

Comparison of 2D-3D finite element numerical models of an embankment on controlled modulus columns (CMC)

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ABSTRACT: Rigid inclusions, among Controlled Modulus Columns (CMC), are increasingly adopted worldwide for ground improvement techniques. They are used to control and reduce settlement as well as increase bearing capacity in soft or loose soils, offering an economical alternative to traditional deep foundation solutions. Rigid inclusions are commonly employed under industrial slabs, road and railway embankments, buildings, tanks and wind turbine foundations. In some countries, such as France and Poland, the rigid inclusions technique has been extensively researched, leading to the implementation of national recommendations. This makes rigid inclusions a reliable solution for soil reinforcement in various geotechnical engineering projects. However, in other regions, this technique is relatively recent and often considered as a soil improvement alternative solution to pile solutions in tender projects and not the conforming solution. Consequently, this raises a couple of questions from clients or consultants, particularly regarding the modelling of CMC projects. One key topic is the modelling of rigid inclusions below embankments projects, where there is often a lack of confidence in simplified two-dimensional numerical (2D) models for validating the ground improvement design (in terms of settlements and stresses within the columns) compared to more realistic three-dimensional (3D) numerical analyses. The purpose of this study is to present different ways to model rigid inclusions and compare them using an example. It reveals that simplified 2D methods or even analytical approaches can yield results similar to those obtained from 3D numerical modelling.

KEYWORDS: Rigid inclusions, numerical methods, finite element model, embankment.

1 INTRODUCTION

Controlled Modulus Columns (CMCs) are rigid inclusions increasingly used in ground improvement to reduce settlement and enhance the bearing capacity of soft soils, particularly beneath embankments. While three-dimensional (3D) numerical modeling provides the most realistic representation of the soil–structure interaction in such systems, it is often complex, time-consuming, and computationally demanding. As a result, two-dimensional (2D) numerical models and analytical approaches are frequently adopted in practice, especially when multiple simulations are required, for example, in sensitivity analyses or preliminary design stages.

In 2D finite element (FE) analysis, rigid inclusions can be represented using various modeling strategies. This study explores several such approaches to simulate the behavior of soft ground reinforced with rigid inclusions beneath embankments. Both local models—focusing on a unit cell beneath the central part of the embankment—and global models—representing the entire embankment—are developed and analyzed. The results from these 2D models, along with those from analytical methods, are compared against 3D numerical simulations to assess their accuracy in predicting ground displacement and stress distribution within the columns.

The objective of this paper is to evaluate the capability of simplified 2D FE models and analytical approaches to accurately represent the behavior of embankments reinforced by rigid inclusions. The study aims to demonstrate their relevance and reliability in the design process, particularly when efficiency and scalability are essential.

2 CONTROLLED MODULUS COLUMNS

2.1 Technique

The Controlled Modulus Column (CMC) technique is a ground improvement method designed to improve the overall mechanical behavior of soft soils by reducing their deformability through the installation of semi-rigid inclusions. Unlike traditional deep foundation systems, CMCs do not bypass compressible soil layers or directly support the full structural load. Instead, they aim to reduce both total and differential settlements by discharging the soil from a part of the load.

These inclusions create a composite system in which the load is shared between the concrete/mortar columns and the surrounding soil. The technique is vibration-free and generates minimal spoil when a displacement installation method is used, thus contributing to cleaner and less disruptive construction sites.

For uniformly loaded structures such as embankments, load transfer is facilitated by a Load Transfer Platform (LTP) positioned between the column heads and the overlying structure. This platform consists of well-compacted granular material, with a typical thickness ranging from 0.3 to 0.8 meters, depending on the type of structure and the underlying soil conditions. The LTP must be compacted in thin layers and exhibit a minimum Young's modulus of 50 MPa (ASIRI recommendations, 2012) to ensure effective load distribution.

The displacement method for installing CMCs is illustrated in Figure 1.

