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Probabilistic non-linear analysis of bearing capacity for shallow foundations

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ABSTRACT: Accurately determining the bearing capacity of shallow foundations is crucial for designing structures that are both reliable and cost-effective, and that meet specified requirements. Significant uncertainty, variability of soil parameters and loads has led to the increasing use of probabilistic methods for calculating foundations based on reliability criteria. This paper presents a reliability-based method to verifying the limit states of shallow foundations, taking into account soil heterogeneity and nonlinear deformation of soil. The method incorporates safety concepts from the Eurocodes, integrating response surface methodology, random finite element simulation and statistical techniques. The computer program is created according to a developed algorithm for the limit state verification of shallow foundations on the base of proposed reliability-based method. For comparison, the probabilistic and deterministic limit state verification is conducted for spread footings in medium, dense, unsaturated sand; stiff loam; and soft clay. A numerical experiment is performed, followed by a correlation-regression analysis of the data. Probabilistic reliability theory is used to derive approximate second-order equations from the statistical evaluation of experimental data, which are then applied in limit state verification of the spread footing. This probabilistic approach allows for the design of shallow foundations that are economically efficient and compliant with the desired reliability standards.

KEYWORDS: shallow foundations, limits states, reliability, probability, soil heterogeneity, action variability.

1 INTRODUCTION

A key point in foundation design methodology is the reliable and economical determination of bearing capacity in accordance with standard requirements. To ensure the safety of shallow foundations, their dimensions are typically determined using limit state design principles, which involve estimating the allowable ground pressure to prevent soil failure and excessive deformation (Gunatatne, 2006). However, in practice, foundation failure may result from design errors, uncertainties in soil properties, and the inherent variability of those properties. Failures can also stem from discrepancies between theoretical calculation models and the actual interaction between the footing and the soil. Structural damage has occurred in many cases due to excessive settlement or loss of bearing capacity, often caused by soil heterogeneity and the variability of loading conditions. Due to the heterogeneity, the coefficient of variation for soil characteristics reaches 30% (Kayser and Gajan, 2014). This can lead to an overestimation of the allowable bearing capacity of foundations. At the same time, the most commonly used methods of calculating the bearing capacity of shallow foundations sometimes result in unreasonably large reserves, leading to uneconomical construction (Kirichek et al., 2015). The overestimation of the reserve of the bearing capacity, which is the difference between the allowable soil pressure and the ultimate bearing capacity, is the result of the foundations being calculated with unjustified accuracy. To design economical foundations, it is necessary to increase the soil pressure beyond the proportionality limit of the load/strain relationship, and this requires non-linear methods to be used for settlement calculations. The variability in load and heterogeneity of non-linear soil characteristics significantly complicate foundation design. Therefore, this problem has not yet been fully resolved and the existing foundation design methodology needs improvement. Probabilistic methods (Baecher & Christian, 2003) allow us to develop a geotechnical design based on statistical data, determining the values of soil characteristics according to structural reliability differentiation. This paper explores the integration of non-linear settlement calculation methods with probabilistic approaches to shallow foundation design.

2 BACKGROUND

It is well known that the serviceability limit states are usually decisive for shallow foundations. The accuracy of deformation analysis depends on the reliability of mechanical models of ground–structure interaction. To this end, the authors conducted a study in which they compared the results of nonlinear settlement analyses obtained using different analytical and numerical methods with test data on foundations (Kirichek and Tregub, 2024). However, when the load exceeds the proportional limit of soil deformation, it is important to understand how close the design load is to the ultimate bearing capacity (Duncan, 2000). When the reserve bearing capacity is smaller than usual, accurately estimating the ultimate bearing capacity and settlements becomes critical. Therefore, reliability-based verification of the limit states of shallow foundations was performed using analytical and numerical analyses, as well as statistical techniques. These included: The Monte Carlo method (Fenton & Griffiths, 2002 & 2008); the random finite element method (Chemali & Tilouine, 2023); reliability-based design (Kayser & Gajan, 2014); the response surface method (Babu and Srivastava, 2007); and the advanced first-order second-moment method (Pereira and Caldeira, 2011). These methods are employed to enhance the probabilistic limit state verification methodology for shallow foundations by considering soil heterogeneity and deformation nonlinearity, thereby improving the reliability of foundation design.

