

# A Smart Constitutive Model and Its Numerical Application Predicting the Cyclic Liquefaction of Sandy Soils through Deep Learning

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**ABSTRACT:** A novel deep learning based constitutive model for liquefiable sandy soils subjected to cyclic excitation is developed using an extensive database of strain-controlled cyclic triaxial tests. This study proposes a model for predicting the cyclic response of in-situ liquefiable sandy soil layers without resorting to complex, plasticity-based theoretical models, which have so far been the current practice. The model predicts the excess pore pressure and deviatoric stress time histories using a few input features consisting of vectorial variables such as the strain-time response variation gathered from a large number of cyclic triaxial experiments. An original numerical tool based on machine learning (ML) has been developed to predict not a single parameter indicating liquefaction, but the entire time-history variations leading to cyclic liquefaction. While classical ML algorithms help predict the number of liquefaction cycles in available models, achieving the desired level of accuracy in predicting the trend and robustness in pore pressure buildup and deviatoric stress requires deep learning. Results indicate that the Long Short-Term Memory (LSTM) model, when combined with Stacked LSTM and the Windowing data processing method, is necessary for making fairly good predictions. Such a ‘smart constitutive model’ (SCM) predicts deviatoric stress reduction and pore-pressure buildup across a wide range of relative densities. A novel finite element program with a finite difference method (FDM) for temporal integration is then developed, and the SCM is incorporated into this poro-inelastic code through an original MATLAB-Python interface. This is followed by site response analyses of an in-situ liquefiable soil under cyclic loading, where the integrated hybrid FEM-FDM-SCM numerical model is employed. Results indicate that the SCM can be successfully used to estimate site liquefaction response under cyclic loading. Research continues to assess the seismic-induced liquefaction of sands.

**KEYWORDS:** Cyclic triaxial test, data-driven study, finite elements, liquefaction, machine learning, site response, smart constitutive model

## 1 INTRODUCTION

Soil liquefaction induced by seismic activity is among the most destructive consequences of earthquakes. Liquefaction is possible in areas underlain by loose, saturated, sandy deposits, with the potential to cause total collapse of buildings. Liquefaction mainly causes a gradual increase in pore pressure until it equals the effective vertical stress. At this point, the soil loses its shear strength and behaves like a fluid, leading to irreversible deformation. This process can result in settlement, tilting, or collapse of upper structures. Therefore, predicting the liquefaction under cyclic loading is a critical topic in geotechnical earthquake engineering. Cyclic triaxial and cyclic simple shear tests are especially popular laboratory methods used to simulate earthquake loading and study dynamic soil behavior. These tests are conducted under either stress-controlled or strain-controlled loading conditions, allowing monitoring of the stress–strain response and excess pore pressure buildup. Constitutive models for soil liquefaction are developed based on these test results to understand soil cyclic behavior. These constitutive models enable more accurate predictions of potential failures in geotechnical systems. However, many models either fail to capture observed behaviors fully or require too many parameters, making them too complex and impractical.

In practice, engineers often prefer fast, simple, yet accurate solutions to complex problems, which can result in oversimplifications or incorrect approaches. Such simplifications may force engineers to use conservative safety factors, thereby increasing the risk of soil failure. The accuracy of FEM analyses largely depends on the reliability of the constitutive model employed. Therefore, in the era of artificial intelligence, there is a growing need to develop, what we can call “smart constitutive models (SCM)” that can learn directly

from laboratory data without relying on theoretical assumptions. A limited number of data-driven studies focus on predicting the cyclic behavior of soils. While some rely on artificially generated data, others utilize real laboratory test results. For instance, Zhang et al. (2020), Guan and Yang (2022), and Drakos (2008) employed artificial data, whereas Chen et al. (2021), Choi and Kumar (2022), and Meng and Pei (2023) based their models on experimental data.

Artificial Neural Network (ANN)-based models are highly advanced and successful in addressing path-dependent material behaviors such as plasticity, viscoelasticity, and failure. In traditional Finite Element Method (FEM), these behaviors are modeled using complex constitutive equations that require iterative solution methods. However, ANNs can learn these behaviors directly from data.

In recent years, several numerical tools and constitutive models have been used to model liquefaction; however, evaluating the required model parameters has become time-consuming. The implementation of ANN models within finite elements is required to adapt ML algorithms to conventional numerical methods. ANN-based models offer a more flexible, data-driven approach, while conventional material models tend to have predefined constitutive relations. This flexibility allows the learning process to simulate complex stress-strain relationships directly by eliminating the need for explicit mathematical functions (Wu & Ghaboussi, 1995). The implementation of the trained ANN into the FE environment enables the use of a smart constitutive model (SCM) instead of a conventional constitutive model, with SCM replacing the material subroutine.

