*Proceedings of the 21st ICSMGE, Vienna, Austria, 14 – 19 June 2026. Pistrol, Adam & Schweiger (eds.)*
*Published by: ÖGG, Austrian Society for Geomechanics, Salzburg, Austria, ISBN 978-3-9503898-4-5*
*https://doi.org/10.53243/ICSMGE2026-597*

# A comparison between numerical and theoretical methods for computing dynamic impedances of pile group in layered soils

**Riccardo Ronci,** Riccardo Conti
***Department of Civil Engineering and Computer Science Engineering, Tor Vergata University of Rome, Italy, riccardo.ronci@uniroma2.it***

ABSTRACT: The seismic response of tall structures, such as bridge piers, can be strongly influenced by the interaction with the foundation system. This is particularly evident in the case of pile groups embedded in soft soils, where dynamic soil-structure interaction modifies both the seismic motion at the foundation level, with respect to the free-field condition (kinematic interaction), and the stiffness and damping properties of the system, with respect to the fixed-base structure (inertial interaction). Under the linearity assumption, it is possible to separate these effects and solve the problem in the frequency domain. When dealing with the inertial interaction problem, the structure is reduced to a single or multi degrees-of-freedom oscillator, while the soil-foundation system is replaced by complex-valued dynamic impedances, associated with each mode of vibration. For pile foundations, dynamic impedances can be computed using either numerical (FEM, FDM or BEM) methods or theoretical approaches, where the pile is typically modelled as a Beam on Dynamic Winkler Foundation (BDWF). The latter provide analytical solutions for the case of a single pile in homogeneous soil, while extensive research on the applicability of BDWF models to pile groups embedded in layered soil deposits is still lacking. With the aim of filling this gap, this work presents a comparison between theoretical BDWF results and those obtained with a 3D numerical FE model developed in ABAQUS, in terms of dynamic impedances at the head of a pile group embedded in a linear elastic layered soil deposit.

KEYWORDS: Soil-structure interaction, inertial interaction, dynamic impedances, pile foundation, finite element modelling.

## 1 INTRODUCTION

The seismic response of tall or heavy structures, such as bridge piers, is strongly influenced by the dynamic interaction with the soil-foundation system, which modifies both the seismic input, with respect to the free-field condition (kinematic interaction), and the deformability and dissipative capacity of the structure itself (inertial interaction).

Under the hypothesis of linear behaviour, it is possible to decouple these two phenomena and to study the inertial interaction problem in the frequency domain. In this case, the structure is modelled as a single or multiple degree-of-freedom system, connected with the ground by means of springs and dashpots that define the impedance of the foundation system (Wolf, 1985).

For a pile group, the elements of the impedance matrix can be calculated by numerical methods, such as FEM or BEM, or simplified analytical methods. In the latter case, the impedance of the single pile is evaluated by the Beam on Dynamic Winkler Foundation method (BDWF), while the interaction between piles is evaluated by exploiting the wave propagation theory (Novak, 1991).

While analytical methods could be applied also to the case of layered soil deposits, the existing literature provides comparison between the results of numerical and analytical methods only in the case of constant or linearly increasing soil stiffness (e.g. Makris & Gazetas, 1992, Mylonakis & Gazetas, 1998).

In order to fill this gap, in this work, theoretical results are compared to those of a numerical FEM model developed in ABAQUS (Abaqus, 2017), with reference to the case study described by Conti et al. (2020) of a pile group foundation for a bridge pier embedded in a layered deposit.

## 2 SIMPLIFIED METHOD

### 2.1 *Dynamic impedances of the single pile*

In BDWF method (Novak, 1974), the single pile is modelled as a Euler beam connected to a rigid support by means of a bed of distributed springs and dashpots, acting both in the vertical and horizontal direction, representing the stiffness and the dissipative capacity of the soil. A scheme of this model, along with the adopted sign convention, is reported in Figure 1.

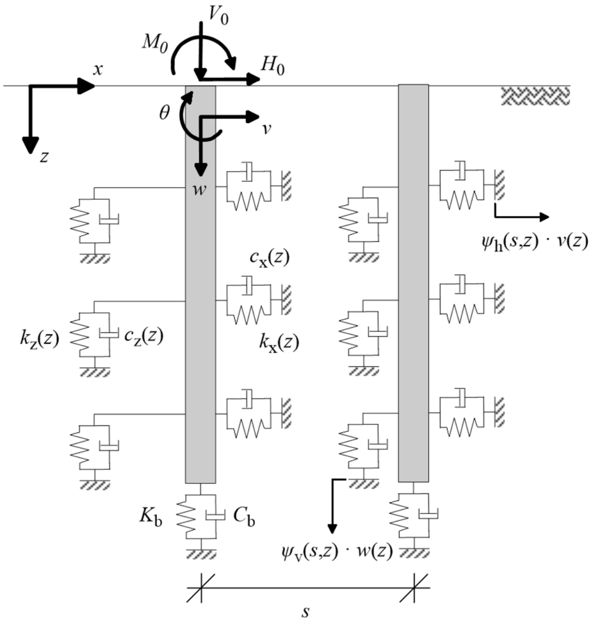

Figure 1. Outline of the BDWF method and sign convention.