3 LIMIT STATE CRITERIA

The reliability of shallow foundations is determined according to limit state criteria (EN 1997-1). Failure or excessive deformation can occur when:

- contact pressure on the ground (P) exceeds the design bearing resistance (Rd);
- estimated settlement (S) exceeds the settlement limit value (Su);
- estimated differential settlement within the footings (ΔS/L) exceeds the differential settlement limit (ΔS/Lu).

The failure probability can be expressed using equations (5):

$$P_i = \frac{n}{k-m}, \tag{1}$$

where n is the number of failures in calculations with random variables of soil characteristics x and pressure $\tilde{P}$;

k is the total number of performed calculations;

m is the number of erroneous soil characteristics that do not meet statistical criteria.

According to EN 1990:2002, the First Order Reliability Method measures the failure probability P_f by the reliability index β, which is related to failure probability by:

$$P_f = \Phi(-\beta)\,, \tag{2}$$

where Φ is the cumulative distribution function of the standardized Normal distribution.

Reliability index:

$$\beta = \frac{\mu_R - \mu_E}{\sqrt{\sigma_R^2 + \sigma_E^2}}\,, \tag{3}$$

where μ_R, μ_E, σ_R, σ_E are the respective mean values and standard deviations of the resistance and the result of the action.

Reliability indices in the context of the methodology under consideration can be determined in the following form:

$$\beta_1 = \frac{\bar{R}_d - \bar{P}}{\sqrt{\sigma_{Rd}^2 + \sigma_{\mathrm{p}}^2}}, \quad \beta_2 = \frac{Su - \bar{S}}{\sigma_S},$$

$$\text{or } \beta_2 = \frac{\bar{P}_{Su} - \bar{P}}{\sqrt{\sigma_{P_{Su}}^2 + \sigma_{\mathrm{p}}^2}} \text{ and } \beta_3 = \frac{\left(\frac{\Delta S}{L}\right)_u - \left(\frac{\overline{\Delta S}}{L}\right)}{\sigma_{\Delta S/L}}, \tag{4}$$

where $\bar{P}$, $\bar{R}_d$, $\bar{S}$, $\left(\frac{\overline{\Delta S}}{L}\right)$, $\bar{P}_{Su}$ are the corresponding values of the root mean square of pressure on the soil, the design value of bearing resistance, the estimated settlement, the estimated differential settlement and the contact pressure on the soil according to the serviceability limit state;

σ_p, σ_S, $\sigma_{\Delta S/L}$, σ_{Rd}, $\sigma_{P_{Su}}$ are the respective standard deviation values for the contact soil stress imposed by the structural load, settlement, differential settlement, design bearing resistance and contact pressure according to the serviceability limit state;

S_u, $\left(\frac{\Delta S}{L}\right)_u$ are the corresponding tolerable settlement and differential settlement.

4 THE PROBABILISTIC NON-LINEAR ANALYSIS

To achieve the set goals of increasing the reliability of design based on approaches of reliability theory the probabilistic non-linear analysis method developed for the limit state of shallow foundations involves the following steps.

1. The numerical test data is analysed using correlation and regression methods.
2. Simulation of random values of soil characteristics ($\tilde{\varphi}$, $\tilde{c}$, $\tilde{E}$, $\tilde{\gamma}$) and contact pressure under the footing ($\tilde{P} = \tilde{F}/bl$) in accordance with the accepted coefficients of variation is performed.
3. Random values of bearing resistance (R_d) are simulated using equations EN 1997-1.
4. The simulation of random values of settlements $\tilde{S}$ and random values of differential settlements $\Delta\tilde{S}/L$ is performed using the Monte Carlo method and regression equations.
5. Mean values μ, standard deviations σ and coefficients of variation v are calculated with respect to R_d, S, $\Delta S/L$ and Psu.
6. Estimation of the failure probability (P_i) and the reliability index (β) is performed.
7. Decision-making about the allowable pressure (P) and foundation dimensions (width b and length l) is based on failure probability analysis.

Figure 1 shows the probabilistic non-linear analysis algorithm created for the limit state of shallow foundations.