Ghaboussi (2018) proposed two different methods for obtaining the tangent stiffness matrix: probing the NN structural model and directly calculating it from the neural

network's connection weights. In the first method, for a plane stress or plane strain problem, three forward passes (or probes) through the NN model are required to assemble the columns of the corresponding tangential stress-strain matrix, regardless of the number of input data points. The other method is an explicit constitutive formulation using weight functions, where functions are defined for each layer in the NN architecture. A more recent study by Liang et al. (2023) used deep learning applications such as PyTorch to integrate ANN models into FEM. The PyTorch deep learning framework demonstrates the advantages of an ML-based approach in large-scale simulations, enabling simultaneous updates to material models based on deformation. Lastly, Birinci et al. (2025) developed a pioneering SCM for soil liquefaction via LSTM combined with the windowing method, using a wide range of actual cyclic test data. Following that work, this study directly integrates the SCM into the FE framework via a Python interface.

## 2 DEVELOPMENT OF THE SMART CONSTITUTIVE MODEL FOR LIQUEFIABLE SANDS

### 2.1 Database of CTX on Loose Sands

The gathered database comprises 56 strain-controlled cyclic triaxial tests from three different sources. Though the tests are conducted on samples with a wide range of distinct characteristics, every sample consists solely of clean sand. Well-recorded laboratory test results with sufficiently frequent intervals are obtained directly from the source. Since the SCM is developed using ML, it is essential to have a diverse range of inputs and varying data patterns to ensure robustness and generalization. That is, both quality and quantity are equally important to develop a successful model. Irregular or noisy data can lead to inaccurate predictions and lower model performance, whereas too little data can lead to underfitting, in which the model cannot learn effectively. A summary of the collected data is shown in Table 1. Time ( $t$ ), strain ( $\epsilon$ ), initial relative density ( $D_{r,0}$ ), initial void ratio ( $e_0$ ), and initial effective confining stress ( $\sigma'_{3,0}$ ) values are the input features in the model. The excess pore pressure ratio ( $r_u$ ) and the deviatoric stress ( $\sigma_d$ ) are the model's outputs. The number of samples used in each source is 36, 11, and 9, respectively. In addition,  $D_{50}$  are 0.22mm, 0.20mm, and 0.14mm;  $C_u$  are 1.71, 1.62, and 1.50;  $C_c$  are 0.86, 1.06, and 1.11 for the references in Table 1 in the respective order.

Table 1. Collected Data for CTX Database.

Source	$\epsilon$ (%)	$D_{r,0}$ (%)	$e_0$	$\sigma'_3$	$t$ (s)	Soil Type
ElGhoraiby et al. (2020)	0.041- 0.301	40- 76	0.58- 0.67	100- 100	600- 6000	Ottawa Sand
Vasko et al. (2018)	0.061- 0.497	55- 55	0.60- 0.60	50- 200	456- 9000	Ottawa Sand
Wichtmann & Triantafylli dis (2016)	0.062- 1.01	29- 101	0.50- 0.72	100- 700	8960- 2180 00	Karlsru he Fine Sand

A data preprocessing method called “windowing” is used to properly and effectively train the model. Windowing is a common ML technique that splits input and output data into fixed-length segments for model training. Windowing creates smaller, more meaningful data segments that the model learns from more effectively. Visualization of this method is shown in Fig. 1, where the red window corresponds to the first, the blue to the second, and the green to the last. The window is shifted on the input data, and the corresponding output data is retrieved. In this study, the window size is set to 50, allowing the output to do one-step prediction for each input window. The window

size was set to 50 based on the test's data collection frequency and the model's optimal capture of graphical details. Therefore, we can conclude that 50 is appropriate based on the test results.

### 2.2 Model Architecture and Optimization Procedure

Assessing liquefaction by predicting the deviatoric stress and the excess pore-water pressure ratio ( $r_u$ ) is a time-series problem. Since classical ML methods do not treat time as a separate feature, this study uses recurrent neural networks (RNNs) to capture the time-dependent nature of the problem. Although there are several RNN types, Long-Short Term Memory (LSTM) (Hochreiter & Schmidhuber, 1997) stands out for its predictive performance and resistance to the vanishing gradient problem during training. LSTM operates on 3-D tensor inputs. The axes of the tensor are: sample number, time steps, and feature number, respectively. Time steps are processed cumulatively, with previous time steps affecting subsequent ones. The LSTM structure consists of three gates and two cells. Cells include the hidden state and the cell state, whereas the gates are the forget gate, the input gate, and the output gate. The model updates its weights by deciding which information to forget and which to keep through the gates and states.