Combining the dynamic equilibrium of a pile element of infinitesimal length in the horizontal direction and the constitutive law of the Euler beam, it is obtained:

$$E_pI_p\frac{\partial^4 v}{\partial z^4}+k_xv+c_x\frac{\partial v}{\partial t}+m\frac{\partial^2 v}{\partial t^2}=0 \tag{1}$$

where $E_p$ = pile Young modulus, $I_p$ = pile moment of inertia, $k_x$ = horizontal distributed stiffness, $c_x$ = horizontal damping coefficient, $m$ = pile mass per unit length, $v$ = horizontal pile displacement.

Using the Fourier transform, Equation (1) can be rewritten in the frequency domain as:

$$E_pI_p\frac{\partial^4 v^*}{\partial z^4}+(k_x+ic_x\omega-m\omega^2)v^*=0 \tag{2}$$

where $\omega$ = circular frequency, $i$ = imaginary unit, $v^*$ = Fourier transform of $v(z,t)$.

In the same way, the equilibrium equation along the vertical direction yields:

$$E_pA_p\frac{\partial^2 w}{\partial z^2} + k_z w + c_z\frac{\partial w}{\partial t} + m\frac{\partial^2 w}{\partial t^2} = 0 \tag{3}$$

where $A_p$ = pile cross-section area, $k_z$ = vertical distributed stiffness, $c_z$ = vertical damping coefficient, $w$ = vertical pile displacement. Equation (3) can be rewritten in the frequency domain as:

$$E_pA_p\frac{\partial^2 w^*}{\partial z^2} + (k_z + ic_z\omega - m\omega^2)w^* = 0 \tag{4}$$

where $w^*$ = Fourier transform of $w(z,t)$. In Equations (2) and (4), it is possible to take into account viscous damping in the pile, multiplying $E_p$ by $(1+2i\xi_p)$, where $\xi_p$ = pile damping ratio.

Static boundary conditions (i.e. defined through internal forces) at the pile head and tip are imposed to solve the ODEs defined by Equation (2) and (4). The BCs for Equation (2) are:

$$\begin{cases} T^*(0) = -E_pI_P\dfrac{\partial^3 v^*}{\partial z^3}(0) = -H_0^* \\ M^*(0) = -E_pI_p\dfrac{\partial^2 v^*}{\partial z^2}(0) = -M_0^* \\ T^*(L) = -E_pI_p\dfrac{\partial^3 v^*}{\partial z^3}(L) = 0 \\ M^*(L) = -E_pI_p\dfrac{\partial^2 v^*}{\partial z^2}(L) = 0 \end{cases} \tag{5}$$

where $L$ = pile length, $T^*$ = Fourier transform of shear force, $M^*$ = Fourier transform of bending moment, $H^*_0$ = Fourier transform of horizontal load at pile head, $M^*_0$ = Fourier transform of moment load at pile head.

In the same way, the BCs for Equation (4) are:

$$\begin{cases} N^*(0) = E_pA_p\dfrac{\partial^2 w^*}{\partial z^2}(0) = -V_0^* \\ N^*(L) = E_pA_p\dfrac{\partial^2 w^*}{\partial z^2}(L) = -(K_b + iC_b\omega)w^*(L) \end{cases} \tag{6}$$

where $N^*$ = Fourier transform of axial force, $V^*_0$ = Fourier transform of vertical load at pile head, $K_b$ = stiffness at pile tip, $C_b$ = damping coefficient at pile tip.

The solutions of Equations (2) and (4) are the pile horizontal, $v$, and vertical, $w$, displacement and the pile rotation $\theta$, defined as:

$$\theta = \frac{\partial v}{\partial z} \tag{7}$$

The pile head displacements can be related to the forces applied at pile head, by means of the single pile impedance matrix, defined as:

$$\begin{pmatrix} w^*(0,\omega) \\ v^*(0,\omega) \\ \theta^*(0,\omega) \end{pmatrix} = \begin{pmatrix} K_z & 0 & 0 \\ 0 & K_x & K_{xr} \\ 0 & K_{xr} & K_r \end{pmatrix} \begin{pmatrix} V_0^*(0,\omega) \\ H_0^*(0,\omega) \\ M_0^*(0,\omega) \end{pmatrix} \tag{8}$$

where $K_z$ = vertical impedance, $K_x$ = horizontal impedance, $K_{xr}$ = cross-coupled impedance, $K_r$ = rocking impedance.