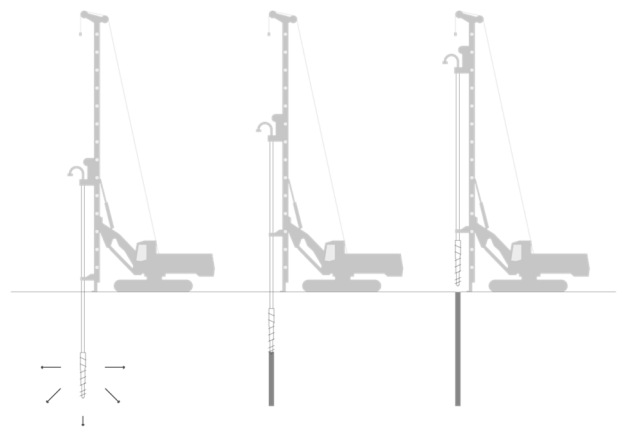


Figure 1. CMC displacement execution method.

A specially designed tool, operated by equipment capable of delivering high torque and static down thrust, is used to laterally displace the soil with minimal vibration and spoil. The tool is advanced to the required depth, increasing the density of the surrounding soil. Upon reaching the target depth or a predefined refusal criterion, a highly workable grout-cement mixture is injected through the hollow auger. As the auger is withdrawn,

the cement mortar/concrete flows under low pressure from its base, forming a high-capacity column suitable for installation near sensitive structures.

2.2 Modeling

2.2.1 FE representation

Rigid inclusions in geotechnical modeling can be represented using four primary types of finite elements, selected based on their geometric characteristics and modeling objectives:

- **Volumetric elements:** These elements capture the full three-dimensional geometry of the inclusion and are defined by continuum mechanics parameters. They are suitable for both 2D and 3D simulations. In the context of 2D Plane Strain (PS) modeling, however, these elements are treated as linear, while the actual inclusions behave as discrete, point-like elements. To accurately reflect the behavior of a network of rigid inclusions, the volumetric elements must be assigned an equivalent Young's modulus E_{Yeq} and an equivalent thickness d_{eq} . These equivalent properties are derived from the actual modulus of the column material, the column radius, and the out-of-plane spacing between inclusions. This homogenization approach ensures that the simplified 2D model captures the global stiffness contribution of the rigid inclusions grid.
- **Line elements:** Appropriate for slender structures where two dimensions are negligible compared to the third, line elements are typically modeled as beams. Their mechanical behavior is characterized by cross section properties (area A and moment of inertia I), and Young's modulus E_Y . These elements are commonly used in 2D PS and 3D models.
- **Surface elements:** Used for thin structures such as plates, surface elements are defined by axial stiffness EA and bending stiffness EI properties. They are particularly suited for 2D PS analyses where one dimension is small relative to the other two.
- **Equivalent soil model:** Suitable for evaluating absolute and differential ground settlements. In this study, it was not adopted, as one of the objectives was to assess the internal forces within the rigid inclusions.

While volumetric elements provide the most accurate representation, the adoption of line or surface elements is primarily motivated by computational efficiency. These simplified representations significantly reduce model complexity while providing results compatible with conventional design methodologies, including internal force diagrams (e.g., vertical force, horizontal force, bending moment). To ensure realistic force transfer and prevent artificial punching effects, the width of line or surface elements at the top—and where applicable, at the bottom—is appropriately adjusted.

2.2.2 Soil-Structure Interaction

Accurate modeling of rigid inclusions in geotechnical structures requires a robust representation of the interaction between the structural elements and the surrounding soil. This interaction is typically formulated through an interaction matrix, which depends on the coupling strategy employed. Two main approaches are mostly used:

- **Interface Elements:** Interface elements provide enhanced flexibility by allowing control over contact conditions and enabling the simulation of non-linear behaviors such as sliding and detachment. Spring-type interfaces, which connect soil and structure nodes via elastoplastic springs, are widely used. A key parameter governing interface behavior is the interface reduction factor R_{int} , (typically

between 0.6 and 1.0), which defines the ratio between the shear strength of the interface and that of the adjacent soil. In this study, they are applied to model lateral friction in both volumetric elements and surface element configurations, with R_{int} set to 1.