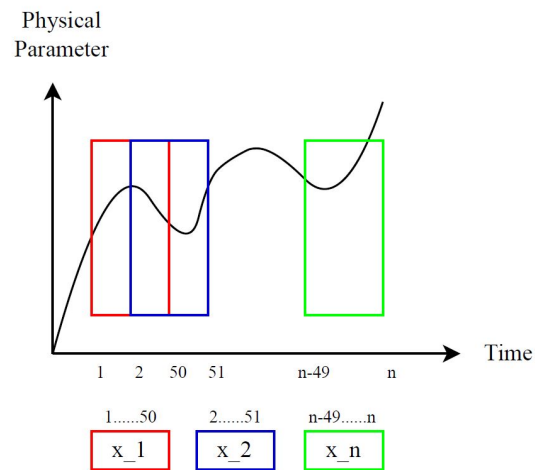


Figure 1. Windowing method.

Different model architectures can be used for time series problems, such as LSTM layers, which can be stacked to form a sequential structure, or encoder-decoder or transformer-based architectures. That said, the increasing complexity of the model is not always beneficial in deep learning. In this study, a typical stacked LSTM network is developed through two LSTM layers. The first one takes 3D input data and generates 3D outputs. Then the other LSTM layer takes these outputs and uses the last layer to make predictions. Outputs are structured as 2D arrays, with the axes corresponding to the sample number and the output feature, respectively.

During ML training, the model's error on the training set should decrease with each iteration, indicating effective learning. Similarly, the validation error should decline in a comparable pattern, demonstrating the model's ability to generalize. Only when both conditions are met can it be concluded that the model learns effectively and generalizes well. Once the final model is obtained, it must be evaluated on the test set. The model's predictions on this set provide the most critical insight into its performance and practical applicability. Among the 56 laboratory data gathered in this study, 35 are allocated to the training set, 9 to the validation set, and 12 to the test set. The loss function is selected as the mean squared error (MSE), and the error metric is the mean absolute error (MAE). The formulae given below, where  $Y_i$  is the ground truth,  $\hat{Y}_i$  is the prediction and  $n$  is the number of samples:

$$MSE = \frac{1}{n} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 \quad (1)$$

$$MAE = \frac{1}{n} \sum_{i=1}^n |Y_i - \hat{Y}_i| \quad (2)$$

In deep learning applications, developing and training the model with random hyperparameters is typically not a valid approach, since even the simplest model has several hyperparameters that must be tuned. Tuning can be done by evaluating every combination, a process called *exhaustive search*. Since our liquefaction problem involves several hyperparameters that must be searched over a wide range, a search algorithm is appropriate. The hyperband method (Li et al., 2018) is used in this study. All operations are performed using the Keras library (Chollet et al., 2015) in Python. The hyperparameters, their search ranges, the search type, and the selected values are given in Table 2.

Table 2. Hyperparameter optimization summary.

Hyperparameter Name	Type of Search	Comments	Chosen Value
LSTM Units	Optimum Number	Min:32 Max:160 Step:16	144
Activation Function in Last Layer	Optimum Choice	Linear or Sigmoid	Linear
Normalization Layers	Existence or Not	LayerNormalization Layer between LSTM Layers	Not to Use LayerNormalization
Recurrent Regularizer	Optimum Choice	L1, L2 or not using any regularizer	Not to use any regularizer
Learning Rate	Optimum Number	Min:0.0001 Max:0.01. Sampling Logarithmically	0.00010085
Optimization Function	Optimum Choice	RMSProp or Adam	Adam
Batch Size	Optimum Number	Min:256 Max:1024 Step:64	896

### 2.3 Model Validation

The loss variation of the final model obtained via hyperparameter optimization is shown in Figure 2. Training has stopped at the 11th epoch due to the underfitting-overfitting trade-off. After deciding on all hyperparameters and the number of epochs, the validation data is added to the training dataset, and all data is used to train the model. Lastly, the final model is tested on the test dataset that has never been used before. These results on the test set are shown in Figures 3 and 4.

