

## Numerical modelling of ground freezing – An overview

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**ABSTRACT:** Ground freezing has been demonstrated to be a reliable and environmentally friendly technique for providing temporary sealing and support in almost all types of water-bearing subsoil. This article addresses the thermal design of freezing measures using numerical methods, with a particular focus on validating these designs through in situ measurements. Firstly, it provides an overview of the various numerical approaches to the thermal design of freezing measures. Selecting the correct (temperature) boundary condition along a freezing pipe is crucial for the accuracy of the numerical calculations. The modelling requirements and their respective impact on the results are discussed using a simple freezing layout and a practical application. Furthermore, the calculation results will be compared with the measured values and evaluated. In numerical modelling, the boundary value problem to be investigated is always simplified for engineering purposes. Although two-dimensional calculations are often used for this purpose, they are not necessarily the most effective approach. Given current computer capacities, it is often possible to model the boundary value problem in three dimensions, taking significant effects into account. This approach is used for both calculation examples in this article. Moreover, in construction practice, it is evident that the actual geometric position of the freezing and temperature lances almost never aligns with the planned target position. Therefore, this article also addresses the topic of recording the actual positions of the freezing and temperature measuring lances and how to account for them in the numerical thermal calculations.

**KEYWORDS:** frost geotechnics, ground freezing, thermal analysis, numerical analysis, temperature measurement.

### 1 INTRODUCTION

In 1883, Friedrich Hermann Poetsch of Germany filed a patent for artificial ground freezing. This technique was successfully employed in a lignite mine in Saxony-Anhalt. This method, known as the freezing shaft technique, became internationally established and was primarily used for constructing shafts in mines until the mid-20th century. Intensive research into the behaviour of frozen soils has expanded the range of applications for freezing methods in general, meaning that they are now also used in civil engineering projects.

As the demand for high-performance infrastructure in densely populated areas grows, the development of underground transport and supply systems is becoming increasingly important. The freezing method is a key construction technique for establishing secure connections and structures below the ground water table. It meets both technical and environmental requirements and is considered a reliable, eco-friendly solution for temporary sealing and stabilization in water-bearing soil.

Through continuous heat extraction, based on the “refrigerator principle”, the water-saturated soil is cooled to such an extent by the freezing method that the pore water freezes. This is achieved by inserting double-walled pipes containing a circulating coolant, such as brine or liquid nitrogen, into the ground. The inner tube is the feed tube, while the annular gap between the inner and outer tube contains the flowback. The resulting ice solidifies the soil, creating a dense, stable frost body that can be used as a temporary sealant and support structure in the ground.

The freezing process and the maintenance of the frost body can be reliably controlled using temperature measurements. The frozen soil structures are completely watertight and thaw slowly, due to the high heat capacity and crystallization heat of water, making the process very safe and reliable. However, disadvantages include installation costs, high energy consumption, and the effort involved in temperature monitoring.

This article focuses on the thermal design of the freezing process using numerical methods. Particular emphasis is placed on validating these designs through in situ measurements.

### 2 GROUND FREEZING DESIGN

The ground freezing design comprises two parts: the static design and the thermal design.

The static design essentially serves to determine the minimum required thickness of the frost body for the temporary support. The necessary input variables are the loads to be considered, such as earth and water pressure, as well as the (time-dependent) mechanical characteristics of the frost body. When determining the shape of a frost body, it must be taken into account that they have only low tensile strength. Therefore, circular icing geometries are the most reasonable option as they have uniform compressive stresses and low tensile stresses. Pressure vaults are another suitable alternative for a simple analytical design. They are designed to transfer most loads via compressive stresses. As a closed frost body is watertight by definition, proof of watertightness is not required. Depending on the complexity of the design task analytical and/or numerical methods are employed. Further details can be found in Neher et al. (2016), for example.

The thermal design, focused herein, determines the number and positions of the freezing pipes required to achieve the minimum thickness of the frost body, as well as the corresponding freezing times and the amount of energy needed. The thickness of the frost body is checked directly in the ground using temperature measurements. As part of the thermal design, the target temperatures to be achieved at the individual temperature measuring points are also determined. In order to describe the propagation of a frost body, the thermal soil parameters have to be known. In particular, the heat capacity of the frozen and unfrozen soil, the thermal conductivity in the frozen and unfrozen state, and the crystallization heat of water must be considered. Furthermore, boundary conditions such as the initial soil and structure temperatures, known heat sources, and groundwater flows play a decisive role. Thermal finite element analysis can address all these aspects more accurately than classical analytical formulas, including those that account for groundwater flow.