- **Indirect Coupling (Embedded Elements):** This approach decouples the structural and soil meshes, allowing structural elements to be embedded within the soil domain. Interaction is modeled via local springs distributed along the element, which relate axial and lateral reactions to relative displacements. This method is computationally efficient and well-suited for 2D PS and 3D models. Adjustments may be necessary to account for head interactions, especially when a Load Transfer Platform (LTP) is present. In this study, indirect coupling is used specifically for line elements.

3 REFERENCE EXAMPLE

A schematic cross-section of the reference configuration used for the numerical analyses is presented in Figure 2.

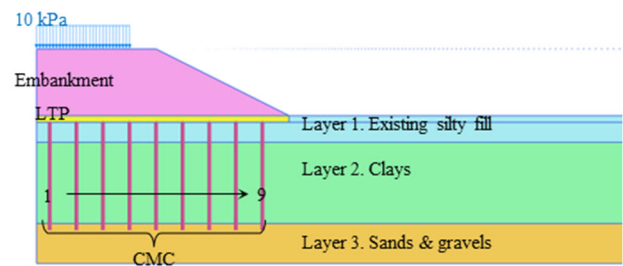


Figure 2. Schematic representation of the modeled embankment profile.

3.1 Geotechnical conditions

The soil stratigraphy consists of three layers, on top of which the embankment fill will be installed:

1. Existing silty fill down to 2 m from Natural Ground Level (NGL);
2. Clays down to 8.1 m / NGL;
3. Sands & gravels beyond.

The assumed geotechnical layer properties and the behavior laws used in numerical models have been summarized in Table 1.

Table 1. Soil profile.

Layer	1	2	3
Bottom depth z_{inf} (m/NGL)	2.0	8.1	11.1
Thickness Δh (m)	2.0	6.1	3.0
Behavior laws	Linear elastic perfectly-plastic		
Failure criterion	Mohr-Coulomb		
Pressuremeter modulus E_M (MPa)	5	4	26
Net limit pressure p_l^* (MPa)	0.50	0.40	2.85
Structural coefficient α	1/2	2/3	1/3
Young's modulus E_Y (MPa)	10	6	78
Poisson's ratio ν	0.3	0.3	0.3
Unit weight γ (kN/m ³)	18	18	18
Friction angle ϕ' (°)	28	24	34
Drained cohesion c' (kPa)	3	5	0
Undrained shear strength s_u (kPa)	75	65	-

The ground water table is located at the surface of layer 2.

3.2 Fill material

The embankment modeled in this study has a total height of 5 meters, with a top width of 9 meters and a base width of 19 meters, corresponding to side slopes of 1 vertical to 2 horizontal (1V:2H). To simulate operational loading conditions, a live load surcharge of 10 kPa is applied at the top of the embankment, positioned 2 meters from the edge, representing typical traffic loads. A 0.5-meter-thick load transfer platform (LTP) is installed above the CMCs network. The LTP is constructed using a dig-and-replace technique. The fill materials are defined using the mechanical parameters and constitutive models summarized in Table 2.

Table 2. Fill parameters.

	Embankment	LTP
Thickness Δh (m)	5	0.5
Behavior laws	Linear elastic perfectly-plastic	
Failure criterion	Mohr-Coulomb	
Young's modulus E_Y (MPa)	40	60
Poisson's ratio ν	0.3	0.3
Unit weight γ (kN/m ³)	18	18
Friction angle ϕ' (°)	35	38
Drained cohesion c' (kPa)	0	0

3.3 Rigid inclusions

The design assumptions for the CMCs are detailed as follows:

- Column diameter: 360 mm
- Pattern configuration: Square grid with 2 m \times 2 m spacing, corresponding to a replacement ratio of approximately 2.5%
- Installation depth: Each column is embedded 0.5 m into the underlying sandy layer, resulting in a total execution depth of 8.6 m /NGL
- Material properties: The mortar used in the rigid inclusions has an unconfined compressive strength of 16 MPa

The parameters adopted for the rigid inclusions are summarized in Table 3.

Table 3. CMC parameters.