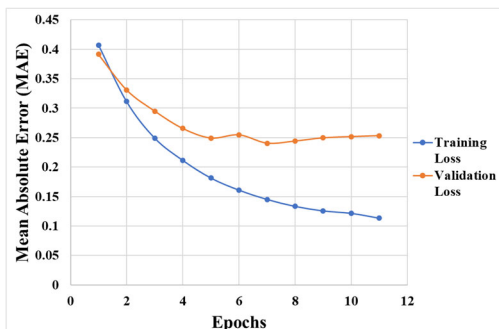


Figure 2. Loss variation of final model

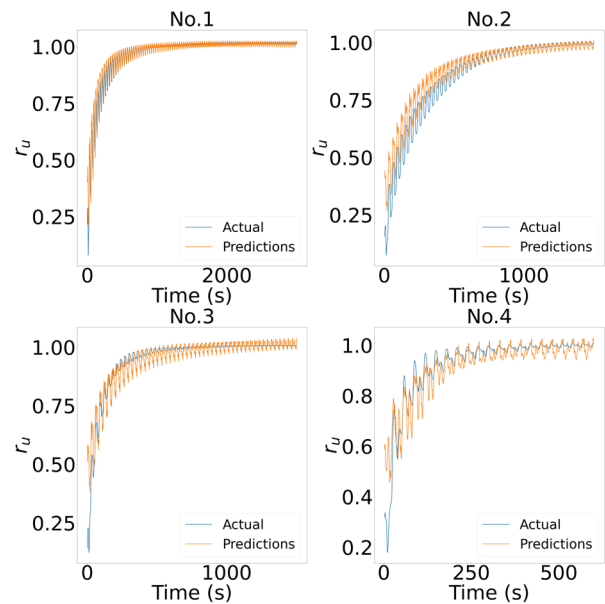


Figure 3. Excess pore water pressure ratio predictions by SCM

From Figures 4 and 5, it can be inferred that the ML model makes successful predictions on the test dataset. Input features and error metrics for the dataset are shown in Table 3. In Test 3, a noticeable decline in model performance is observed compared to other tests. This is likely due to the high cyclic strain ratio, significantly longer test duration, and higher confining pressure. The result highlights the importance of dataset quantity and diversity. We believe that had the training set included more samples representative of this test condition, the prediction accuracy would have been higher.

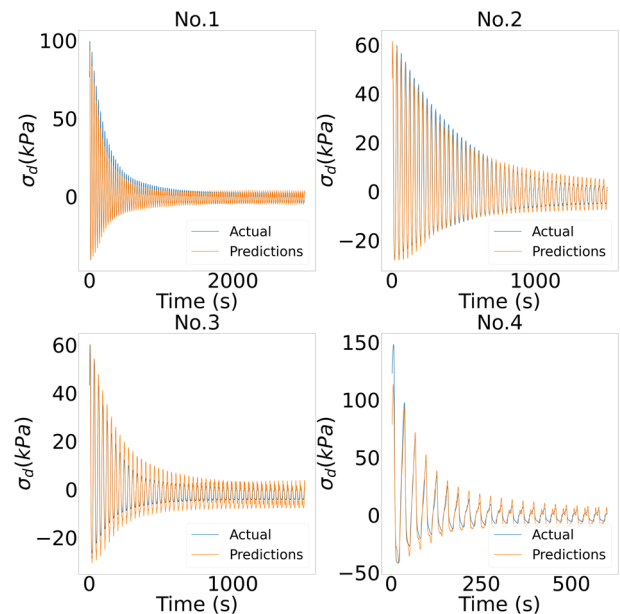


Figure 4. Deviatoric stress predictions by SCM

Table 3. Input features and error metrics in the test dataset

Test No.	$\varepsilon$ (%)	$D_r$ (%)	$\sigma'_3$ (kPa)	$e$	MAE*	$r^2$ *	MAE <sub>x</sub>	$r^{2x}$
No.1	0.11	63.8	100	0.61	0.012	0.96	1.534	0.97
No.2	0.05	40.0	100	0.67	0.036	0.92	2.439	0.93
No.3	0.50	63.0	200	0.62	0.069	0.54	8.737	0.73
No.4	0.10	63.8	100	0.61	0.016	0.97	1.749	0.98

\* The parameter is associated with  $r_u$ . \*The parameter is for  $\sigma_d$ .

### 3 NUMERICAL FORMULATION

The mathematical formulation and the numerical solution of the dynamic response of poro-elastic granular soil are presented in this section. Poroelasticity theory (Biot, 1942, 1956) is employed, along with the SCM developed in this study, which is incorporated into the formulation. Neglecting the inertial terms associated with pore-water motion, Ulker (2009) developed a partially dynamic (PD) form that was further elaborated by Ulker and Rahman (2009) and is used in this study to analyze the dynamic response of liquefiable sand. This mathematical form, now called poro-inelasticity, is then discretized using a hybrid FEM and finite difference method (FDM) and implemented in a coupled computer program using MATLAB and Python.

To carry out the integration, FEM is used for spatial discretization, while central finite differences are used for temporal integration of the coupled equation of motion.

#### 3.1 Hybrid FEM-FDM Discretized Formulation

The final form of the coupled equation of motion employing the poro-inelasticity equations discretized through FEM is given below:

$$\begin{bmatrix} M_s & 0 \\ M_f & 0 \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{p} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ C^T & C_f \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{p} \end{bmatrix} + \begin{bmatrix} K_s & -C \\ 0 & K_f \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \quad (3)$$

where

$M_s = \int N^u T \rho N^u d\Omega$  and  $M_f = \int B^p T \frac{k}{g} N^u d\Omega$  are well-known mass matrix of the solid and fluid parts, respectively,

$K_s = \int B^u T \underline{D} B^u d\Omega$  and  $K_f = \int B^p T \frac{k}{\rho_f g} B^p d\Omega$  are the stiffness matrices of the solid and fluid parts, respectively,