Symmetry of the impedance matrix stems from the hypothesis of linear viscoelastic behaviour of both pile and soil, while the presence of zero-value elements is due to the decoupling of Equations (2) and (4).

Values of the stiffness ($k_x$, $k_z$) and damping ($c_x$, $c_z$) of the distributed springs and dashpots are calibrated against the numerical solutions obtained for the case of a single pile embedded in a homogenous half-space or in a homogeneous layer overlying a rigid base. These solutions relate the distributed stiffness and damping to the soil Young modulus $E_s$, Poisson ratio $\nu_s$, damping ratio $\xi_s$ and density $\rho_s$.

In the case of a pile embedded in a layered soil deposit, the distributed stiffness and damping for each layer can be calibrated using the solutions for a half-space and the properties of the corresponding soil layer (Mylonakis, 1995).

### 2.2 *Dynamic interaction factors between piles*

Interaction effects between piles modifies the overall stiffness of the pile group, due to the additional displacement imposed by a loaded pile (source pile) to the others (receiver piles). This can be expressed through interaction factor as:

$$u_k(0) = u_{kk}(0) + \alpha_{jk}u_{jj}(0) \tag{9}$$

where $u_k$ = generic displacement at the head of pile $k$, $u_{kk}$ = generic displacement at the head of pile $k$ due to his own load, $\alpha_{jk}$ = interaction factor between pile $j$ and $k$, $u_{jj}$ = generic displacement at the head of pile $j$ due to his own load. Hence,

$$u_{kj}(0) = \alpha_{jk}u_{jj}(0) \tag{10}$$

is the displacement imposed by the source pile to the unloaded receiver pile.

Interaction factors can be evaluated in the framework of the BDWF method, considering that the base of the distributed springs and dashpots of the receiver pile are displaced by a fraction $\psi$ (attenuation function) of the corresponding displacement of the source pile at the same depth, as outlined in Figure 1. Therefore, the equilibrium of an unloaded receiver pile in the horizontal direction in the frequency domain yields:

$$\begin{aligned} E_pI_p\frac{\partial^4 v^*_{jk}}{\partial z^4} + (k_x + ic_x\omega - m\omega^2)v^*_{jk} \\ = -(k_x + ic_x\omega)\psi_h v^*_{jj} \end{aligned} \tag{11}$$

since, for an unloaded receiver pile $u_{kk} = u_{jk}$ and where $\psi_h$ = horizontal attenuation function. Similarly, the equilibrium equation along the vertical direction takes the form:

$$\begin{aligned} E_pA_p\frac{\partial^2 w^*_{jk}}{\partial z^2} + (k_z + ic_z\omega - m\omega^2)w^*_{jk} \\ = -(k_z + ic_z\omega)\psi_v w^*_{jj} \end{aligned} \tag{12}$$

where $\psi_v$ = vertical attenuation function.

From Equation (10), imposing the boundary conditions of no loads at the head of the receiver pile and a unit displacement at the head of the source pile, the solutions of Equations (11) and (12) in terms of $v_{jk}$ and $w_{jk}$, respectively, at $z = 0$, are the interaction factors. Therefore, the boundary conditions for Equation (11) are:

$$\begin{cases} T_k^*(0) = -E_pI_P\dfrac{\partial^3 v^*_{jk}}{\partial z^3}(0) = 0 \\ M_k^*(0) = -E_pI_p\dfrac{\partial^2 v^*_{jk}}{\partial z^2}(0) = 0 \\ v_{jj}(0) = 1 \\ \theta_{jj}(0) = 0 \\ -E_pI_p\dfrac{\partial^3 v^*_{jj}}{\partial z^3}(L) = 0 \\ -E_pI_p\dfrac{\partial^2 v^*_{jj}}{\partial z^2}(L) = 0 \end{cases} \tag{13}$$