### 3 NUMERICAL THERMAL DESIGN

In this section first the different types of boundary conditions “Dirichlet”, “Neumann” or “Robin” used in thermal finite element analysis are explained. In case of thermal analysis for ground freezing “Dirichlet” or “Robin” boundary conditions are mostly used. This two are compared in section 4.

#### 3.1 Dirichlet boundary condition - prescribed temperature

In finite element analysis, the terms “Dirichlet” and “nodal” boundary condition are used interchangeably. A “Dirichlet” boundary condition specifies the exact temperature value at a boundary and therefore at nodes. This makes it ideal for modelling contact with a thermal reservoir or a surface held at a constant temperature.

For example, from an engineering perspective, one can provide a constant temperature inside the freezing pipes in thermal analyses of ground freezing, assuming that the temperature of the pipe wall is the same as that of the fluid inside the pipe.

The following mathematical formula applies at the boundary, where  $T_0$  is the known temperature.

$$T(x) = T_0 \quad (1)$$

#### 3.2 Neumann boundary condition - prescribed heat flux

The “Neumann” boundary condition specifies the heat flux, or the rate of heat transfer per unit area, across a boundary and is therefore a pure flux condition. It is typically used to model insulated surfaces with zero heat flux, or surfaces with a known heat input or output.

The following mathematical formula applies at the boundary, where  $k$  is the thermal conductivity,  $\frac{\partial T}{\partial n}$  is the temperature gradient normal to the surface and  $q$  is the specified heat flux.

$$-k \frac{\partial T}{\partial n} = q \quad (2)$$

#### 3.3 Robin boundary condition - convective heat transfer

The “Robin” boundary condition is used to model convective heat transfer between a surface and a surrounding fluid, such as air or water. It combines aspects of the aforementioned “Dirichlet” and “Neumann” boundary conditions.

In thermal analyses of ground freezing using the “Robin” boundary condition, the input parameters are the initial ground temperature, the temperature of the freezing fluid (e.g. brine), and its convective heat transfer coefficient. The latter is linked to the flow regime (laminar or turbulent) and the diameter of the freezing pipe. Although this approach is more realistic than the “Dirichlet” boundary condition, incorrect results may be produced if the convective heat transfer coefficient is not selected appropriately.

The difference between the fluid temperature inside the pipe and the pipe wall temperature depends on the convective heat transfer coefficient. For high convective heat transfer coefficients, the difference to the prescribed temperature boundary condition (“Dirichlet”) vanishes.

The following mathematical formula applies at the boundary, where  $k$  is the thermal conductivity,  $\frac{\partial T}{\partial n}$  is the temperature gradient normal to the surface,  $\alpha_T$  is the convective heat transfer coefficient,  $T$  is the surface temperature and  $T_\infty$  is the ambient fluid temperature.

$$-k \frac{\partial T}{\partial n} = \alpha_T (T - T_\infty) \quad (3)$$

#### 3.4 Convective heat transfer coefficient

The convective heat transfer coefficient must be determined carefully, as it significantly affects the required freezing times and the temperature of the frost body. Several empirical correlations have been developed for forced convective heat transfer coefficients in double-walled pipes also known as tube-in-tube systems. The coolant, which is mostly brine, circulates through an inner tube to the end of the freezing pipe. It then flows back through the annular gap between the inner pipe and outer pipe to the head of the freezing pipe. The flow regime within this gap is crucial for determining the forced convective heat transfer coefficient. The equivalent diameter of the flow regime in an annular gap is  $d_{eq} = d_{o,i} - d_{i,o}$ ,  $d_{o,i}$  is the inner diameter of the outer tube of the freezing pipe and  $d_{i,o}$  is the outer diameter of the inner tube.

The convective heat transfer coefficient is defined by using the dimensionless Nusselt number  $Nu$ , the thermal conductivity of the fluid  $k$  and the equivalent diameter of the pipe  $d_{eq}$ .

$$\alpha_T = \frac{Nu \cdot k}{d_{eq}} \quad (4)$$

The Nusselt number depends on the flow conditions and the thermal parameters of the fluid.