$C = \int B^u T m N^p d\Omega$  is the coupling matrix between displacement and pore pressure,  $C_f = \int N^p T \frac{k}{K_f} N^p d\Omega$  is the compressibility matrix of the fluid.  $N^u$ ,  $N^p$ ,  $B^u$  and  $B^p$ , are the shape function and their derivative matrices.  $F_1$  and  $F_2$  represent the surface tractions, body forces, and boundary tractions. We neglect the  $M_f$  matrix and the associated effect of the fluid part on the soil solid motion. The above form is written in a nonlinear form giving:

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{R}_{int} = \mathbf{F} \quad (4)$$

where  $\mathbf{M}$  and  $\mathbf{C}$  are the mass and damping matrices of Equation (3), and  $\mathbf{F}$  is the right-hand side force vector. Here,  $\mathbf{R}_{int}$  is the internal unbalanced force containing the effective stress terms. Equation (4) is subsequently discretized in temporal domain using the central difference FDM through a second-order approximation:

$$\hat{k} = \frac{1}{\Delta t^2} \mathbf{M} + \frac{1}{2\Delta t} \mathbf{C} \quad (5)$$

$$\hat{f}_i = \mathbf{F}_i - \left[ \frac{1}{\Delta t^2} \mathbf{M} - \frac{1}{2\Delta t} \mathbf{C} \right] X_{i-1} + \left[ \frac{2}{\Delta t^2} \mathbf{M} \right] X_i - \mathbf{R}_{int,i} \quad (6)$$

where Equations (5) and (6) are the effective algorithmic stiffness and force,  $\Delta t$  is time increment and  $i$  is the time step. Using Equations (5) and (6), the solution for the unknown degree of freedom vector can be shown as:

$$X_{i+1} = \hat{f}_i \hat{k}^{-1} \quad (7)$$

An explicit time integration procedure is used to incorporate the SCM due to its robustness and suitability for such stability problems. Such a now hybrid FEM-FDM integration is then implemented in a fully coupled MATLAB-PYTHON environment employing the SCM, replacing the constitutive model formulation.

### 4 FINITE ELEMENT ANALYSIS OF LIQUEFACTION

#### 4.1 Verification of SCM

To verify that the SCM predicts the actual stresses developed at a given site under specified strains, a set of FE analyses is conducted for different soil conditions under the 1989 Loma Prieta earthquake Loma Gilroy #2 station records. Here, to govern the loose sandy soil's liquefaction behavior, the UBCSAND model is employed. The model parameters are derived based on the clean sand correction adopted from SPT results suggested by Beaty and Byrne (2011). Then, site response analyses are conducted with PLAXIS-2D. The strains obtained from the FE analyses are used as a part of the input for SCM in the PYTHON module. Then the predicted deviatoric stresses and pore pressure ratio,  $r_u$ , from SCM are compared with those from the FE model, as shown in Figures 5 and 6.