Since displacements v and θ are coupled, Equation (11) allows to find the interaction coefficients relative to the horizontal displacement of the source pile. To evaluate the coefficients relative to the source pile rotation, third and fourth condition in Equation (13) have to be changed as:

$$\begin{cases} v_j(0) = 0 \\ \theta_j(0) = 1 \end{cases} \tag{14}$$

In the same way, boundary conditions for Equation (12) are:

$$\begin{cases} N_k^*(0) = E_p A_P \dfrac{\partial^2 w_{jk}^*}{\partial z^2}(0) = 0 \\ w_{jj}^*(0) = 1 \\ E_p A_p \dfrac{\partial^2 w_{jj}^*}{\partial z^2}(L) = -(K_b + iC_b\omega)w^*(L) \end{cases} \tag{15}$$

### 2.3 *Dynamic impedances of the pile group*

The dynamic impedance matrix of a pile group can be evaluated assuming that the pile heads are connected by a rigid raft not in contact with the soil and by enforcing:

- equilibrium between loads on raft and pile reactions;
- compatibility of the pile heads displacements with the kinematic constraint imposed by the raft.

Moreover, it is possible to consider that for the interaction is valid the superposition method, so the interaction factors derived for a system of two piles are still valid in a wider group.

Under these hypotheses, the impedance matrix of the pile group can be evaluated as (Mylonakis, 1995):

$$K_G^* = T^t E^{-1} T \tag{16}$$

where:

$$T^t = \begin{pmatrix} 1 & 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 1 & -x_1 & \cdots & 0 & 1 & -x_N \\ 0 & 0 & 1 & \cdots & 0 & 0 & 1 \end{pmatrix} \tag{17}$$

$$E = \begin{pmatrix} A_{11} \cdot K_1^{-1} & \cdots & A_{1N} \cdot K_N^{-1} \\ \vdots & \ddots & \vdots \\ A_{N1} \cdot K_1^{-1} & \cdots & A_{NN} \cdot K_N^{-1} \end{pmatrix} \tag{18}$$

$$A_{jk} = \begin{pmatrix} \alpha_{jk}^{ww} & 0 & 0 \\ 0 & \alpha_{jk}^{vv} & \alpha_{jk}^{v\theta} \\ 0 & \alpha_{jk}^{\theta v} & \alpha_{jk}^{\theta\theta} \end{pmatrix} \tag{19}$$

and $x_j$ = j-th pile coordinate along an axis parallel to the horizontal load and with reference system centred on the pile group barycentre; $K_j$ = j-th pile impedance matrix. For the interaction factors, the subscript indicates the source and receiver pile, while the superscript indicates the linked displacements.

## 3 MODELLING

### 3.1 *Case study*

The adopted case study is the pile group foundation of a highway bridge pier, described by Conti et al. (2020). The geotechnical soil model, outlined in Figure 2, features a sequence of clayey silts (CS) up to a depth of 92 m from the pile heads, where the seismic bedrock (B) is located. This sequence is interrupted by a peat layer (P), at a depth of 6 – 8 m, and a silty sand layer (SS), at a depth of 16 – 21 m.

Table 1 summarises the physical and mechanical properties for each layer, in terms of: small strain shear stiffness $G_0$, mobilized shear stiffness $G_s$, damping ratio $\xi_s$ and density $\rho_s$. The mobilized soil shear stiffness $G_s$ and damping ratio $\xi_s$ were evaluated using 1D linear equivalent site response analyses, considering 7 earthquakes, following the Italian Building Code (NTC 2018). For the Poisson ratio, a constant value of $\nu_s = 0.3$ was adopted for each layer. Further details on geotechnical characterization and site response analyses are available in the paper by Conti et al. (2020).

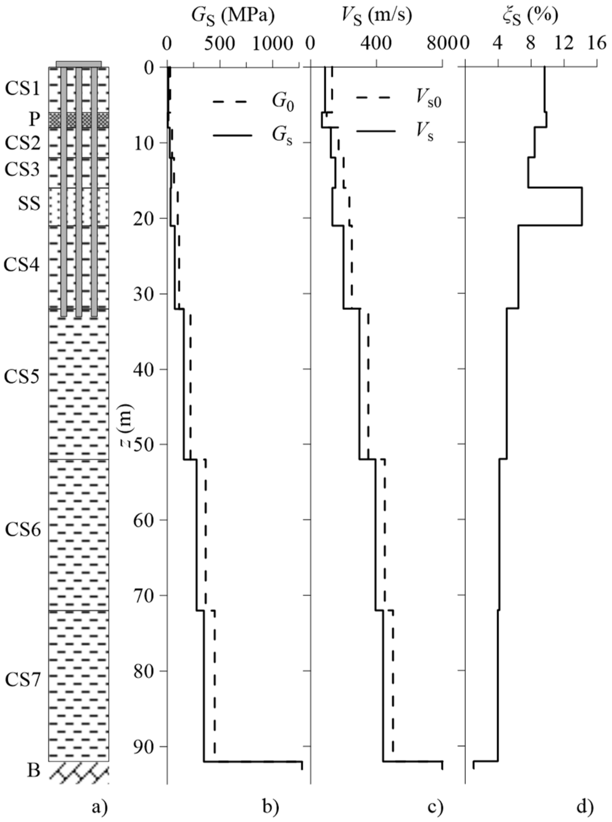

Figure 2. Geotechnical soil model adopted: a) geometric outline, b) shear stiffness, c) shear wave velocity, d) damping ratio.