Firstly, the dimensionless Reynolds number  $Re$  is used to distinguish between a laminar flow regime ( $Re < 2300$ ), a turbulent flow regime ( $Re > 10000$ ) and a transition regime ( $2300 < Re < 10000$ ). The Reynolds number  $Re$  can be calculated using the flow velocity of the fluid  $v$ , the equivalent diameter of the pipe  $d_{eq}$  and the temperature-dependent kinematic viscosity of the fluid  $\nu_f$ .

$$Re = \frac{v \cdot d_{eq}}{\nu_f} \quad (5)$$

The Nusselt number also depends on the velocity profile of the annular gap flow. In the hydrodynamic entrance region, the velocity profile develops, whereas in the fully developed region it remains constant. For typical brine freezing, the length of the hydrodynamic entrance region is approximately two metres.

As with the flow profile, the Nusselt number depends on the temperature profile. For typical brine freezing, the length of the thermal entrance region is about 200 metres long.

According to Schüller (2015), based on the work of Martin (1990) and Gnielinsky (2013), the mean Nusselt number for annular gap flow can be calculated as follows.

$$Nu_{mRR} = \sqrt[3]{Nu_{mRR1}^3 + Nu_{mRR2}^3 + Nu_{mRR3}^3} \quad (6)$$

Where  $Nu_{mRR1}$  is the Nusselt number of a hydrodynamic and thermal developed flow,

$$Nu_{mRR1} = 3.66 + \left[ 4 - \frac{0.102}{\left( \frac{d_{i,o}}{d_{o,i}} + 0.02 \right)} \right] \cdot \left( \frac{d_{i,o}}{d_{o,i}} \right)^{0.04} \quad (7)$$

$Nu_{mRR2}$  is the Nusselt number of a hydrodynamic developed flow and a thermal entrance flow,

$$Nu_{mRR2} = 1.615 + \left( 1 + 0.14 \cdot \left( \frac{d_{i,o}}{d_{o,i}} \right)^{0.1} \right) \cdot \sqrt[3]{Re \cdot Pr \cdot \frac{d_{eq}}{L_{Gr}}} \quad (8)$$

and  $Nu_{mRR3}$  is the Nusselt number of a hydrodynamic and thermal entrance flow.

$$Nu_{mRR3} = \sqrt[3]{\frac{2}{1 + 22 \cdot Pr}} \cdot \sqrt{Re \cdot Pr \cdot \frac{d_{eq}}{L_{Gr}}} \quad (9)$$

In the above equations,  $d_{i,o}$  is the outer diameter of the inner tube of the freezing pipe, and  $d_{o,i}$  is the inner diameter of the outer tube, whereas  $Pr$  is the Prandtl number, and  $L_{Gr}$  the length of the freezing pipe.

For practical frost body calculations, the Nusselt number should be averaged over the length of the freezing pipe. To further simplify the calculation, the forced convective heat transfer coefficient is kept constant throughout.

Using the above equations, the resulting forced convective heat transfer coefficient lies between  $90 \frac{W}{m^2 \cdot K}$  and  $200 \frac{W}{m^2 \cdot K}$  for a laminar flow regime. For a turbulent flow regime, the forced convective heat transfer coefficient would be greater than  $500 \frac{W}{m^2 \cdot K}$ .

The fluid flow inside the freezing pipes is usually laminar.

#### 4 COMPARISON OF DIRICHLET AND ROBIN BOUNDARY CONDITION FOR A FIELD TRIAL

To illustrate the practical impact of the two different boundary conditions, “Dirichlet” and “Robin”, on the expected freezing times, a small-scale ground freezing layout involving three circular arranged brine pipes, one temperature pipe, and a centrally arranged freezing target zone for soil sampling is considered. All pipes are arranged on a circle with a radius of one metre.

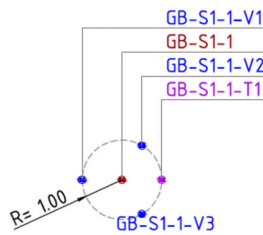


Figure 1. Freezing layout with three brine pipes (GB-S1-1-V1 to GB-S1-1-V3) and one temperature pipe (GB-S1-1-T1) and a planned soil sample location (GB-S1-1).