Bottom depth z_{mf} (m/NGL)	8.6
Length L (m)	8.1
Diameter D_c (m)	0.36
Behavior laws	Linear elastic perfectly-plastic
Young's modulus E_Y (MPa)	6 800
Poisson's ratio ν	0.2
Unit weight γ (kN/m ³)	22

The rigid inclusions are modeled using four distinct approaches: as volumetric elements in 2D axisymmetric (AXI) and full 3D configurations, as volumetric elements with equivalent properties implemented in 2D PS model; as plate elements and as embedded beam rows (EBR) integrated within a 2D PS model.

3.3.1 2D AXI and 3D volumetric configurations

In both the 2D AXI and 3D volumetric models, the columns are represented as solid elements using the full set of material properties detailed in Table 3.

3.3.2 2D PS volumetric approach

Under the PS assumption, equivalent properties are derived to account for the actual circular geometry and out-of-plane spacing s of the CMC columns, as illustrated in Figure 3.

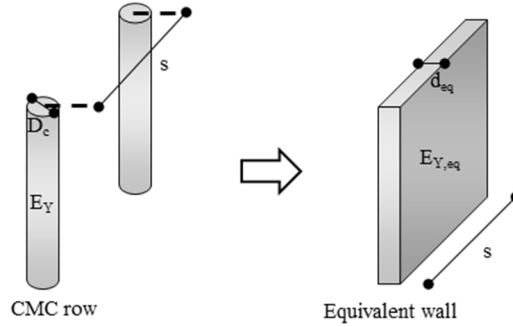


Figure 3. Typical 2D plane-strain conversion model.

In this approach, rows of CMC columns are transformed into an equivalent continuous wall with matching axial stiffness $E.A$ and flexural rigidity $E.I$. The following equations define the conversion:

$$(E \cdot A)_{CMC\ row} = E_Y \times \frac{\pi D_c^2}{4} \quad (1)$$

$$(E \cdot I)_{CMC\ row} = E_Y \times \frac{\pi D_c^4}{64} \quad (2)$$

$$(E \cdot A)_{wall} = E_{Y,eq} \times s \times d_{eq} \quad (3)$$

$$(E \cdot I)_{wall} = E_{Y,eq} \times \frac{s \times d_{eq}^3}{12} \quad (4)$$

Equating the stiffness $E.A$ and rigidity $E.I$ of the CMC row and the equivalent wall

$$(E \cdot A)_{CMC\ row} = (E \cdot A)_{wall} \quad (5)$$

$$(E \cdot I)_{CMC\ row} = (E \cdot I)_{wall} \quad (6)$$

Solving equations (1) to (4) within (5) and (6) yields the equivalent wall thickness and modulus:

$$d_{eq} = \frac{D_c \sqrt{3}}{2} \quad (7)$$

$$E_{Y,eq} = E_Y \times \frac{\pi D_c}{2s\sqrt{3}} \quad (8)$$

The parameters used for the rigid inclusions in this approach are summarized in Table 4.

Table 4. CMC parameters in 2D PS volumetric approach.

Bottom depth z_{mf} (m/NGL)	8.6
Length L (m)	8.1
Thickness d_{eq} (m)	0.312
Behavior laws	Linear elastic perfectly-plastic
Young's modulus $E_{Y,eq}$ (MPa)	1 110
Poisson's ratio ν	0.2
Unit weight γ (kN/m ³)	22

3.3.3 Plate element representation in 2D PS model

This modeling approach, rigid inclusions are represented as an equivalent continuous wall as illustrated in Figure 3 using plate elements with equivalent axial stiffness $(E.A)_{eq}$ and flexural rigidity $(E.I)_{eq}$ expressed per linear meter. These equivalent properties are calculated using the following equations:

$$(E \cdot A)_{eq} = \frac{(E \cdot A)_{CMC \text{ row}}}{s} = E_Y \times \frac{\pi D_c^2}{4s} \quad (9)$$

$$(E \cdot I)_{eq} = E_Y \times \frac{\pi D_c^4}{64s} \quad (10)$$

The parameters adopted for the rigid inclusions in this plate element approach are summarized in Table 5.

Table 5. CMC parameters in 2D PS plate approach.

