2m with encapsulated rockfill as shown in Figure 13 was investigated in this study.

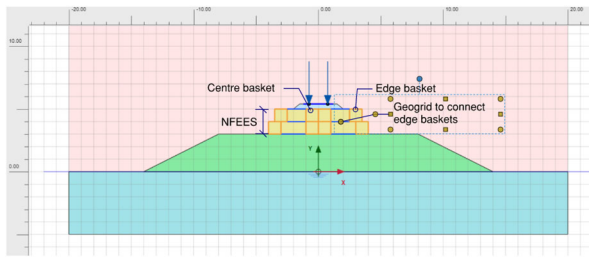


Figure 13. Analytical model

As shown in Figure 13, the edge baskets are assumed to be installed outside of the zone of influence of rail loading, which is defined as a 1V : 1H spread from the edge of the sleeper. The edge baskets are connected with geogrid reinforcement, which are placed horizontally between the rock fill layers, ensuring a uniform distribution of reinforcement throughout the layer.

The axle load configuration was used in the assessment in accordance with Section 9.2 of Australian Standard AS5100.2-2017 assuming a conservative 300LA rail traffic load.

4.2.2 Rail dynamic loading

The dynamic effect of the passing train has been considered based on AS5100.2-2017 via a dynamic load factor (DLF). Specifically the method of applying stringer elements (longitudinal beams that support the bridge deck) to consider the additional stresses caused by dynamic loading has been used in this study. Based on a horizontal reinforcement spacing of <300mm, the DLF has been assessed as 1.67. AS5100.2-2017 further stipulated that for all parts of the structure below ground, the DLF shall be linearly transitioned from the ground level value to zero (0) at a cover depth of 2 m. For structures in embankments, the ground level shall be taken as the underside of the ballast. Based on this the average DLF has been adopted as 1.335 for this study. Based on the above the point load applied on the model shown in Figure 13 is assessed as 137kN/m.

4.2.3 Analysis results:

The results of the analysis are shown in Figure 14. It can be seen that under the current configuration, the transient vertical deflections of the rail under dynamic loading, the assessed deflection of 6.0mm meets the specified deflection requirements of 3.3 mm to 6.4 mm.

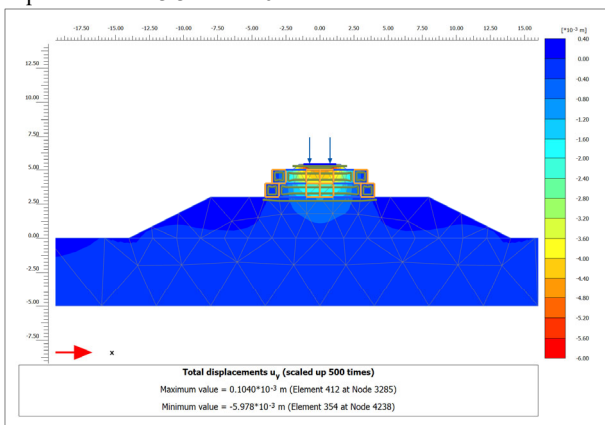


Figure 14. Results of analysis

5 CONCLUSIONS

This paper provides the results of research into the feasibility and possible use of no-fines encapsulated rail embankments for flood resilience. The research is motivated by a search for innovative design solutions that can be easier, quick and cost

effective to implement. The feasibility of NFEES was analysed and demonstrated for a high embankment case where the NFEES allows flow to percolate through a porous top section of the embankment, while accommodating the rail dynamic load during its design life.

Beyond investigating the viability of NFEES, this paper presents a systematic analytical methodology to assess the vulnerability of rail embankments to flooding incidents.

6 ACKNOWLEDGEMENTS

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