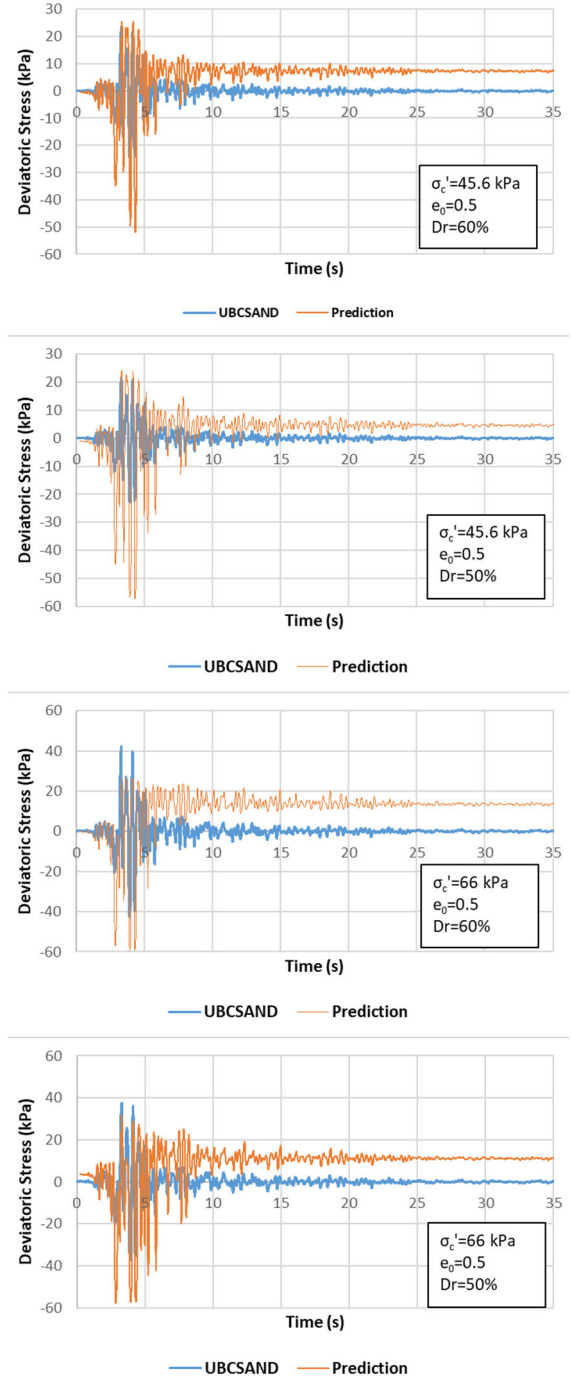


Figure 5. Deviatoric stress predictions under irregular loading

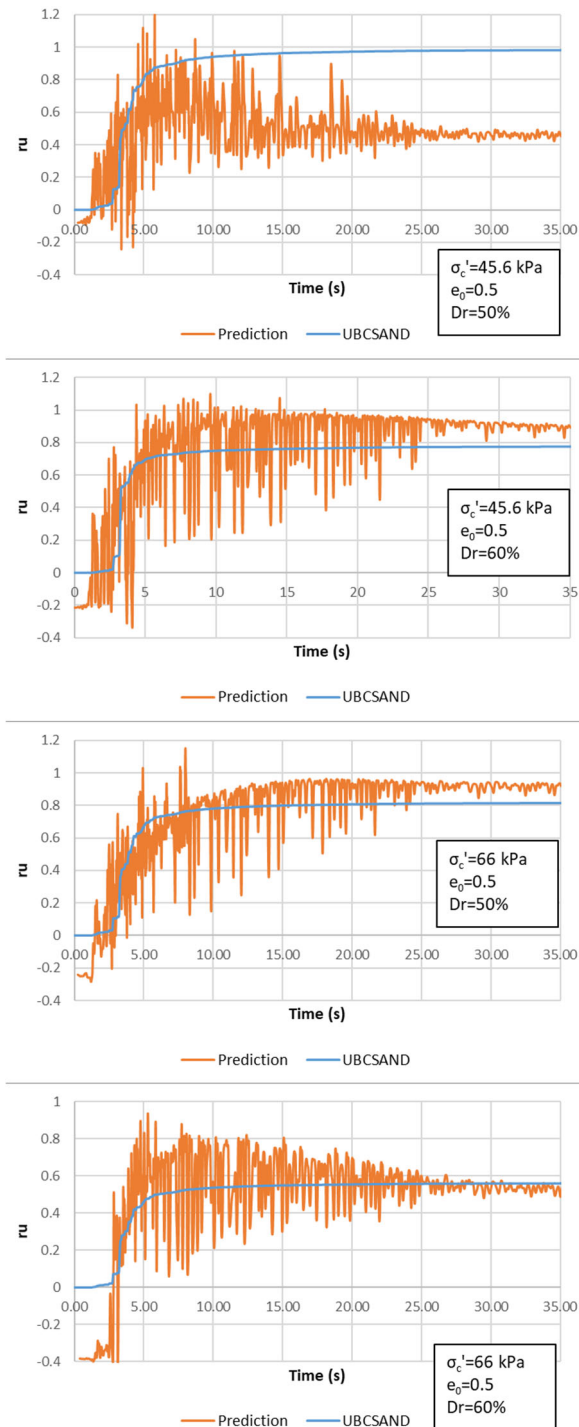


Figure 6.  $\tau_u$  predictions under irregular loading

Though the response trends match fairly well across all considered soil states, the discrepancy in the magnitudes of the predicted deviatoric stresses is associated with the strain levels used to obtain the SCM response for those particular soil tests. We also need to mention here that the UBCSAND model includes features that aim to modify the given strain vector to obtain stable FE solutions. It is believed to have played a role in such magnitude differences, especially the peak response.

#### 4.2 Integration of the SCM into the FEM

SCM is integrated into the FE framework as a material subroutine and predicts stresses and internal variables using strains from the FE solver and other input features. At each time step or load increment, the FE solver asks the SCM to update

the tangent stiffness matrix and material behavior for predicting displacements and other variables. This process is repeated at each Gauss point to predict material response locally. The corresponding code algorithm is presented in Figure 7.