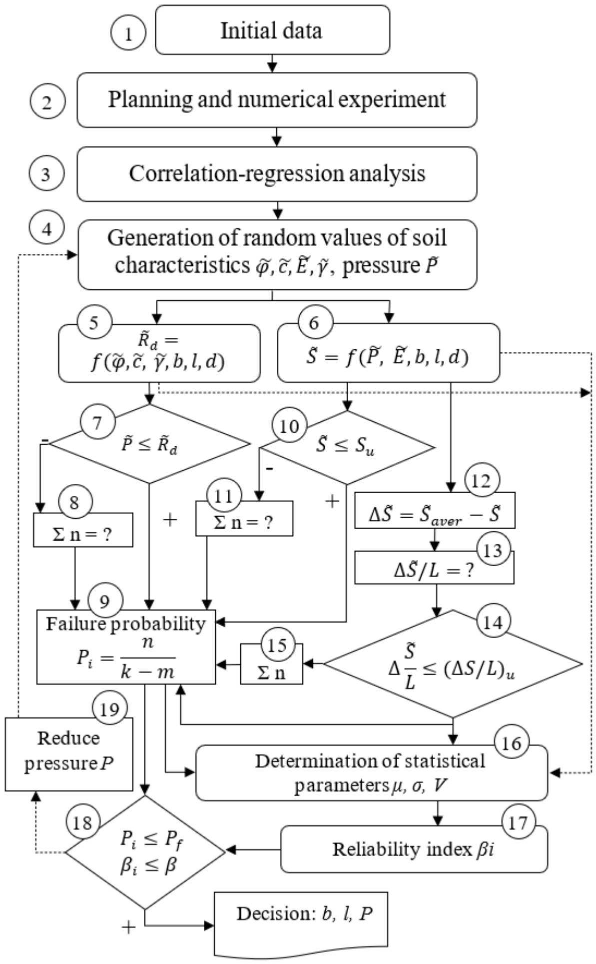

Figure 1. The probabilistic non-linear analysis algorithm for the limit state of shallow foundations.

Input data for the probabilistic non-linear analysis algorithm are:

- width, length and depth of the footing (b, l, d);
- distance between footings (L);
- vertical static load (F) and coefficient of variation;
- representative values of the soil characteristics (φ, c, E, γ) and the coefficients of variation;
- tolerable settlement (Su);
- tolerable differential settlement within the footing $(\Delta S/L)_u$;
- recommended values of the reliability index (β);
- probability of failure (P_f) depending on the reliability differentiation according to EN 1990:2002.

The probabilistic analysis is carried out iteratively until the limit state condition is reached. If the recommended values for the reliability index and failure probability are exceeded, the contact pressure on the soil is reduced and another calculation is performed.

A correlation-regression analysis is performed on the resulting data, and a numerical experiment is conducted. Second-order approximation equations are obtained based on the statistical evaluation of the experimental data within the framework of probabilistic reliability theory, and these are used for limit state verification.

This probabilistic method allows for the design of economically efficient shallow foundations with the recommended reliability values.

5 NUMERICAL EXPERIMENT

For example, the proposed method is used to calculate the settlement of spread footings with dimensions of bxl = 3x3m and a depth of d = 3 m. The footings are designed for framed constructions. They are constructed on single-layer soils consisting of medium, dense, unsaturated sand; stiff loam; and soft clay. The soil characteristics are given in Table 1.

Table 1. Soil characteristics.

Soil	E_k, MPa	φ_k, degree	c_k, kPa	γ_k, kN/m^3
Medium, dense, unsaturated sand	29.8	33.7	-	16.45
Stiff loam	13.64	20.9	22.4	17.38
Soft clay	8.81	-	32.4	17.73

The design bearing resistances R_d are determined in accordance with the requirements of EN 1997-1. Settlements are modeled using a finite element analysis in PLAXIS 3D with the Hardening Soil Model. Figure 2 shows the results of the numerical modeling of foundation settlements. This model has previously demonstrated satisfactory agreement between estimated settlement values and foundation test data. A tolerable settlement of 5 cm is assumed, with a differential settlement of $(\Delta S/L)_u=0.002$.

The results of the deterministic methods are shown in Table 2. The conditions for the ultimate state are met with the allowable load P not exceeding the design bearing resistance R_d, and the maximum settlement value S not exceeding the tolerable value S_u.

Table 2. Results of deterministic methods.

Soil	P, MPa	S, cm	R_d, MPa
Medium, dense, unsaturated sand	0.44	5.0	1,006
Stiff loam	0.27	4.9	0,493
Soft clay	0.17	5.0	0,276

A computer program based on the above proposed algorithm is developed to automate probabilistic analyses of the limit states of spread footings.

