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Discovering the shape of jet-grouting columns by thermal back analysis – application of ANN surrogate models

Marek Wojciechowski
Division of Geotechnics and Engineering Structures, Lodz University of Technology, Poland, mwojc@p.lodz.pl

ABSTRACT: Jet grouting is a widely used ground improvement technique, but the geometry of the resulting columns often remains uncertain due to the ground variability and complexity of the injection process. This study investigates a novel methodology for identifying the shape of jet-grouted columns by analyzing their thermal response during cement hydration. In this approach, a transient heat conduction model is formulated and repeatedly solved using the finite element method (FEM) to determine the column shape that best matches field temperature measurements. To overcome the high computational cost of repeated FEM simulations in the identification process, an artificial neural network (ANN) surrogate model is developed. The ANN is trained on a dataset of temperature profiles corresponding to various column geometries and enables rapid, differentiable temperature predictions. This facilitates efficient gradient-based optimization in the shape identification process and allows for incorporation of Monte Carlo analysis to account for measurement uncertainty. The proposed approach significantly reduces computational time – by several orders of magnitude – compared to the FEM-based identification method, while maintaining high accuracy in identifying column shapes. These results demonstrate the potential of ANN surrogate models as a powerful tool for data-driven geotechnical analysis.

KEYWORDS: jet-grouted column, heat transfer, inverse problem, surrogate model, ANN.

1 INTRODUCTION

Jet grouting is a widely used ground improvement technique that involves the injection of high-pressure fluids into the subsoil to form rigid columns. Due to the inherent variability in soil properties and the complexity of the jet grouting process, achieving a column with a precisely defined diameter is not feasible. Consequently, verifying the actual geometry of the constructed columns is a critical component of quality assurance. Among various indirect verification methods (Croce, Flora and Modoni, 2014; Bearce, Mooney and Kessouri, 2015; Guerreros et al., 2016), one promising approach involves analyzing the thermal response of a freshly formed column. The exothermic nature of cement hydration leads to heat generation, which increases the temperature in the column. By employing a numerical model of transient heat conduction, it is possible to formulate an inverse problem: determining the column geometry that best reproduces the observed temperature evolution. This methodology was previously applied by Adam et al. (2010) to estimate the diameter of the column at the representative depth, and by Wojciechowski (2023) to discover the shape of a column with axial variation. However, the approach requires repeated solutions of an axisymmetric initial-boundary value problem (I-BVP) using finite element method (FEM), which is computationally intensive. Each iteration of the shape optimization process necessitates the generation of a new finite element mesh and a complete thermal simulation.

To mitigate the computational burden, the present study proposes a surrogate modeling approach (Asher et al., 2015; Alizadeh, Allen and Mistree, 2020; Kudela and Matousek, 2022) based on artificial neural networks (ANNs). This method replaces the repeated FEM simulations with a trained ANN that approximates the thermal behavior of columns with varying geometries. The ANN is trained on a dataset generated through the pre-processed numerical simulations, capturing the temperature data for a range of different column shapes. Once trained, the ANN surrogate model enables instantaneous evaluation of temperature profiles, greatly accelerating the column shape identification process and facilitating statistical treatment of measurement uncertainties through Monte Carlo simulations.

The following sections introduce the heat conduction model used in this study and describe the generation of training data, the learning process of the artificial neural network, and the column shape identification procedure utilizing the surrogate model. Computational effort metrics are provided for each stage of ANN development and application. The results show that the proposed methodology is several orders of magnitude faster than the conventional FEM-based shape identification process.

2 I-BVP OF HEAT CONDUCTION

The central point of the proposed methodology is a theoretical model of heat conduction in the surrounding of the freshly executed column. The geometrical setup is presented in Figure 1a) – the vertical, axisymmetric jet-grouted columns with smooth side surfaces are considered here.