Bottom depth z_{inf} (m/NGL)	8.6
Length L (m)	8.1
Thickness d_{eq} (m)	0.312
Behavior laws	Linear elastic perfectly-plastic
Axial stiffness $(E \cdot A)_{eq}$ (MN/m)	364
Bending rigidity $(E \cdot I)_{eq}$ (MN.m ² /m)	2.8
Poisson's ratio ν	0.2

3.3.4 EBR in 2D PS model

The rigid inclusions are represented as EBR superimposed on the soil mesh. These are line elements that interact with the surrounding soil through indirect coupling via specialized interface elements. The interaction behavior is governed by elastoplastic laws, accounting for both skin friction along the shaft and tip resistance at the base.

The definition and implementation of EBR parameters follow the guidelines provided in the PLAXIS 2D Reference Manual, including the treatment of interface behavior, stiffness scaling, and bearing capacity formulation.

The parameters adopted for the EBR representation are as follows:

Table 6. CMC parameters in embedded beam row.

Bottom depth z_{inf} (m/NGL)	8.6
Length L (m)	8.1
Diameter D_c (m)	0.36
Out-of-plane spacing s (m)	2
Behavior laws	Linear elastic perfectly-plastic
Young's modulus E_Y (MPa)	6 800
Unit weight γ (kN/m ³)	22
Axial skin resistance	Layer dependent
Lateral resistance	Unlimited
Base resistance F_{max} (kN)	621

4 MULTI-SCALE MODELING OF THE EMBANKMENT-RIGID INCLUSIONS SYSTEM

To comprehensively assess the performance of the ground improvement system, both local and global finite element models are developed and analysed.

All simulations were performed using the widely recognized FE softwares PLAXIS 2D and PLAXIS 3D, which provide robust and accurate tools for modelling complex two-dimensional and three-dimensional soil-structure interactions.

4.1 Model presentation

4.1.1 Local models

The local models focus on a representative unit cell located beneath the central portion of the embankment, allowing for a detailed investigation of the stress distribution, load transfer mechanisms, and soil-column interaction at a finer scale. The various local models, each incorporating a different representation of the CMCs, are illustrated in the Figure 4.

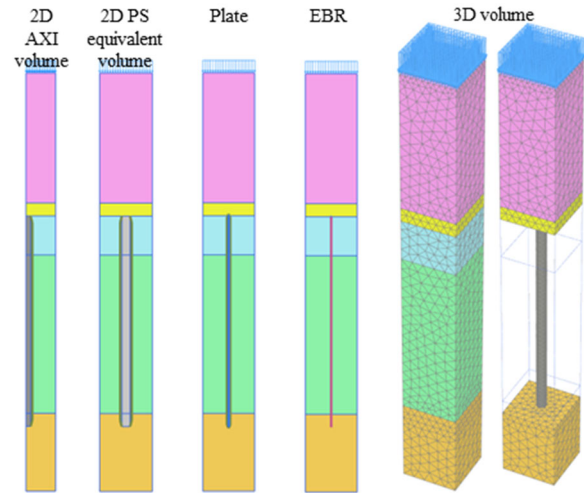


Figure 4. Local FE models with different CMC representations beneath the embankment.

4.1.2 Global models

In contrast, the global models represent the entire embankment structure, including the surrounding ground and boundary conditions, thereby capturing the overall behaviour of the system under operational loading. This approach enables the evaluation of large-scale responses such as total and differential settlements, global stability, and the effectiveness of the rigid inclusions network in reducing deformations. The different global models used in the study are shown in the Figure 5.

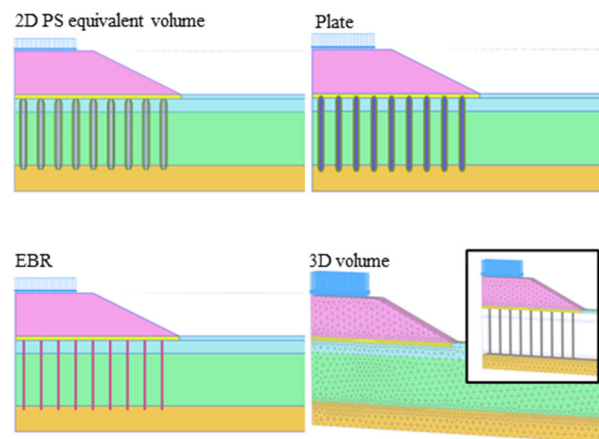


Figure 5. Global FE models with different CMC representations beneath the embankment.