Lab tests of soil samples are the basis for the statistical data relating to soil characteristic heterogeneity and distribution functions. The study modeled a random sample of $k=10^4$ soil characteristics and contact pressure values P using a normal distribution. The following values of the coefficient of variation (v_x) are assumed when modeling the random variables of:

- friction angle, deformation modulus for sand: $v(\varphi,E)=0.05$;
- friction angle, cohesion, deformation modulus for loam and clay: $v(\varphi, c, E) = 0.1$;
- specific gravity for soil: $v(\gamma) = 0.02$;
- contact pressure under the footing: $v(P) = 0.1$.

The random variables of the soil characteristic values ($\tilde{\varphi}$, $\tilde{c}$, $\tilde{\gamma}$ and $\tilde{E}$) and the contact pressure values $\tilde{P}$ are modeled according to the Normal distribution. The random variables of foundation design resistance $\tilde{R}_d$ and settlement $\tilde{S}$ are then determined after which the limit state conditions are verified.

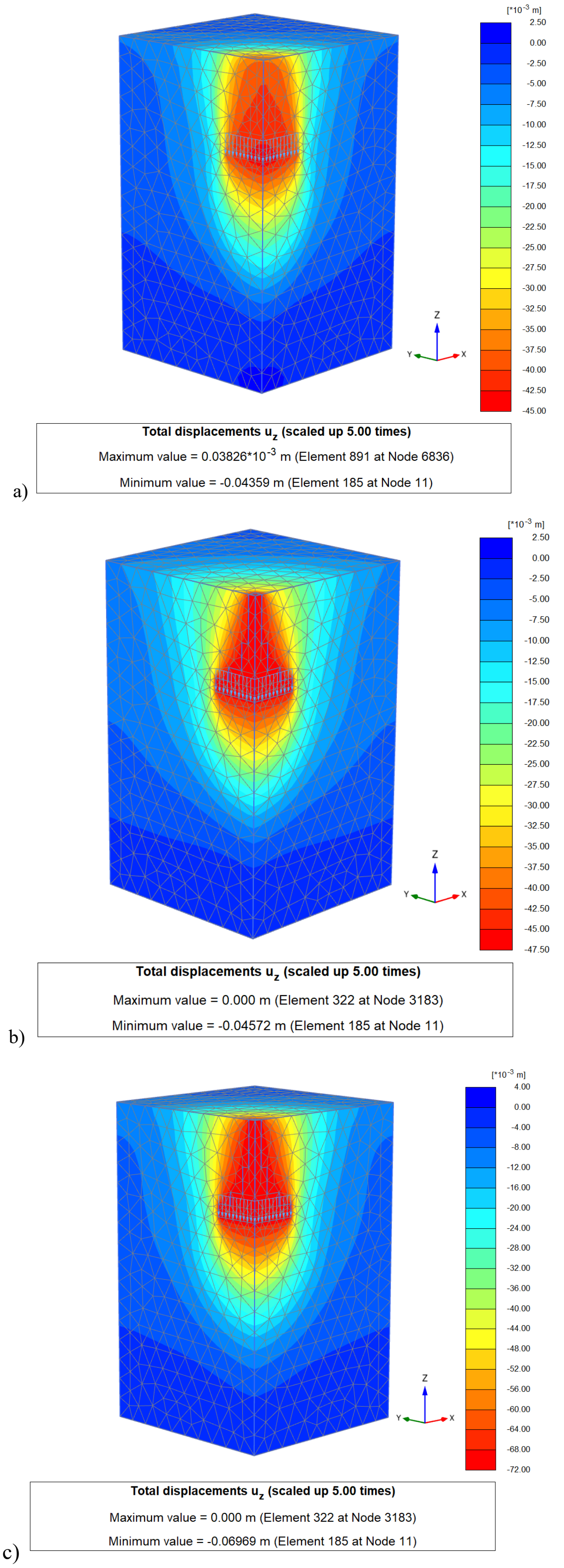

Figure 2. Numerical modeling of foundation settlement in PLAXIS 3D: a - in medium, dense sand (contact pressure under the footing: P=0.41 MP; b - in stiff loam (contact pressure under the footing: P=0.25 MP; c - in soft clay (contact pressure under the footing: P=0.20 MPa).

The method of estimating the load capacity of shallow foundations takes a probabilistic approach, involving response surface methodology and the creation of an approximating equation. To this end, a numerical experiment is designed and carried out.