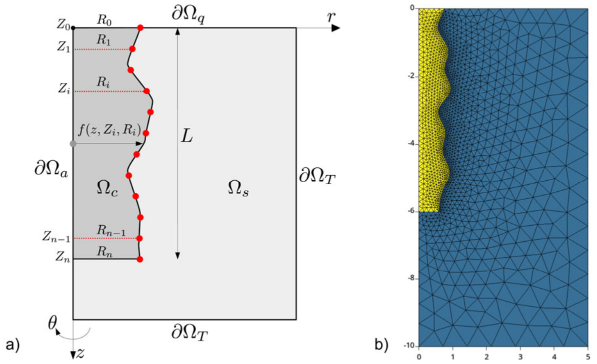

Figure 1. Two-dimensional axisymmetric model of the jet-grouted column: a) definitions for the initial-boundary value problem of heat conduction; b) example of the finite element discretization.

The physics of the problem is ruled by the following set of equations:

$$\rho c_p \dot{T} = -q_{i,i} + h \quad \text{on} \quad \Omega = \Omega_s + \Omega_c \tag{1}$$

$$q_i = -kT_{,i} \quad \text{on} \quad \Omega = \Omega_s + \Omega_c \tag{2}$$

$$h = \begin{cases} 0 & \text{on} \quad \Omega_s \\ h_c & \text{on} \quad \Omega_c \end{cases} \tag{3}$$

$$q_i n_i = \begin{cases} 0 & \text{on} \quad \partial\Omega_a \\ \kappa(T - T_\infty) & \text{on} \quad \partial\Omega_q \end{cases} \tag{4}$$

$$T = T_0 \quad \text{on} \quad \partial\Omega_T \tag{5}$$

$$t = 0\text{: } T = T_0 \quad \text{on} \quad \Omega \tag{6}$$

Above, the symbol ρ denotes the material density, c_p represents the specific heat capacity, and k is the isotropic thermal conductivity. The temperature field is denoted by T, while q_i refers to the heat flux vector. The term h corresponds to the volumetric heat source, which in this context arises from the exothermic chemical reactions associated with cement hydration. The unit normal vector to the boundary is indicated by n_i. The derivative of temperature with respect to time t is expressed as $\dot{T}$, and spatial derivatives are represented using index notation with commas. At the external boundary, heat transfer is modeled using a heat exchange coefficient κ and the ambient air temperature T_∞. Initial temperature distribution (at time $t = 0$) is denoted as T_0. Equations from (1) through (6) are further rewritten in weak (integral) form and finite element method is applied to find solution, that is a temperature evolution throughout time and domain Ω. Fine meshes with triangular elements and linear shape functions have been utilized. Figure 1b) shows example finite element discretization used in computations.

The presented axisymmetric heat transfer problem has been implemented in the framework of *fempy* project (Wojciechowski, 2013). Meshes used in computations were generated using the *gmsh* software (Geuzaine and Remacle, 2009), and for time integration, the backward differentiation formula (BDF) implementation from *scipy* package was utilized (Jones et al., 2023).

3 DATABASE CREATION

The primary objective of the present study is to develop an ANN surrogate model that replicates FEM simulations for predicting temperature distributions inside the jet-grouted columns with predefined geometries. Thus, a database which relates column shape with temperature profiles is necessary for training the ANN model.

In this work, it is assumed, that the side surface of the column of length L can be approximated by a cubic spline revolved around the column axis. This spline is uniquely determined by the set of radii R_i, specified at the corresponding, evenly spaced depths Z_i, with $i = 1 \dots N$. It is further assumed that, for the purpose of identifying the column shape, it is sufficient to consider temperature measurements taken along the column axis at a single time point t following column installation. Accordingly, the dataset used to train the artificial neural network (ANN) will consist of input-target pairs of the form:

$$\{(z_j, R_{i;k}), T_{j;k}\} \tag{7}$$

where z_j denotes the depths at which axial temperatures $T_{j;k}$ are recorded, with $j = 1 \dots M$ and $k = 1 \dots K$. For K different realizations of the column shape $R_{i;k}$, the total number of entries in the training dataset is $K \times M$. Databases for validating and testing the trained neural network can be created in the same way.

