

# Multidirectional and irregular high-cyclic loading using the high-cycle accumulation model

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**ABSTRACT:** The high-cycle accumulation (HCA) model typically requires grouping load cycles and applying Miner's rule to estimate cumulative deformations. However, Miner's rule is inadequate for partially drained or undrained conditions and allows predicting the final response to an irregular (changing with time or cycles) cyclic action only. This paper presents an approach that allows for the simulation of irregular and multidirectional cyclic loading through the use of individual, cycle-specific strain amplitudes. This method allows to consider more complex sequences of cyclic loading, varying in direction, frequency and amplitude, to be represented without grouping cycles. The implementation preserves partly the efficiency of the conventional HCA formulation by requiring only one increment per cycle. The extension is applied to simulate model tests on monopile foundations subjected to multi-directional and irregular cyclic loading.

**KEYWORDS:** high-cyclic loading, Miner's rule, HCA model, offshore

## 1 INTRODUCTION

The high-cycle accumulation (HCA) model introduced by Niemunis et al. (2005) is a constitutive model designed to simulate the accumulation of permanent strains in granular soils under cyclic loading. It computes the rate of accumulated strain as a product of several functions, one of which is dependent on a scalar strain amplitude  $\varepsilon^{ampl}$ . This scalar is derived from the strain loop and can reflect its intensity and multidimensionality.

Since the strain amplitude is required as input for the HCA model, it is required to define (a) representative cycle(s). In practical applications such as monopile foundations for offshore wind turbines, the loading direction, frequency and magnitude may vary continuously. To simulate such conditions with classical high-cycle models, it is common to group load cycles of similar amplitude and apply the so-called Miner rule as shown in Fig. 1. Originally developed in the context of fatigue analysis, Miner's rule assumes that the total damage from cyclic loading is a linear superposition of the damage caused by each load amplitude group. As shown in Fig. 1, the packages with identical amplitude  $\varepsilon_{Ni}^{ampl}$ , where  $i$  is the number of the package, are treated sequentially, without considering their original order. The experimental investigation of the validity of the Miner's rule in geotechnics is an ongoing research topic (Stark et al. 2022; Hagemann & Grabe, 2024).

In the context of partially drained or undrained soil conditions, the Miner rule is not generally valid (Tafili et al. 2023; Liu et al., 2022; Staubach & Wichtmann, 2020). Grouping cycles into packages and assuming equivalent accumulation does not adequately capture the coupled hydro-mechanical behaviour and may lead to erroneous predictions. In general, using a descending order of amplitudes results in the largest final deformations. In addition, the application of Miner's rule only allows for the prediction of the final response after an equivalent number of cycles has been considered. It does not permit the estimation of the response during the loading process.

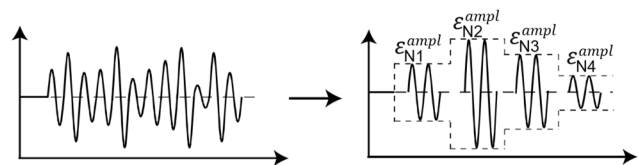


Figure 1. Schematic depiction of grouping an irregular cyclic load history in packages with similar strain amplitude  $\varepsilon_{Ni}^{ampl}$ . The index of the strain amplitude corresponds to the package number.

## 2 MODELLING IRREGULAR LOADING WITH THE HCA MODEL

To avoid the necessity to group irregular cyclic loading in packages with similar characteristics, the HCA model is supplemented in the present work to directly incorporate multiple, cycle-specific strain amplitudes. This allows each cycle to be treated individually, with its own unique calculated  $\varepsilon_{Ni}^{ampl}$  from the strain path recorded during that cycle.

The underlying concept of this approach is illustrated in Figure 2. Unlike the conventional strategy (as shown in Figure 1) where cycles with similar amplitudes are grouped together, the enhanced model retains the actual sequence and amplitude of each cycle. Instead of grouping cycles with similar amplitude, the strain amplitude is evaluated individually for each cycle, allowing the irregular loading history to be fully preserved.

This method introduces two principal differences compared to conventional HCA simulations:

1. Cycle-specific preprocessing of strain amplitudes: Prior to applying the HCA model, strain amplitudes for all cycles must be computed. Although this step could seem computationally demanding when analysing hundreds or thousands of cycles with different characteristics, practical examples later in the paper demonstrate that computing individual amplitudes for hundreds of cycles is feasible.