##### 4.1 Effect of different boundary conditions: Dirichlet and Robin

When a “Dirichlet” boundary condition is used with a constant brine temperature of  $-33 \text{ }^\circ\text{C}$  and thus an identical prescribed nodal temperature, the expected freezing time to reach a soil sample temperature of  $-10 \text{ }^\circ\text{C}$  is 28 days. This increases to 44 days when using a “Robin” boundary condition with a forced convective heat transfer coefficient of  $\alpha_T = 100 \frac{W}{m^2 \cdot K}$ , as shown in Figure 2. A forced convective heat transfer coefficient of  $\alpha_T = 600 \frac{W}{m^2 \cdot K}$  for an assumed turbulent flow regime leads to similar required freezing times to those with a prescribed nodal temperature. However, a forced convective heat transfer coefficient between  $90 \frac{W}{m^2 \cdot K}$  and  $200 \frac{W}{m^2 \cdot K}$  for a typical laminar flow significantly increases the required freezing time.

As the expected freezing times differ, so do the freezing pipe wall temperatures, too. (see Figure 3). A forced convective heat transfer coefficient of  $\alpha_T = 600 \frac{W}{m^2 \cdot K}$  for an assumed turbulent flow regime leads to similar freezing pipe wall temperatures to those with a prescribed nodal temperature. However, a forced convective heat transfer coefficient between  $90 \frac{W}{m^2 \cdot K}$  and  $200 \frac{W}{m^2 \cdot K}$  for a typical laminar flow results in

significant temperature differences between the fluid and the freezing pipe wall.

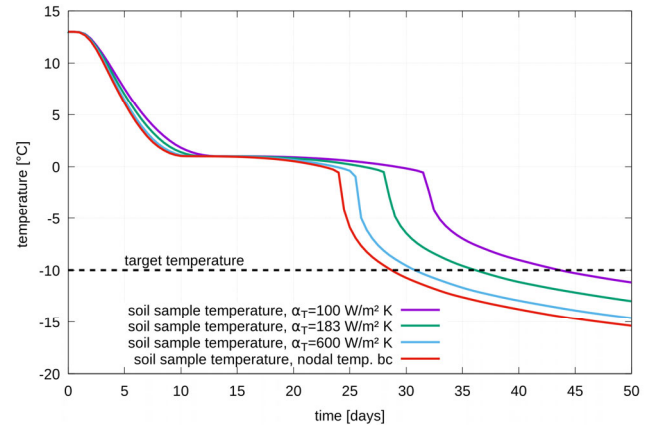


Figure 2. Influence of the boundary conditions on the expected freezing times.

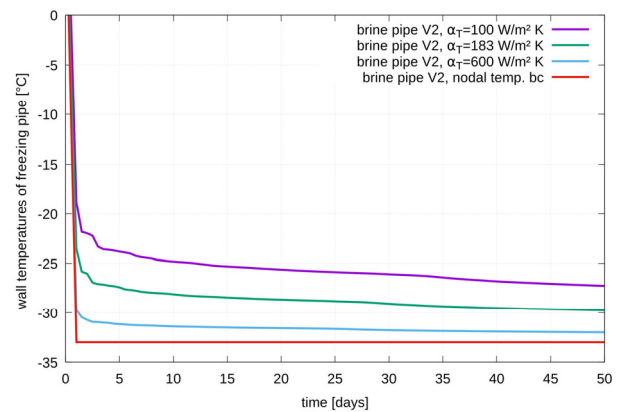


Figure 3. Calculated wall temperatures of freezing pipe GB-S1-1-V2 for different boundary conditions.

##### 4.2 As-Built analysis

After drilling the freezing and temperature pipes, a survey of all pipes was conducted. Based on the actual pipe arrangement, an as-built thermal design calculation has been performed, determining the estimated freezing time and target temperature values (see Figure 4), as well as the expected shape of the frost body. Figure 5 shows the shape of the frost body after 50 days of freezing at a limit temperature of  $-10 \text{ }^\circ\text{C}$ .