The influence of random variables is considered at three levels: 1) $E = \bar{E} - 3\sigma_E$, $P = \bar{P} - 3\sigma_p$; 2) $E = \bar{E}$, $P = \bar{P}$; 3) $E = \bar{E} + 3\sigma_E$, $P = \bar{P} + 3\sigma_p$ (where $\bar{E}$ and $\bar{P}$ are the root mean square of the deformation modulus $\tilde{E}$ and contact pressure under the footing $\tilde{P}$). The graphs in Figure 3 are based on the results of a numerical simulation performed using the finite element method. These show the range of probable settlements $\tilde{S}$ based on calculations for settlements ($S_1…S_9$) according to matrix 3^2 of the experimental plan. These calculations take into account the non-linear properties of the soil and the variability of the pressure $\tilde{P}$.

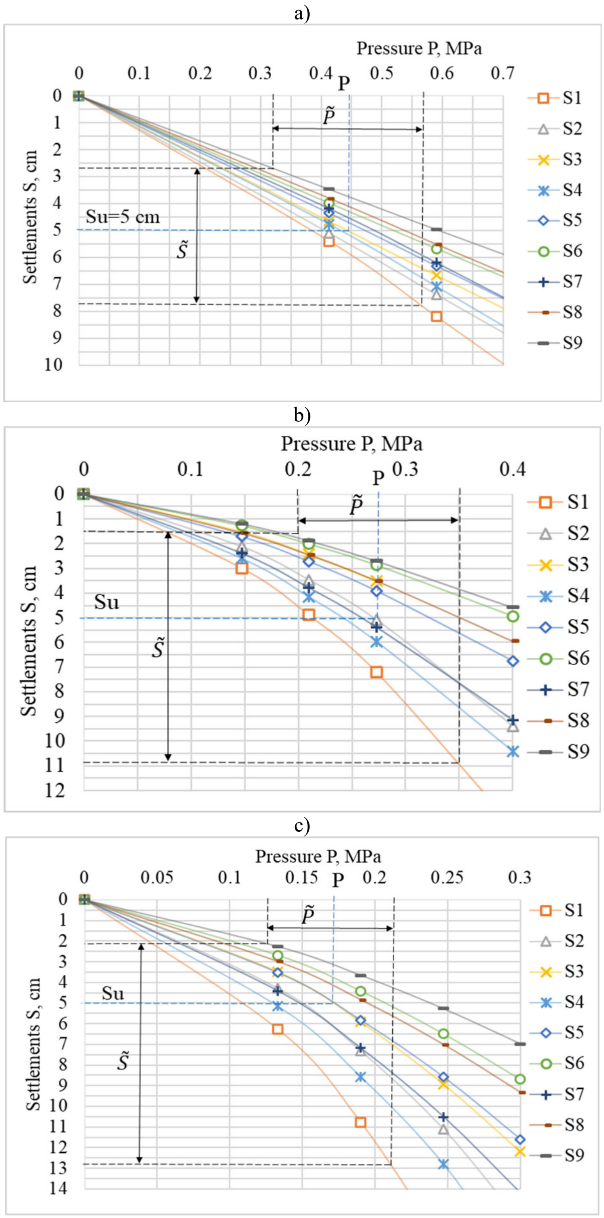

Figure 3. Probabilistic settlement analysis of the spread footings in dense sand (a), stiff loam (b), soft clay (d).

Correlation analysis of the data confirmed the most influential factors to be included in the regression models. Second-order approximation equations for calculating foundation settlements in sand are obtained based on the regression analysis results of numerical modeling data (5):

$$S = -1.770 + 32.822 \cdot P - 0.108 \cdot E - \\ -1.646 \cdot P^2 - 0.571 \cdot P \cdot E + 0.004 \cdot E^2 . \qquad (5)$$

where P is the contact pressure under the footing; E is deformation module.

The regression equation for the settlement of the spread footing in loam is:

$$S = 4.715 + 21.724 \cdot P - 0.557E + \\ +17.737 \cdot P^2 - 1.143 \cdot P \cdot E + 0.019 \cdot E^2 . \qquad (6)$$

The regression equation for the settlement of the spread footing in clay is:

$$S = 7.315 + 77.103 \cdot P - 1.752 \cdot E - \\ -41.356 \cdot P^2 - 3.978 \cdot P \cdot E + 0.098 \cdot E^2 . \qquad (7)$$

In formulas (5) to (7), settlements (S) and pressures (P) are correlated with the value of S_{01} set to 1 cm and P_{01}=1MPa. The range of the coefficients of determination is from 0.95 to 0.99, indicating a satisfactory level of agreement between the numerical and modeled values.