2. Reduced time increment: The time increment during the HCA simulation phase must be limited to the period of a single load cycle (i.e.,  $\Delta N = 1$ ) to ensure sufficient resolution of the evolving strain amplitudes. This means that, at most, one cycle is simulated per increment. While this somewhat limits the advantage of the HCA model, namely, the ability to simulate millions of cycles in large increments, it remains vastly more efficient than simulating every cycle using a conventional constitutive model that resolves the entire stress-strain path per cycle.

As also illustrated in Figure 2, the method allows the reuse of strain amplitudes if cycles share identical characteristics. In the numerical implementation, a list of precomputed strain amplitudes is stored, and at each increment, the corresponding amplitude is assigned to the current cycle index. The strain amplitudes are derived from an auxiliary simulation using a conventional constitutive model, as is discussed in Section 3.

This approach eliminates the need for cycle grouping or application of Miner's rule. Load histories with continuously varying amplitude and direction can be simulated directly, thereby representing the actual sequence of events. This significantly improves model fidelity, especially in cases involving undrained or partially drained conditions, where the cyclic response is strongly dependent on the loading history.

However, the approach comes at the cost of neglecting the influence of cyclic history on the strain amplitude. For instance, a cyclic loading sequence involving several thousand cycles may affect the strain amplitude of a subsequent cycle with different characteristics. Such effects cannot be considered with the present approach. In the conventional HCA model, these influences are typically addressed through the concept of update cycles. However, standard constitutive models such as the hypoplastic model with intergranular strain extension used in conjunction with the HCA model generally cannot account for the influence of the cyclic history on the strain amplitude either. They can only capture changes in density and stress state resulting from the previous cyclic loading considered with the HCA model.

The practical applicability and relevance of the approach is demonstrated in the next section, where it is validated against physical model tests involving monopile foundations subjected to complex, multidirectional cyclic loading.

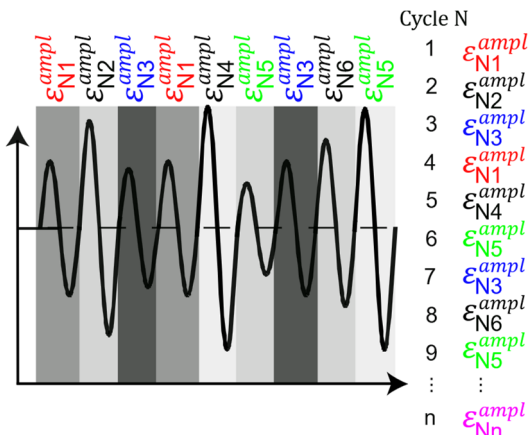


Figure 2. Keeping the irregular time history of the cyclic loading by using multiple strain amplitudes. A stack of strain amplitudes is pre-computed and then used during the HCA calculation. The index of the strain amplitude corresponds to the cycle number.

### 3 EXAMPLE AND VALIDATION

To validate HCA model incorporating multiple strain amplitudes, numerical simulations were performed for the physical model tests reported by Richards et al. (2020). In these tests, piles embedded in dry sand were subjected to multidirectional lateral cyclic loading with variable direction and amplitude. These experiments serve as a benchmark to evaluate the ability of the enhanced HCA model to capture strain accumulation under irregular cyclic loading conditions. Figure 3 shows the numerical model used in this work, the dimensions, loading and a photo of the model tests.

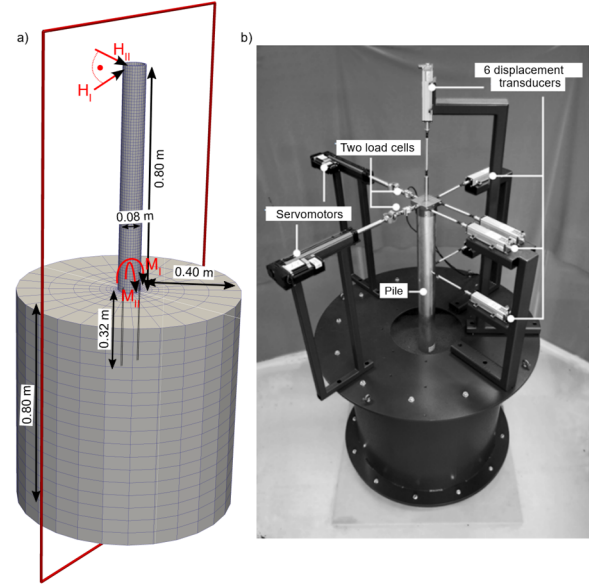


Figure 3. Dimensions and mesh of the finite-element model (a) of the small-scale model tests depicted in (b), reprinted from Richards et al. (2020)