4.2 Results

4.2.1 Local models

The settlement distribution obtained from the local models is shown in Figure 6. Figure 7 compares key parameters across the models, including soil settlements, axial load within the rigid inclusions and skin friction along . These results indicate

that the 3D models produce outcomes consistent with those of the EBR and 2D AXI volume models, confirming their reliability for simulating reinforced ground behavior.

In contrast, the 2D PS equivalent volume and plate models exhibit significant deviations, highlighting their limitations in capturing certain interaction mechanisms, particularly in the mobilization of friction at the interface. The interface in these models appears overly rigid, suggesting that R_{int} should be adjusted to better align with the behavior observed in the 3D model. For the plate model, setting $R_{int} = 0.21$ produces a comparable response. However, such adjustment is not applicable in the 2D PS equivalent volume model due to its inherent limitations.

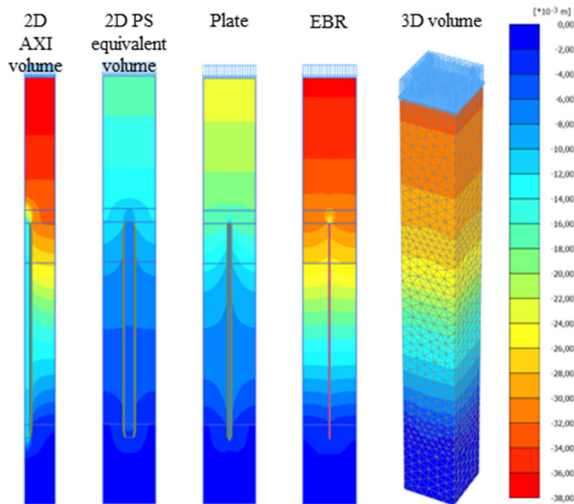


Figure 6. Settlement distribution in local models.

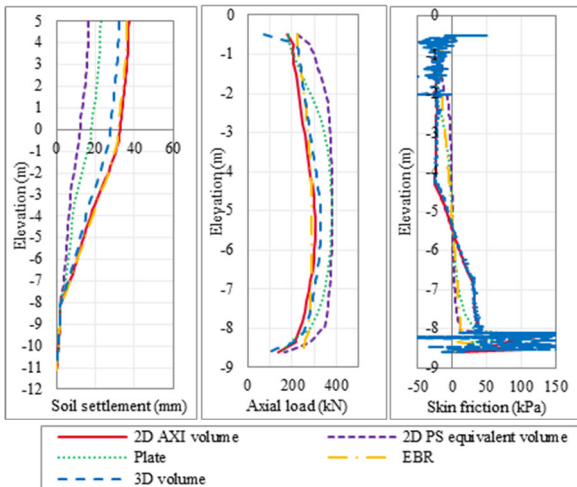


Figure 7. Comparison of soil settlement, axial load in CMCs and skin friction profiles.

4.2.2 Comparison with analytical approach

The ASIRI (2012) recommendations propose simplified analytical methods for typical reinforced ground configurations, such as grids of rigid inclusions subjected to uniform vertical loads in the central section of an embankment.

This study uses the MV2 model, a biphasic analytical approach that distinguishes between the CMC domain (rigid inclusions) and the SOIL domain. The iterative calculation begins with an assumed load distribution, followed by successive adjustments to achieve stress equilibrium beneath the embankment.

Key principles governing the interaction between the two domains include:

- Frank and Zhao's interface behavior laws for SOIL-CMC interaction.
- Combarieu's fictitious inclusion method (1988) for evaluating negative skin friction and modeling load transfer above the inclusion heads.

Figure 8 illustrates the analytical results, which closely align with those obtained from the 2D AXI volume. Analytical models like MV2 offer quick and reliable insights for designing local configurations. They are sufficiently accurate, simpler to implement, and useful for validating numerical simulations.