Figure 6 shows a comparison between the calculated and measured temperatures at depths of about 10 m and 30 m. While there is sufficient agreement between the calculated and measured temperatures at a depth of 10 m, the measured temperature at a depth of 30 m drops significantly faster than the calculated temperature. The main reason for this difference is most likely the variation in soil properties and initial ground temperature. Some ground temperature measurements indicate an initial ground temperature of about  $8$  to  $9 \text{ }^\circ\text{C}$  instead of the assumed  $13 \text{ }^\circ\text{C}$ .

In addition to monitoring the temperature with the temperature pipe, the temperature inside the drilled hole was measured after the soil sample was taken. After 50 days of freezing, the measured temperature at the soil sample location was approximately  $-14 \text{ }^\circ\text{C}$ , compared to a calculated temperature of  $-14.7 \text{ }^\circ\text{C}$ .

Initial tests to measure the pipe wall temperature revealed some discrepancies between the brine and the freezing pipe wall temperature, but no reliable trends emerged. Further tests are required to improve our understanding of the actual forced

convective heat transfer coefficients between brine and pipe walls in ground freezing projects.

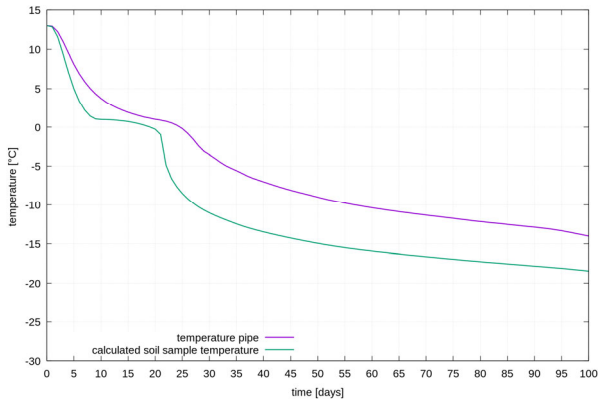


Figure 4. Calculated temperature at soil sample location (green) and for the temperature pipe location (pink) for a forced convective heat transfer coefficient of  $\alpha_T = 130 \frac{W}{m^2 \cdot K}$  in about 10 m depth.

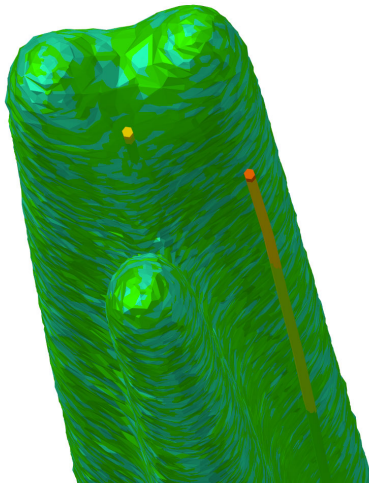


Figure 5. Calculated shape of frost body after 50 days of freezing at a limit temperature of  $-10 \text{ }^\circ\text{C}$ ; view from down under.

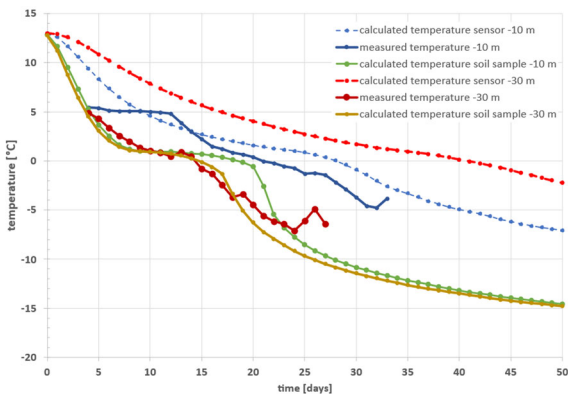


Figure 6. Comparison between calculated and measured temperatures for two different depths.

## 5 PRACTICAL APPLICATION

### 5.1 General overview

Three-dimensional modelling offers significant advantages for numerical thermal design, particularly for projects involving complex geometries. Figure 7 shows the planned layout of a frost body for constructing an additional passageway to an existing subway station. In this case, ground freezing was

required to enable the construction of the access tunnel from the construction pit to the subway tunnel. As can be seen, the ground freezing results in a complex three-dimensional frost body. Consequently, the entire thermal design process was conducted in 3D.