Table 1. Physical and mechanical properties of soil layers.

| Layer | $G_0$ (MPa) | $G_s$ (MPa) | $\xi_s$ (%) | $\rho_s$ (kg/m³) |
|---|---|---|---|---|
| CS1 | 27.0 | 12.0 | 9.65 | 1600 |
| P | 11.3 | 5.6 | 9.86 | 1200 |
| CS2 | 46.2 | 24.0 | 8.43 | 1600 |
| CS3 | 64.0 | 36.2 | 7.65 | 1600 |
| SS | 99.4 | 31.3 | 14.17 | 1800 |
| CS4 | 112.5 | 71.4 | 6.46 | 1800 |
| CS5 | 220.5 | 157.5 | 5.04 | 1800 |
| CS6 | 364.5 | 278.3 | 4.15 | 1800 |
| CS7 | 450.0 | 347.6 | 3.96 | 1800 |
| B | 1280.0 | 1280.0 | 1.05 | 2000 |

The foundation is a 3×3 group of reinforced concrete piles. Each pile has a length $L = 33$ m, diameter $D = 1.2$ m, Young modulus $E_p$ = 30 GPa, Poisson ratio $\nu_p$ = 0.1, density $\rho_p = 2500$ kg/m³ and damping ratio $\xi_p = 0$. The spacing between the piles is $s = 3.6$ m (ratio $s/D = 3$) and pile head is located at the soil surface. The piles are connected by a rigid raft not in contact with the ground and, therefore, acting only as a kinematic constraint.

### 3.2 *FEM model*

Dynamic impedances of the pile group were evaluated by implementing a FEM model in the frequency domain, using the commercial code ABAQUS (ABAQUS, 2017).

Taking advantage of the symmetry of the problem, two different numerical models were analysed. The vertical impedance, $K_z$ was evaluated modelling only one quarter of the domain and applying a vertical load at the centre of the pile group. The other components of the impedance matrix were evaluated modelling half of the domain and applying (at the centre of the pile group) either a horizontal load along the $x$-direction or a moment around the $y$-axis.

Vertical boundaries were located at 36 m from the axis of the most external pile in group, whereas the bottom boundary was located at a depth of 93 m, i.e. 1 m inside the bedrock layer. The domain was discretized by means of eight-node 3D elements with reduced integration (C3D8R). The element height is 0.30 m in CS1 layer, 0.25 m in P layer, 0.50 m for the layers between CS2 and CS5, 1 m for layers CS6, CS7 and B. The element length in the horizontal plane increases linearly from 0.3 m up to 2 m.

Horizontal displacements were restrained for the nodes located on the vertical symmetry planes. On the other vertical boundaries and at bottom boundary, eight-node 3D infinite elements with reduced integration (CIN3D8R) were applied. For static conditions ($\omega = 0$), these elements behave like the infinite elements proposed by Zienkiewicz et al. (1983), whereas, for dynamic conditions ($\omega > 0$), they behave like viscous dampers, providing the silent boundary defined by Lysmer & Kuhlemeyer (1969). Nodes located at the pile heads were constrained to a rigid body motion.

No interfaces were defined between the piles and the soil, so perfect bonding was assumed.

The load was applied at the centre of the pile group, at the level of the pile heads, with a frequency between 0 and 25 Hz, representative of the frequency content of real earthquakes, and intensity of 10 kN (forces $V_0$ and $H_0$) or 10 kNm (moment $M_0$).

For the same case study, also a single pile was analysed as reference.

Impedances were evaluated from the calculated displacements at the loaded point, applying Equation (8).

### 3.3 *BDWF model*

Equations (2), (4), (11), (12) are solved exactly, using the transfer matrix approach proposed by Mylonakis (1995).

For the horizontal direction, distributed stiffness and damping are evaluated following Roesset (1980):

$$\begin{cases} k_x = 1.2 \cdot E_s \\ c_x = 5 \cdot \rho_s V_s D \end{cases} \tag{20}$$

where Vs = shear wave velocity.