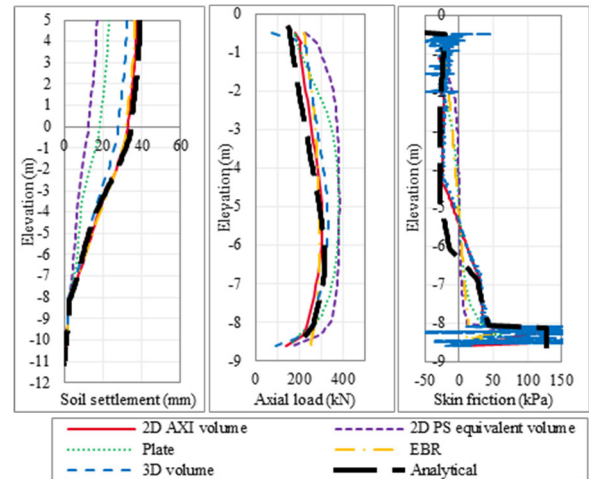


Figure 8. Comparison of soil settlement, axial load in CMCs and skin friction profiles including analytical calculation.

4.2.3 Global models

The settlement and lateral deformation distributions obtained from the global models are presented in Figure 9 and in Figure 10, respectively.

Figure 11 compares model outputs, including surface settlements of the embankment, horizontal displacement at embankment base, and axial forces and bending moments within rigid inclusions 1, 5 and 9.

The results show that all models produce values of the same order of magnitude. Regarding settlement, the 2P PS equivalent volume model predicts lower values compared to the 3D model, while the plate and EBR models yield slightly higher values. For horizontal displacements, the 3D model provides the lowest values, indicating that the 2D approaches are conservative in estimating lateral movement. Unlike the local models, no adjustment of the R_{int} parameters is required for the 2P PS equivalent volume and plate models in the global configuration.

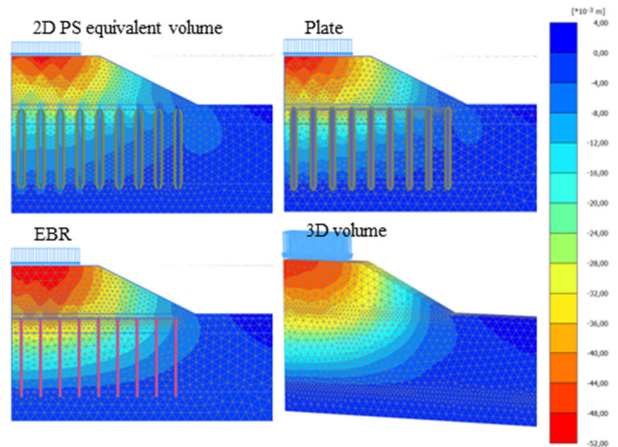


Figure 9. Settlement distribution in global models.

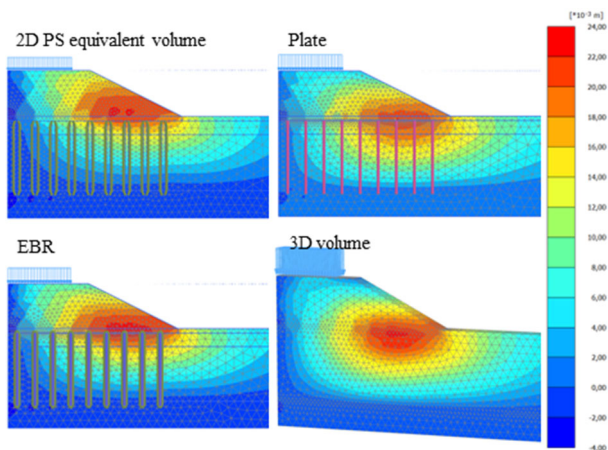


Figure 10. Horizontal displacement distribution in global models.