The initial layout of the freezing pipes was created using a three-dimensional CAD model featuring idealised cylindrical frost bodies around each freezing pipe (see Figure 7). Based on this model, the numerical thermal design model was fully meshed and calculated in 3D using the finite element program Simulia Abaqus. Figure 8 shows the overall dimensions and the two different soil layers. The model was meshed using approximately 3.2 million elements and 570 thousand nodes.

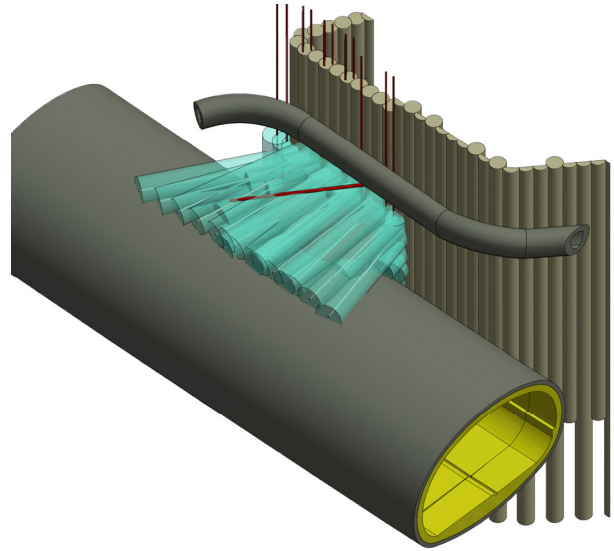


Figure 7. Frost body for constructing a new access to an existing subway station.

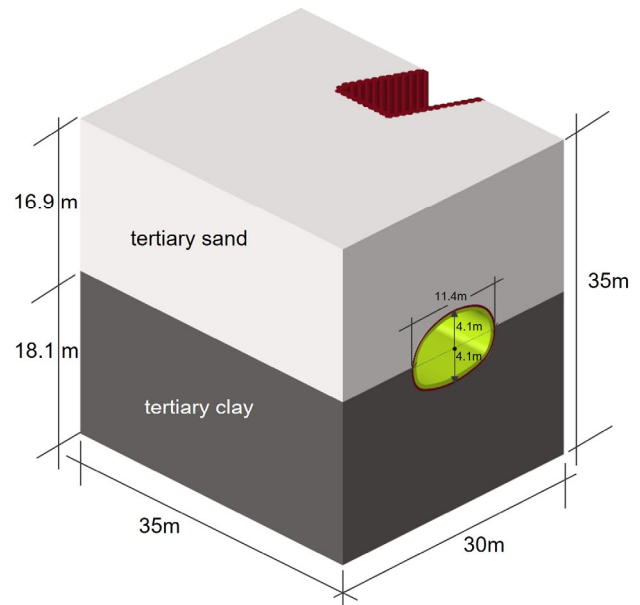


Figure 8. Three-dimensional numerical thermal design model.

As ground freezing was planned to create a new access to an existing subway station, modelling of the tunnel lining was also required. In order to achieve sufficiently low temperatures in the contact zone with the tunnel, the internal surfaces of the subway station that would come into contact with the frost body were thermally insulated.

The numerical thermal design modelled and calculated not only the initial freezing of the soil, but also all relevant construction steps. The main steps are summarised in Table 1.

Table 1. Main construction steps and their duration.

Step	Description	Duration [days]	Total [days]
1	Initial frost body	40	40
5	Temporary lining part I	2	47
6	Permanent lining part I	21	68
12	Temporary lining part II	2	89
13	Permanent lining part II	21	110

During the calculation, some elements had to be disabled (e.g. excavation) and others activated (e.g. temporary and permanent linings). The hydration heat development of the concrete was considered for all linings (temporary and permanent). Additionally, any surface in contact with air was modelled using the “Robin” boundary condition.

### 5.2 Calculated shape of frost body for selected construction steps

The following figures (Figure 9 to Figure 12) show the development of the frost body over the main construction steps.