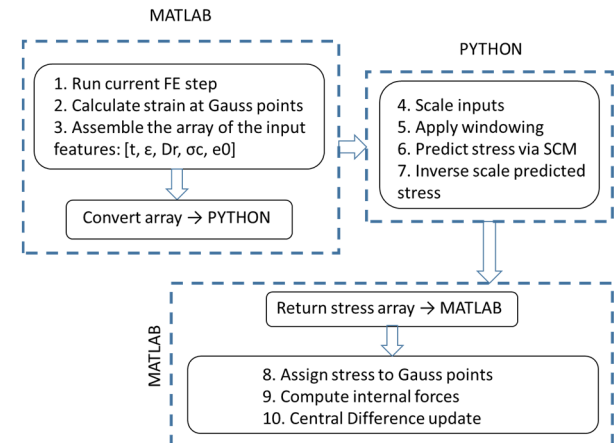


Figure 7. Algorithm of the coupled FEM-FDM-SCM system with the original MATLAB-PYTHON interface

#### 4.3 Poro-Inelastic Analyses with SCM

The performance of the SCM in the FE framework is investigated using a 20 m-high soil column that is harmonically loaded at the top (Figure 8). The soil profile is derived from a liquefied site during the 2023 Kahramanmaraş earthquakes. The groundwater table is located at the top of the profile.

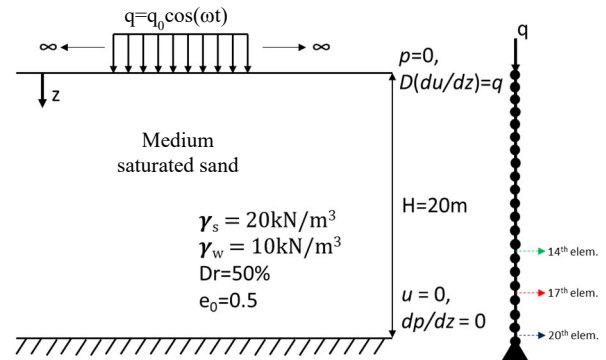


Figure 8. Soil layer properties, load, boundary conditions, and the 1-D FE model

As shown in Figure 9, deviatoric stress, as evaluated at several locations in the soil, decreases over time, consistent with the expected behavior of liquefiable soils. The reduction in stress occurs almost instantly due to the sandy soil sample's predictive cyclic behavior under SCM. That is, a medium sand soil, which liquefies fairly quickly in the first few cycles of loading has been chosen. In other words, the cyclic triaxial test set used to train the SCM contains axial strain levels that lead to such liquefaction response, and that is essentially captured in this finite element analysis. Locations closer to the ground surface exhibit a large tensile response during the first few load cycles, indicating liquefaction, for which solutions are not given here due to brevity.

#### 5 DISCUSSION

Cyclic loading-induced soil liquefaction is modeled through a data-driven study, where an ML-based soil constitutive model, now called the "Smart Constitutive Model," is employed. SCM essentially replaces an otherwise complex cyclic elastoplastic model to govern the liquefaction behavior of saturated medium dense sand. SCM is then implemented into a classical 1-D FEM

program developed in the study, which is integrated in the temporal domain using the central difference FDM. This coupled FEM-FDM-SCM hybrid model is then used to simulate a liquefiable sand soil subjected to a few cyclic load cycles. Discussion goes such that:

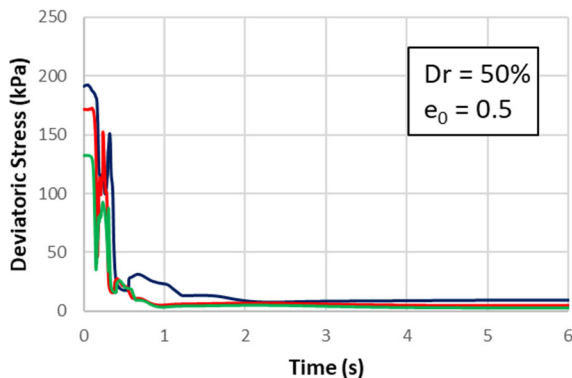


Figure 9. Deviatoric stress predictions through the FE framework, plots correspond to the nodal point locations given in Figure 8

- SCM can be considered a pioneering data-driven model based on deep learning to capture the dynamic soil liquefaction response of granular soils.
- Under irregular seismic loading, given the actual strain vectors, SCM tends to follow the expected trend coming from the otherwise dynamic elasto-plastic response. Regarding the magnitudes of deviatoric stress, there is a slight difference between the elasto-plastic response and the SCM predictions. However, that is still on the conservative side for the SCM.
- FEM-FDM-SCM hybrid model can successfully capture the field liquefaction of medium dense sands under large load amplitudes in terms of pore pressure buildup and corresponding effective stress reduction.

## 6 CONCLUSIONS

The assessment of the liquefaction potential of soils during an earthquake is an important issue that requires the adoption of suitable computational tools for prediction. Therefore, it is highly important to evaluate the liquefaction susceptibility of sandy soil deposits using appropriate theoretical constitutive relations within the nonlinear FE framework. In this study, we attempt to answer this call by introducing deep learning tools into the material framework to replace such complex models essentially. That is, unlike many other common studies, where liquefaction is quantified by a single parameter, such as the factor of safety, the entire time-history variations in pore pressure and effective stress during cyclic loading are predicted. For that, a comprehensive database of cyclic triaxial test results is gathered with the actual test data. Then, a deep-learning-based LSTM framework is used to train ML models on the database and to make predictions. SCM is successfully tested and then implemented in an in-house 1-D FE code via a novel integrated hybrid MATLAB-PYTHON interface. Such coupling is proposed for the first time in this study and subsequently tested in the field cyclic liquefaction response of medium-dense sand. Initial results are successful at large load amplitudes within the first few cycles. Further studies continue to predict the field liquefaction of other liquefiable soils with different densities and under various cyclic loads, including irregular seismic excitations.