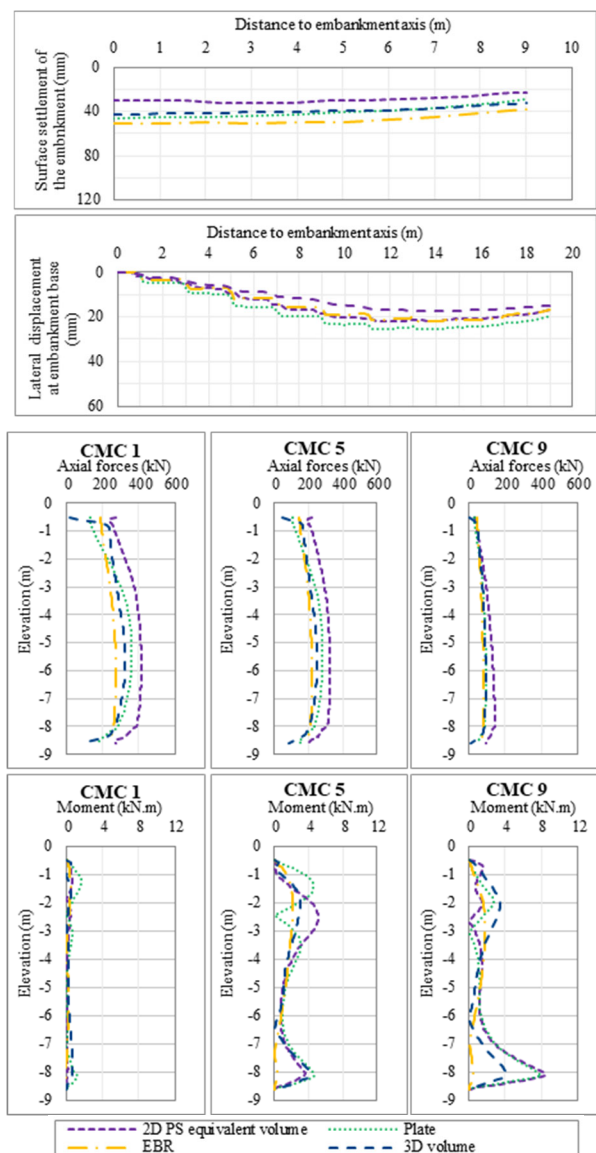


Figure 11. Comparison of displacements, axial force and bending moments within the rigid inclusions.

Regarding internal forces within the rigid inclusions, both the 2D PS equivalent volume and plate models tend to overestimate axial forces and bending moments compared to the 3D model. In contrast, the EBR model provides comparable results for axial loads but underestimates bending moments, particularly at

the interface with the anchoring layer. This limitation could be addressed by adjusting the lateral stiffness factor within the EBR model.

5 CONCLUSIONS

The comparative evaluation of CMC modeling approaches highlights the specific strengths and limitations of each configuration.

The 3D model offers the most realistic representation of reinforced ground behavior, offering accurate predictions of settlements, horizontal displacements, and internal forces. It serves as a reliable reference for validating other modeling approaches.

The 2D AXI volume model also performs well, closely matching the 3D results in local configuration while being significantly more efficient computationally.

The 2D PS equivalent volume model cannot be calibrated against axisymmetrical or 3D models due to its simplified geometry, assumptions and wall-like behavior. To align with the global 3D model, interface reduction factors R_{int} close to 1 seem to be required. This model tends to underestimate settlements—more significantly in local models, and slightly in global ones while overestimating lateral displacements. Additionally, retrieving internal forces within the rigid inclusions is difficult and may require the addition of a fictitious plate element inside the columns. For these reasons, this element is not the most well-suited for accurately representing rigid inclusions.

The Plate model requires different R_{int} values depending on the calibration target with lower values needed for elementary cell calibration and higher values (0.85-1) for global model calibration. The wall-type element used in this model introduces discontinuities in soil displacement on either side of the plate. Despite this, the plate model is appropriate for representing rigid inclusions in axisymmetric global configurations, such as tank foundations.

The EBR model shows good agreement with both elementary and global 3D models. It provides consistent predictions for settlements and internal forces, making it the most suitable element for representing rigid inclusions when properly defined.

Analytical models, such as MV2, offer a fast and reliable alternative for local configurations. They provide sufficiently accurate results for preliminary design and standard cases, while being simple to implement and computationally efficient. Their ability to replicate key behaviors observed in numerical models also makes them useful for validation purposes.

6 REFERENCES

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