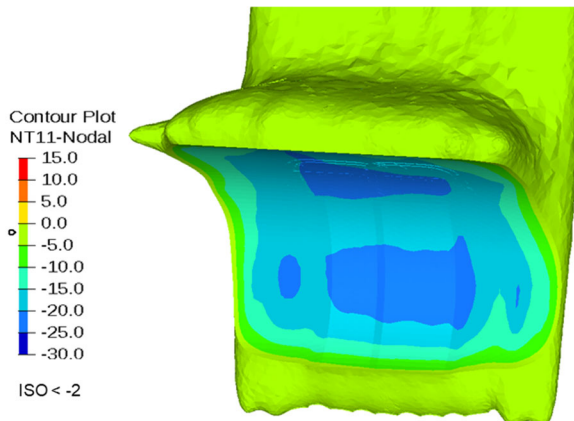


Figure 9. Initial frost body, step 1

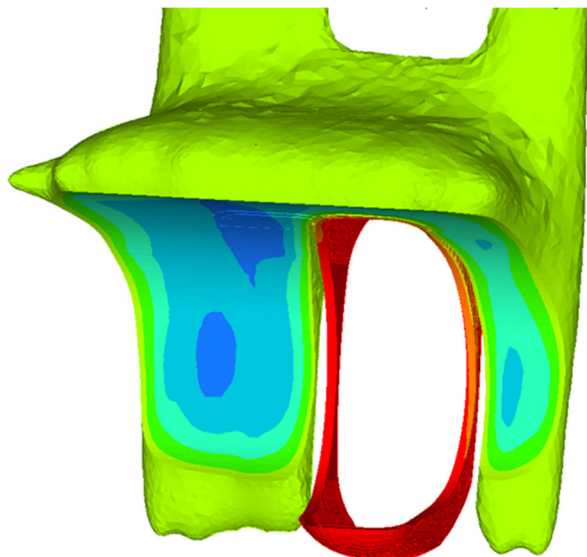


Figure 10. Frost body after the application and hardening of the temporary lining part I, step 5

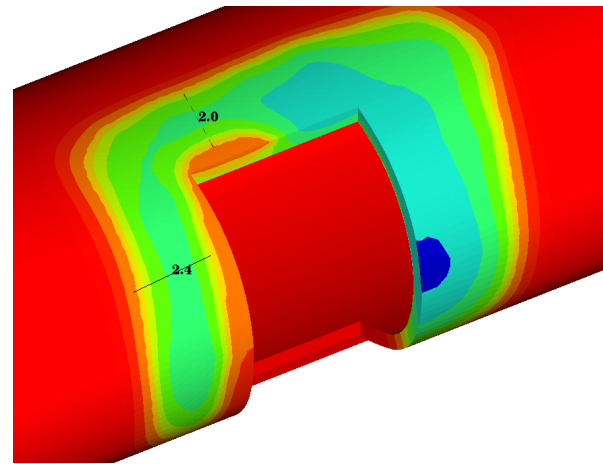


Figure 11. Connection to existing tunnel: concrete temperatures and frost body thickness for construction step 5

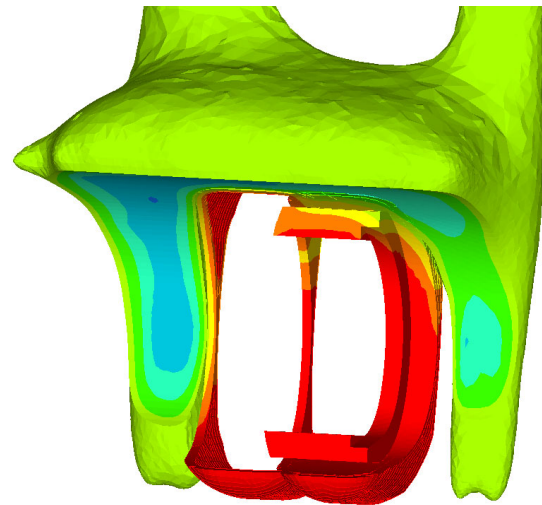


Figure 12. Frost body after the application and hardening of the temporary lining part II, step 12

### 5.3 Effect of different boundary conditions: Dirichlet and Robin

Figure 13 illustrates the impact of the “Robin” (left) and “Dirichlet” (right) boundary conditions on both the thickness and temperature of the frost body in a section. With the chosen conservative forced convective heat transfer coefficient of  $\alpha_T = 70 \frac{W}{m^2 \cdot K}$ , the frost body is significantly smaller and warmer.