Instead, for the vertical direction, the formulation by Makris & Gazetas (1993) is used:

$$\begin{cases} k_z = 0.6\left(1 + 0.5\sqrt{\omega D/V_s}\right) \cdot E_s \\ c_z = 1.2\pi\sqrt[4]{V_s/\omega D} \cdot \rho_s V_s D \end{cases} \tag{21}$$

These formulations provide results very close to the numerical ones, in terms of single pile impedances, as shown in Section 4.

Concentrated stiffness and damping coefficient applied at the pile tip are evaluated following Roesset (1980):

$$\begin{cases} K_z = E_s D/(1 - \nu_s^2) \\ C_z = K_b(0.425 \cdot D/V_s + 2\xi_s/\omega) \end{cases} \tag{22}$$

The attenuation function $\psi_h$ by Dobry & Gazetas (1988) is used for the horizontal direction:

$$\psi_h(r,\phi) = \psi_h(r,0) \cdot \cos^2\phi + \psi_h\left(r,\frac{\pi}{2}\right) \cdot \sin^2\phi \tag{23}$$

where $\phi$ = angle between $x$-axis and the line from source pile to receiver pile, and:

$$\begin{cases} \psi_h(r,0) = \sqrt{\dfrac{D}{2s}} \exp\left[-(i + \xi_s)\left(\dfrac{s}{D}\right)\alpha_c \cdot \dfrac{\omega D}{V_s}\right] \\ \psi_h\left(r,\dfrac{\pi}{2}\right) = \sqrt{\dfrac{D}{2s}} \exp\left[-(i + \xi_s)\left(\dfrac{s}{D}\right) \cdot \dfrac{\omega D}{V_s}\right] \end{cases} \tag{24}$$

where $\alpha_c = \pi(1-\nu_s)/3.4$.

For the vertical direction, the attenuation function $\psi_v$ proposed by Mylonakis & Gazetas (1998) is used:

$$\psi_v(r,\phi) = \sqrt{\frac{D}{2s}} \exp\left(-(i + \xi_s)\left(\frac{s}{D} - \frac{1}{2}\right) \cdot \frac{\omega D}{V_s}\right) \tag{25}$$

As $\psi_h$ and $\psi_v$ depend on $V_s$, which is a function of depth $z$, the attenuation functions will also depend on $z$.

## 4 RESULTS

The numerical results for the single pile model are reported in Figure 3 in terms of horizontal, cross-coupled, rocking and vertical impedance, respectively, together with the results of the BDWF model.

The results of the two methods are in very good agreement in the entire frequency range, both for the real and the imaginary part. Only for $K_v$, the BDWF method slightly overestimates the real part of the impedance. In all cases, the BDWF method is unable to reproduce the decrease in the real part of the impedance and the rapid increase in the imaginary part at a frequency of about 2 Hz, due to resonance effect of the soil deposit, according to Anoyatis & Mylonakis (2012).

In Figure 4 are reported the results of the numerical and BDWF models for the pile group, in terms of horizontal, cross-coupled, rocking and vertical impedance, respectively.

For horizontal and cross-coupled impedances, the results of the two models are in good agreement only for low ($f < 7$ Hz) and high frequencies ($f > 20$ Hz). In the intermediate range, both models provide a peak in the real and imaginary parts, related to the destructive interference of the waves scattering from the piles, but the peak value and the frequencies at which the peak occurs are different. As the BDWF method provides single pile impedances close to those of numerical one, the cause of this inconsistency has to lie in the pile interaction model.

Instead, for the rocking and vertical impedance, the results provided by the two methods are in good agreement in the whole frequency range, albeit some minor differences exist in the case of $K_v$. This can be explained by the fact that the interaction function $\psi_v$ describes better the interference phenomenon. Moreover, the rocking impedance of the pile group depends more on the vertical than the rocking impedance of the single pile.

In all the cases, it can be seen that the soil deposit resonance at 2 Hz has a negligible effect on the pile group impedance. Hence, the fact that the BDWF method cannot describe the modification in the single pile impedance at this frequency does not influence the results for the pile group.

## 5 CONCLUSIONS

In this work, a simplified BDWF method has been implemented and used for the evaluation of the dynamic impedances of a single pile and a pile group embedded in a layered soil deposit, under the hypothesis of linear viscoelastic behaviour of pile and soil. The results have been compared to those of a FEM model implemented in Abaqus.