The model tests were conducted at Oxford University and build upon earlier work by Leblanc et al. (2010). The test setup consists of a cylindrical container with 800 mm diameter and height, filled with dry Yellow Leighton Buzzard sand at an initial relative density of approximately  $D_{r0} = 60\%$ . A scaled monopile made of aluminum was used, with a length  $L = 320$  mm, an outer diameter  $D = 80$  mm, and a wall thickness  $t = 5$  mm. Aluminum was selected to approximate the reduced stiffness in the 1g model scale.

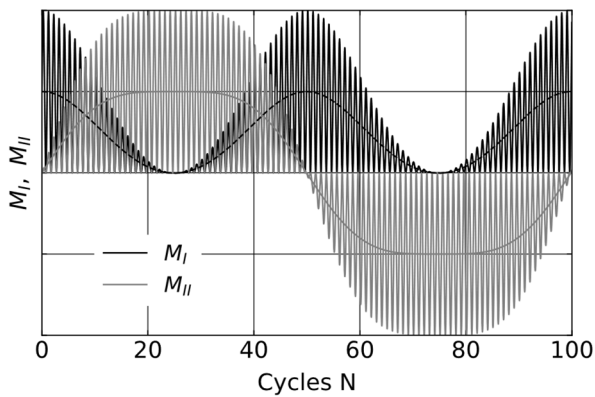
The pile was installed using a drop-weight method to simulate impact driving, resulting in densification around the pile tip. Following installation, lateral cyclic loading was applied 800 mm above the soil surface by two orthogonal servomotor-controlled actuators. These generated horizontal forces  $H_I$  and  $H_{II}$ , inducing two orthogonal moment components  $M_I$  and  $M_{II}$  at the ground surface. Figure 3 shows the load components.

A so-called fan-type loading scheme was applied, in which the direction and magnitude of the lateral loading changes continuously over time. This was achieved by introducing a phase shift between the two lateral force components  $H_I$  and  $H_{II}$ . As a result, the resulting loading direction rotates during the test. Each fan-type loading block, or sweep, consisted of 100 individual cycles differing in amplitude. During the sweep, both the average value and the amplitude of the moment components varied as a function of the cycle number. The complete time history of one sweep is illustrated in Figure 4. Part b) of the figure presents a magnified view of cycles 18 to 36. Two cycle-specific strain amplitudes are marked as representative examples.

This form of loading produces an irregular and multidirectional cyclic history that cannot be easily captured using classical high-cycle accumulation models based on cycle grouping. It therefore provides an ideal basis to evaluate the performance and advantages of the extended HCA model which uses individually assigned strain amplitudes for each cycle.

The numerical analysis is divided into three phases. In the first phase, the pile installation was simulated using the Coupled Eulerian-Lagrangian (CEL) method. The soil was modeled using a hypoplastic constitutive model (von Wolffersdorff, 1996) with intergranular strain extension (Niemunis & Herle, 1997) to capture the substantial densification and stress changes around the pile tip during penetration. The results of these simulations are presented in detail in Staubach et al. (2025).

a)



b)

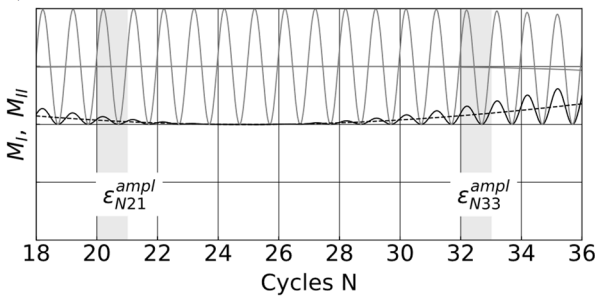


Figure 4. Fan-type loading applied in the physical model tests (Richards et al., 2020): a) Evolution of the two moment components  $M_I$  and  $M_{II}$  during one sweep of 100 cycles, showing both amplitude and average value variation; b) Detail view of a subset of cycles (cycles 18 to 36) with indicated cycle-specific amplitudes  $\epsilon_{N21}^{ampl}$  and  $\epsilon_{N33}^{ampl}$ .