A comparison between the measured and calculated temperatures for several temperature sensors is given in Figure 14. As can be seen, the chosen forced convective heat transfer coefficient is too low. The measured temperatures fall faster than the calculated temperatures. A comparison with a nodal temperature (“Dirichlet”) boundary condition shows better agreement with the measured temperatures. This indicates a high forced convective heat transfer coefficient, which could only be achieved with turbulent flow. However, since the actual flow regime was laminar, it suggests that the thermal soil parameters were chosen conservatively.

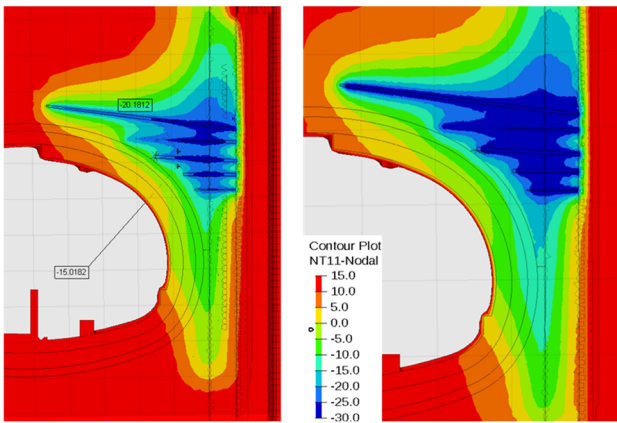


Figure 13. Comparison of “Robin” boundary condition (left) and “Dirichlet” boundary condition (right).

## 6 CONCLUSIONS

Artificial ground freezing is a reliable and proven construction method for temporarily improving and sealing the ground. The main advantage is its temporary nature. Once thawed, the ground's hydrological conditions return to a state similar to that prior to construction. By contrast, diaphragm and bored pile walls remain in the ground, permanently altering the flow of groundwater.

Three-dimensional numerical thermal design calculations are especially useful for complex frost body geometries. The accuracy of estimated freezing times depends on several factors, primarily representative thermal soil parameters, thorough drilling followed by accurate as-built surveys, and detailed numerical thermal design calculations based on the as-built geometry.

When performing these calculations, it is important to consider the thermal soil parameters and provide a realistic and reliable description of the heat transfer between the brine and the pipe wall. This can be achieved by determining the convective heat transfer coefficient based on the work of Schüller (2015). However, as these correlations are purely empirical, averaging values for different flow conditions, further research is required to develop improved correlations for determining forced convective heat transfer coefficients for ground freezing applications.

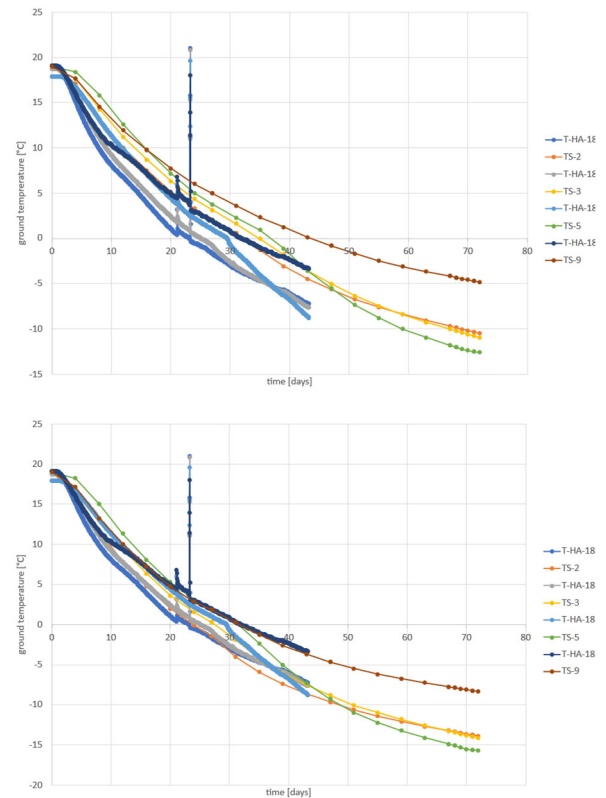


Figure 14. Comparison of “Robin” boundary condition with  $\alpha_T = 70 \frac{W}{m^2 \cdot K}$  (top) and “Dirichlet” boundary condition (bottom); measurement: big points; calculation: thin lines with small points

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