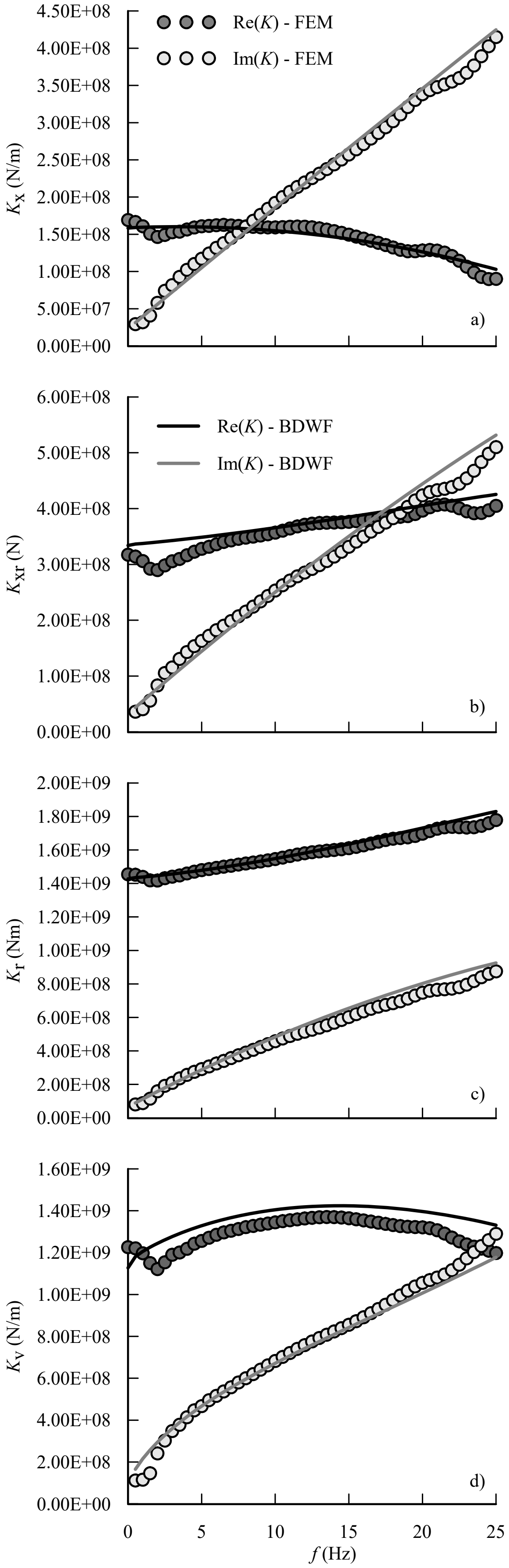

Figure 3. Single pile impedances a) horizontal, b) cross-coupled, c) rocking, d) vertical.