## 7 ACKNOWLEDGEMENTS

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## 8 REFERENCES

- Beatty, M. H. and Byrne, P. M. 2011. UBCSAND constitutive model version 904aR, Itasca UDM Web Site, 69.
- Birinci, Ö. T., Ülker, M.B.C. and Taşkın, G. 2025. Prediction of pore water pressure generation of liquefiable clean sands under cyclic shear loading through deep learning. *Geomechanics and Geoengineering: An International Journal* 1-22.
- Chen, W., Olarte, A. A. P., and Cudmani, R. 2021. Modelling the monotonic and cyclic behaviour of sands using Artificial Neural Networks. *EPJ Web of Conferences* 249, 11015.
- Choi, Y., and Kumar, K. 2023. A machine learning approach to predicting pore pressure response in liquefiable sands under cyclic loading. *Geo-Congress 2023*, Los Angeles, 202-210.
- Chollet, F. & others, 2015. Keras. [Online] Available at: <https://github.com/fchollet/keras>.
- Drakos, S. 2008. *Applications of artificial intelligence in constitutive modelling of soils*. M.Phil. Thesis. Swansea University (United Kingdom).
- ElGhoraiby, M. A., Park, H., and Manzari, M. T. 2020. Stress-strain behavior and liquefaction strength characteristics of Ottawa F65 sand. *Soil Dynamics and Earthquake Engineering* 138, 106292.
- Ghaboussi, J. 2018. *Soft Computing in Engineering*, CRC Press, Taylor & Francis Group.
- Guan, Q. Z., and Yang, Z. X. 2023. Hybrid deep learning model for prediction of monotonic and cyclic responses of sand. *Acta Geotechnica* 18(3), 1447-1461.
- Hochreiter, S., and Schmidhuber, J. 1997. Long short-term memory. *Neural Computation* 9(8), 1735-1780.
- Li, L., Jamieson, K., DeSalvo, G., Rostamizadeh, A., and Talwalkar, A. 2018. Hyperband: A novel bandit-based approach to hyperparameter optimization. *Journal of Machine Learning Research* 18(185), 1-52.
- Liang, L., Liu, M., Elefteriades, J., and Sun, W. 2023. PyTorch-FEA: Autograd-enabled finite Element Analysis Methods with Applications for Biomechanical Analysis of Human Aorta. bioRxiv: The preprint server for biology. 10.1101/2023.03.27.533816.
- Meng, F., and Pei, H. 2023. Cyclic shear stress-strain prediction of saturated sand based on the unrolled seq2seq model and scheduled sampling. *Soil Dynamics and Earthquake Engineering*, 165, 107665.
- Smith, E. R. C. 2019. *Conduits of invasive species into the UK: The angling route?* Ph. D. Thesis. University College London.
- Ulker, M. B. C. 2009. *Dynamics of Saturated Porous Media: Wave-Induced Response and Instability of Seabed*. Ph. D. Thesis. North Carolina State University, Raleigh, NC.
- Ulker, M. B. C., and Rahman, M.S. 2009. Response of saturated porous media: Different formulations and their applicability. *Int. J. Analy. Numer. Mthds in Geomech* 33(5), 633-664.
- Vasko, A., ElGhoraiby, M., and Manzari, M. 2018. 'LEAP-GWU-2015 Laboratory Tests', doi:10.17603/DS2TH7Q, Available at: <https://www.designsafe-ci.org/data/browser/public/designsafe.storage.published/PRJ-1780> (Accessed July 2025).
- Wichtmann, T., and Triantafyllidis, T. 2016. An experimental database for the development, calibration and verification of constitutive models for sand with focus to cyclic loading: Part II tests with strain cycles and combined loading. *Acta Geotechnica* 11(4), 763-774.
- Wu, X. and Ghaboussi, J. 1995. *Civil engineering studies: Neural network-based material modeling* (Report No. UILU-ENG-95-2002). Department of Civil Engineering, University of Illinois at Urbana-Champaign. <https://www.ideals.illinois.edu/items/14237>
- Zhang, P., Yin, Z. Y., Jin, Y. F., and Ye, G. L. 2020. An AI-based model for describing cyclic characteristics of granular materials. *International Journal for Numerical and Analytical Methods in Geomechanics* 44(9), 1315-1335.