4 ANN SURROGATE MODEL

In this study, the ANN surrogate model is implemented as a fully connected multi-layered feed-forward neural network, taking at input depth z and components of vector R_i, and returning at output temperatures $\tilde{T}$ approximating results generated by FEM solutions (see Figure 2). The choice of feed-forward architecture is motivated by the well-established universal approximation property of such models for multivariate functions (Hornik, Stinchcombe and White, 1990). ANN models with identity transfer functions at input and output layers and non-linear *softplus* activation function of the form $g(x) = ln(1 + e^x)$ at two hidden layers have been considered. The output of the network is therefore a continuous function and differentiable with respect to the input variables.

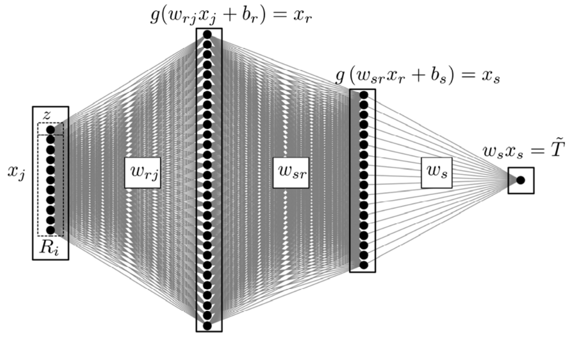

Figure 2. Architecture of the ANN surrogate model.

Approximation realized by the ANN model can be expressed as $\tilde{T} = \tilde{T}(z, R_i, \mathbf{w})$, where value $\mathbf{w} = [w_{rj}, w_{sr}, w_s, b_r, b_s]$ denotes parameters of the network, which are weights of connections and biases of the hidden nodes. Parameters are initialized randomly at network creation. Training the model involves searching for the components of $\mathbf{w}$ that minimize the mean squared error (MSE) computed over the training dataset:

$$E(\mathbf{w}) = \frac{1}{K \cdot M} \sum_{k=1}^{K} \sum_{j=1}^{M} \left[\tilde{T}(z_j, R_{i;k}, \mathbf{w}) - T_{j;k}\right]^2 \tag{8}$$

In this study minimization of cost function (8) is performed using efficient quasi-Newton BFGS method (Nocedal and Wright, 2006; Jones et al., 2023). During training the mean square error for the validation database is also computed at every BFGS iteration. To avoid model overfitting, training is terminated if the validation MSE does not decrease over a predefined number of iterations. Additionally, for achieving best learning results, the multi-start method is used. Once trained, the dependence of the ANN on parameters $\mathbf{w}$ is dropped and the model is expressed simply by $\tilde{T} = \tilde{T}(z, R_i)$.

In this study, the ANN model is implemented using *spinn* package designed for simple training of physics-informed artificial neural networks (Wojciechowski, 2012; 2024). This package relies on the famous *JaX* framework for numerical computations with automatic differentiation and just-in-time compilation on different devices (Bradbury et al., 2018).

5 SHAPE IDENTIFICATION PROCEDURE

The shape identification task is formulated as the following optimization problem:

$$R_i^{\mathrm{opt}} = \arg\min_{R_i \in \Pi} \left[\frac{1}{P} \sum_{p=1}^{P} \left(\tilde{T}(z_p^\star, R_i) - T_p^\star\right)^2\right] \tag{9}$$

where P denotes the number of temperature measurements taken along the column axis at a specific time t after column installation – the same time used to generate the training dataset. The objective is to determine the set of radii R_i that minimizes the mean squared error between the ANN-predicted temperatures $\tilde{T}$ and the observed temperatures $T_p^\star$ at the corresponding depths $z_p^\star$. The search domain Π is defined here by simple bound constraints $R_i^{\mathrm{min}} \leq R_i \leq R_i^{\mathrm{max}}$. One note, that because of the differentiability of ANN surrogate model $\tilde{T}(z, R_i)$ this optimization problem can be solved using gradient optimization techniques. L-BFGS-B method is used in this study – a variant of BFGS algorithm for bound constrained problems. Possibility of using gradient optimization techniques is considered as a huge advantage over the FEM-based identification, where heuristic, evolutionary algorithms have to be utilized.