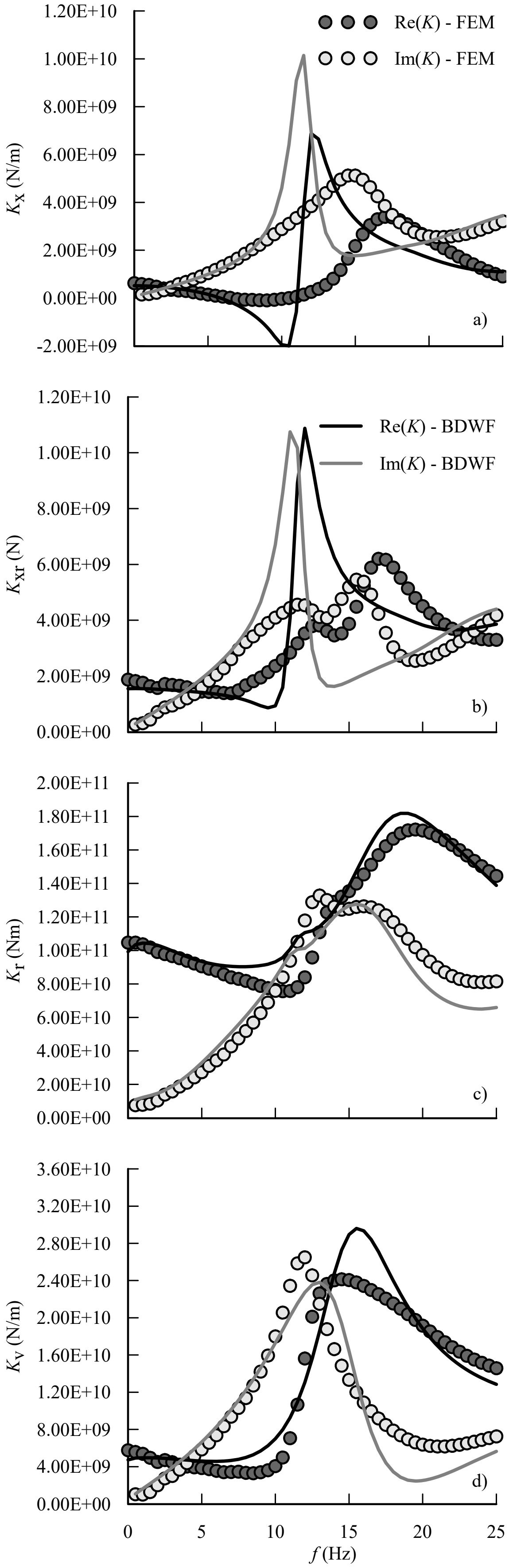

Figure 4. Pile group impedance a) horizontal, b) cross-coupled, c) rocking, d) vertical.

With regard to the single pile, the two methods yield results that are in very good agreement, over the entire frequency range investigated. Minor differences at low frequencies are due to the resonance of the soil deposit, that cannot be reproduced by the simplified method.

For the pile group, instead, the BDWF and the FEM methods agree only in the case of vertical and rotational impedances. For the horizontal and cross-coupled impedances, the two methods provide different results in proximity of the resonance frequency of the group.

This finding is in contrast with the results by Dobry and Gazetas (1988), who found only minor differences in the case of a 3×3 group, embedded in a homogeneous half-space and with spacings 2, 5 and 10 times the pile diameter.

Since the rocking impedance of the pile group is mainly related to the vertical response of the single pile, this leads to the conclusion that the model used to determine the pile-to-pile interaction along the horizontal direction has to be refined.

Moreover, resonance effects within the soil deposit have only a minor influence on the dynamic impedances of the pile group, therefore their inaccurate estimation has no practical influence on the results.

Based on these considerations, the currently available BDWF formulation provides a proper evaluation of the dynamic impedances of single piles and pile groups embedded in layered soil deposits, except for the case of the horizontal and cross-coupled impedances of the pile group close to the resonance frequency.

This work is a preliminary step for the development of simplified methods for evaluating the dynamic impedance matrix of a piled raft foundation embedded in layered soils. In turn, a correct evaluation of dynamic impedances allows for the assessment of dynamic interaction effects on structures subjected to seismic loads.

Further analyses are needed to refine the pile-to-pile interaction model and to verify the applicability of the linearity hypothesis under seismic loads strong enough to make the soil behaviour markedly non-linear.

## 6 REFERENCES

Anoyatis, G., and Mylonakis, G., 2012. Dynamic Winkler modulus for axially loaded piles. *Gèotechnique*. 62(6), 521-536.

Conti, R., *et al*, 2020. Seismic performance of bridge piers: caissons vs pile foundations. *Soil Dynamics and Earthquake Engineering*, 130 (2020), 105985.

ABAQUS, 2017. *Abaqus Analysis User's manual*. Dassault Systèmes Simulia Corp.

Dobry, R., and Gazetas, G., 1988. Simple method for dynamic stiffness and damping of floating pile groups. *Géotechnique*, 38(4), 557-574.

Lysmer, J., and Kuhlemeyer, R. L., 1969. Finite dynamic model for infinite media, *Journal of the Engineering Mechanics Division*, 95(4), 859-877.

Makris, N., and Gazetas, G., 1992. Dynamic pile-soil-pile interaction. Part II: lateral and seismic response. *Earthquake Engineering and Structural Dynamics*, 21(2), 145-162.

Mylonakis, G., 1995. *Contributions to static and seismic analysis of piles and pile-supported bridges*. PhD Thesis. Buffalo: State University of New York.

Mylonakis, G., and Gazetas, G., 1998. Vertical vibrations and additional distress of grouped piles in layered soils. *Soils and Foundations*, 38(1), 1-14.

Novak, M., 1974. Dynamic stiffness and damping of piles. Canadian Geotechnical Journal. 11(4), 574-598.

NTC, 2018. *Aggiornamento delle Norme tecniche per le costruzioni*. DM 17-1-2018 (in italian).

Roesset, J. M., 1980. Stiffness and damping coefficients in foundations. In: O'Neil, O., and Dobry, R. (ed). *Dynamic response of pile foundations*. New York: ASCE.

Wolf, J. P., 1985. *Dynamic Soil-Structure Interaction*. Englewood Cliffs: Prentice-Hall.

Zienkiewicz, O. C., *et al*., 1983. A novel boundary infinite element. *International Journal for Numerical Methods in Engineering*, 19(3), 393-404.