The pressure equation corresponding to the serviceability limit state, *P(Su)=f(E, Su)* can be obtained from Equations (5), (6) and (7), by assuming *S=Su*. This equation is used to calculate the probability that the applied pressure (P) exceeds the value $P(S_u)$, at which the settlement reaches its limiting value. The probabilistic analysis of non-linear settlements (P_S) and differential settlements ($P_{\Delta S/L}$) is performed using the statistical testing method and the approximating equations. Design bearing resistances ($\tilde{R}_d$) are calculated according to EN 1997-1, using simulated random values for soil characteristics. Figures 4, 5 and 6 show the probability of the contact pressure on the soil under the footings (P) reaching critical values according to the ultimate limit state criterion (P_f) and serviceability limit state criteria (P_S) and ($P_{\Delta s/L}$).

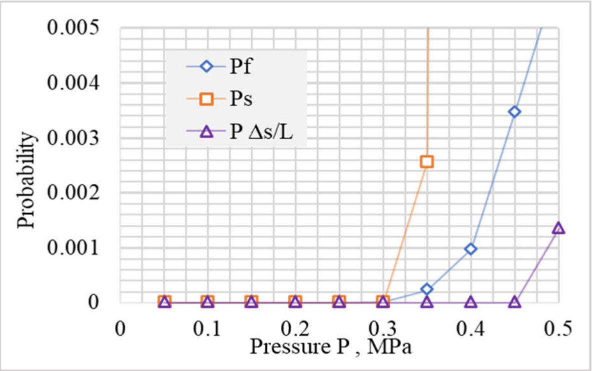

Figure 4. Probabilistic limit state analysis of the spread footing in dense sand.

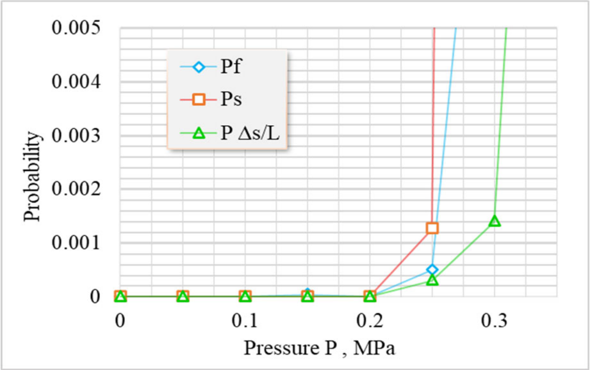

Figure 5. Probabilistic limit state analysis of the spread footing in stiff loam.

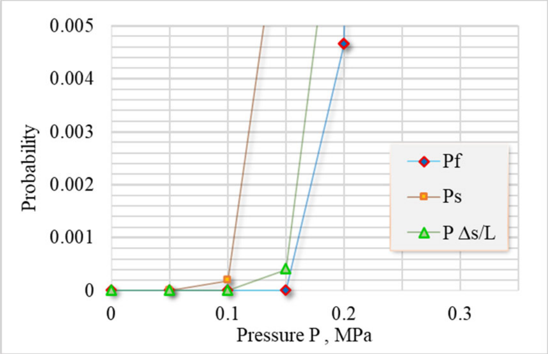

Figure 6. Probabilistic limit state analysis of the spread footing in soft clay.

Meanwhile, μ, σ and v are obtained in relation to R_d, S, $\Delta S/L$ and $P(Su)$. These values are then used to calculate the reliability index β. Figures 7, 8 and 9 show the relationship between the reliability index and the estimated pressure value according to the limit states. Here, β_1 shows dependence on the contact pressure according to the ultimate limit state, β_2 and β_3 show dependence on the contact pressure according to the serviceability limit state, respectively on settlements P_S and differential settlements $P_{\Delta s/L}$.

The allowable load is determined according to Consequence class using failure probability charts and reliability indices, taking the requirements of EN 1997-1into account.

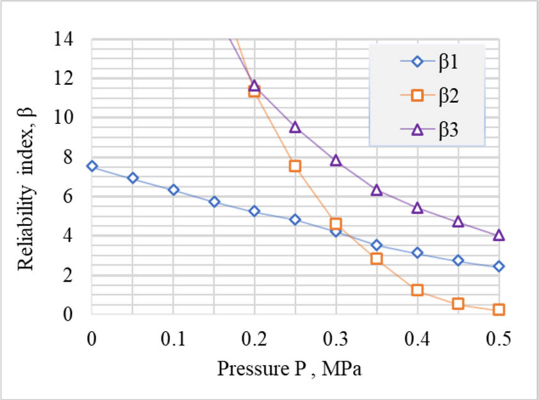

Figure 7. The reliability index dependence on the contact pressure of the footing on dense sand.