In the second phase, 100 different strain amplitudes were calculated using the hypoplastic model with intergranular strain extension. This was implemented in the finite element software numgeo (Machaček and Staubach; see www.numgeo.de, Machaček, 2020; Staubach, 2022). The pile installation-induced changes were considered in these analyses. The strain amplitudes were derived from a strain path recorded in an auxiliary simulation that employed the same loading history as the physical test. Each strain amplitude corresponds to one of the 100 individual cycles within a fan-type loading sweep. Since the hypoplastic model with intergranular strain extension is not suited for simulating strain accumulation over a large number of cycles, the high-cycle accumulation (HCA) simulation was not performed as a continuation of this initial analysis. Instead, a second, separate model was used. The initial state of this second model corresponds to the state immediately following pile installation and does not include the updated

stress or void ratio resulting from the 100 cycles simulated with the hypoplastic model with intergranular strain extension. The precomputed 100 strain amplitudes from the first simulation were used as input to the HCA model. In conventional HCA simulations using a single constant amplitude, this simplification would typically be acceptable, as the accumulation of strain from only one cycle is very small and the hypoplastic model with intergranular strain extension is suitable to capture the plastic deformations. However, when 100 cycles are involved, as is the case here, this would lead to an approximately linear accumulation of deformations with number of loading cycles. For that reason, the decoupled approach adopted in this study ensures that the full set of variable amplitudes can be considered without distortion of the cyclic accumulation behavior.

In the third phase, lateral cyclic loading was simulated using the HCA model. The simulation covered 10 loading sweeps, corresponding to a total of 1,000 individual load cycles. In the enhanced HCA formulation, each of the 100 unique cycles is assigned a specific precomputed strain amplitude.

The results of the numerical analysis using the extended HCA model are shown in Figure 5. The figure presents the evolution of the normalized pile head rotation  $\frac{\Delta\theta_M}{\theta_R}$  over 1,000 loading cycles, where  $\theta_R$  corresponds to the reference rotation of  $2^\circ$  at ultimate lateral resistance. The grey line represents the experimental data obtained from the physical model tests by Richards et al. (2020), while the black line shows the outcome of the HCA simulation incorporating multiple strain amplitudes.

The overall agreement between simulation and experiment is good. Both the magnitude and the trend of the accumulated rotation are captured. Notably, the characteristic rippling pattern in the experimental curve, which arises due to the continuous change in loading direction during fan-type loading, is reproduced in the simulation. This confirms that the use of individually assigned strain amplitudes allows the model to accurately reflect the irregular loading history.

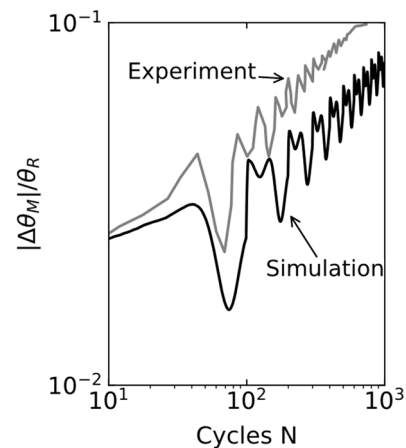


Figure 5. Normalized change in pile head rotation relative to the reference rotation at ultimate capacity  $\frac{\Delta\theta_M}{\theta_R}$  as a function of the number of load cycles  $N$ . Comparison between experimental data of Richards et al. (2020) and numerical simulation using the HCA model with cycle-specific strain amplitudes.

## 4 CONCLUSIONS

This paper presented an extension of the high-cycle accumulation (HCA) model that incorporates individually assigned, cycle-specific strain amplitudes. The main objective was to enable the simulation of complex, multidirectional cyclic loading histories without relying on cycle grouping and the application of Miner's rule. The proposed method allows arbitrary variation in load direction, frequency and amplitude to be represented in a numerically efficient manner, using a single calculation increment per cycle.

The validation against model tests involving monopile foundations under fan-type lateral loading has shown that the formulation can reproduce the observed accumulation of rotation with satisfactory accuracy. In particular, the irregular pattern and cumulative trends of the measured response were captured well.

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