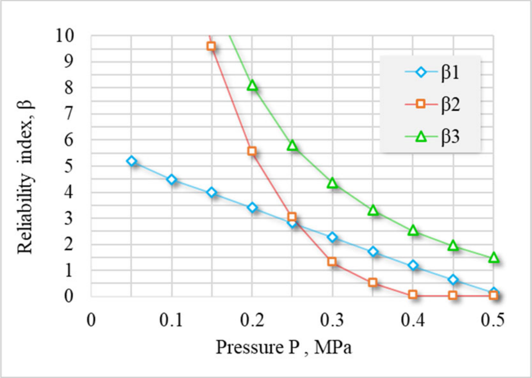

Figure 8. The reliability index dependence on the contact pressure of the footing on stiff loam.

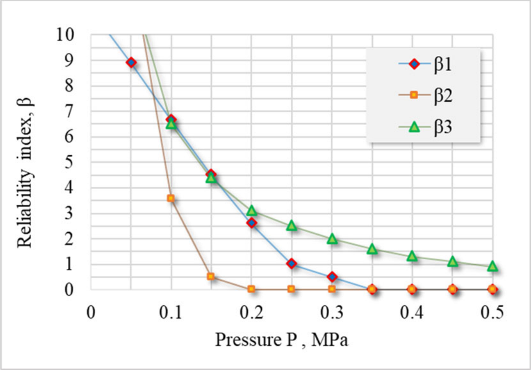

Figure 9. The reliability index dependence on the contact pressure of the footing on soft clay.

This analysis shows that the probability of failure of the considered foundations increases rapidly once the pressure reaches 0.2–0.3 MPa.

The results of the statistical analysis showed that the densities of the random variables R_d and $P(Su)$ could be approximated by functions closely resembling the Normal distribution. As the load increases, the probability of failure also increases, as shown in Figure 10 for stiff loam, where the distribution curve of the random variables of contact pressure $\tilde{P}$ approaches the curves representing the random variables of foundation bearing resistance $\tilde{R}_d$ and the contact pressure for limiting settlement $\tilde{P}_{Su}$,.

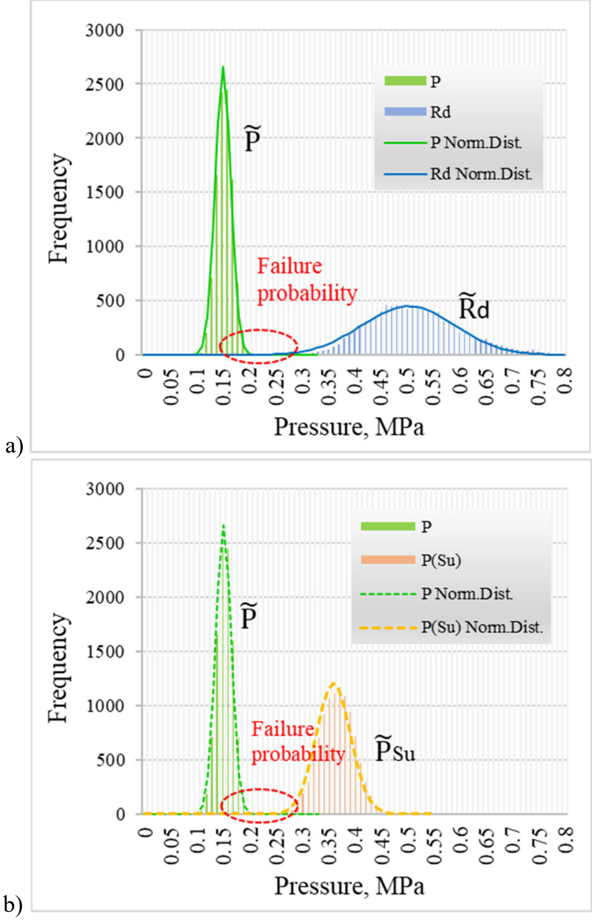

Figure 10. Distribution of random variables of contact pressure under the footing$\tilde{P}$, bearing resistance $\tilde{R}_d$ (a) and the contact pressure according to the serviceability limit state $\tilde{P}_{Su}$ (b).

The distribution of settlement values exhibits an asymmetry of up to 0.61, which is primarily associated with a disproportionate increase in non-linear settlements as loads increase (see Figure 11).

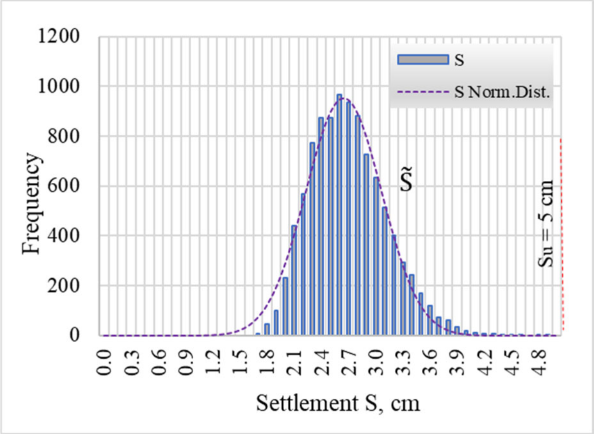

Figure 11. Distribution of settlement of the footing in stiff loam.

The probabilistic analysis revealed that the root mean square differential settlement (ΔS/L) did not exceed the limit values, as illustrated in Figure 12.

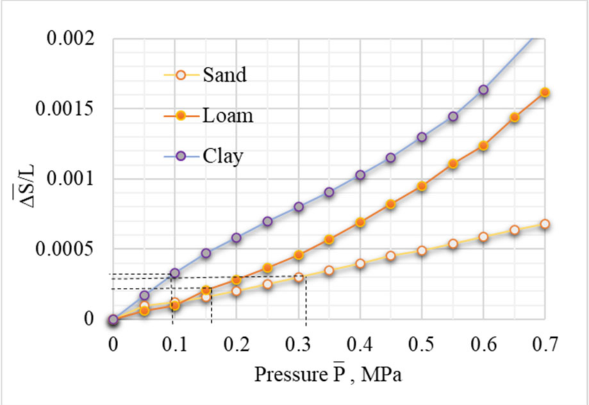

Figure 12. Relationship between root mean square differential settlements $\Delta\bar{S}/L$ and contact pressure$\bar{P}$.

The allowable pressure is checked in terms of the ultimate and serviceability limit states using the First Order Reliability Method which assumes a normal distribution (5):

$$P = \bar{R}_d - \beta \cdot \sqrt{\sigma_{Rd}^2 + \sigma_{\mathrm{p}}^2}\,,$$
$$P = \bar{P}_{Su} - \beta \cdot \sqrt{\sigma_{Psu}^2 + \sigma_{\mathrm{p}}^2}\,. \qquad (8)$$

Table 3 shows the allowable pressures for CC2 structures over a 50-year period. Failure probability values and reliability indices can be used to calibrate partial factors when comparing the reliability levels of structures.

Table 3. Results of probabilistic analysis of the strip footings.

Soil	P, MPa	Coefficients of variation			$\underline{P_f}$ Ps	$\underline{\beta_1}$ β_2
		v(R$_d$)	v(S)	v (ΔS/L)		
Medium, dense, unsaturated sand	0.31	0.19	0.17	0.74	$\underline{0}$ $2\cdot10^{-4}$	$\underline{4.0}$ 4.2
Stiff loam	0.16	0.18	0.15	0.75	$\underline{0}$ $1\cdot10^{-4}$	$\underline{3.8}$ 9.5
Soft clay	0.09	0.09	0.16	0.79	$\underline{0}$ $1\cdot10^{-4}$	$\underline{7.1}$ 4.6

6 CONCLUSIONS

This paper presents a method for estimating the bearing capacity of shallow foundations – an essential criterion in foundation design. A statistical approach is employed to assess the heterogeneity of soil characteristics, thereby enhancing the reliability of the foundation design. A justified increase in soil pressure is achieved by applying validated methods for calculating non-linear foundation deformations, supported by experimental data. A probabilistic, non-linear limit state analysis method has been developed to evaluate both the allowable soil pressure and the design reliability of shallow foundations. This comprehensive approach integrates response surface modeling, finite element approximation, Monte Carlo simulation, and first-order reliability analysis. By combining finite element modeling with a probabilistic framework, the method accounts soil deformation heterogeneity, ground nonlinearity, and variability in applied loads. Implemented through custom-developed software, the proposed algorithm determines bearing capacity of shallow foundations using probabilistic limit state analyses in accordance with Eurocodes requirements. This probabilistic approach enables the design of cost-effective shallow foundations with a defined level of reliability, tailored to the significance and requirements of the structure.

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