An important aspect which should be taken into account at the stage of shape identification is the random nature of measurements. In this study, a simple Monte Carlo (MC) analysis is performed to account for the uncertainty associated with the measured temperatures. The temperatures are treated as random variables following a normal distribution $\mathcal{N}$ so that:

$$\mathcal{N}\left(T_p^\star, \sigma_T^2\right) = T_p^\star + \mathcal{N}(0, \sigma_T^2) \quad (10)$$

where σ_T denotes known (or assumed) standard error of the mean $T_p^\star$. Fixed number of realizations Q of this random variable is then generated and optimization problem (9) is solved Q times, resulting in a set of optimized column shapes R_i^{opt}. As a final result of the identification process the mean values $\bar{R}_i^{\text{opt}}$ of the column radii are computed – where the overbar denotes averaging over the Q optimization results – along with the corresponding standard errors σ_{R_i}. It is expected, that identified shape $\bar{R}_i^{\text{opt}}$ closely approximates the reference shape $R_i^\star$, for which the temperature measurements were obtained.

6 IDENTIFICATION EXAMPLE

Let us consider an axisymmetric jet-grouted column with a length of $L = 6$ m, radius at the ground surface equal to 0.75 m and unknown shape below the surface, embedded in a homogeneous soil. The thermal properties of both the grout and the surrounding soil are assumed to be known. The adopted values are provided in Table 1. The heat production rate for the column has been illustrated in Figure 3 – it is based on a simplified hydration model that approximately reflects the thermal behavior of columns containing 500 kg/m³ of Portland cement. These settings qualitatively reflect conditions that may occur in real-world scenarios, which is considered sufficient for demonstrating the identification methodology. However, it is important to emphasize that a realistic setup suitable for actual field conditions can also be applied at this stage (with layered soils of variable conductivity, real initial temperatures, etc.).

Table 1. Adopted thermal parameters of soil and jet-grouted column.

Symbol	Column	Soil	Unit
ρ	2300	1800	kg/m³
c_p	900	1200	J/(kg·K)
k	1.8	0.8	W/(m·K)
h	*Figure 3*	0	W/m³
T_0	293.15	283.15	°K
T_∞	283.15	283.15	°K
κ	8	8	W/(m²·K)

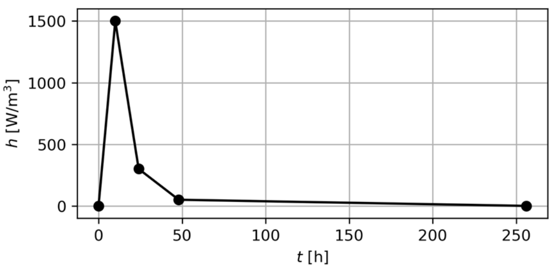

Figure 3. Adopted heat generation rate in a fresh jet-grouted column.

It is further assumed that the column's side surface can be represented by a cubic spline defined by $N = 10$ unknown radii collected in vector R_i, specified at intervals of 0.6 m along the vertical z axis (excluding the surface radius which is predefined). It is also assumed that the radii R_i can take any value from the range $R_i^{\min} = 0.5$ m to $R_i^{\max} = 1.0$ m, with uniform probability.

To create a database for ANN training a number of $K = 100$ different shapes (vectors $R_{i;k}$) were drawn and the relevant I-BVP of heat conduction was solved. Computations were stopped when the modeling time reached $t = 36$ hours. At this time point, the temperatures $T_{j;k}$ at $M = 41$ depths z_j, spaced every 0.15 m along the column axis, were extracted from the FEM model and added to the training dataset as specified by equation (7). Total number of training patterns in the dataset is then equal to $K \times M = 4100$. For validation purposes, a dataset of exactly the same size was generated. Creation of these two databases takes approximately 35 seconds when using *fempy* software and running on a mobile processor Intel i7-12700H in 5 parallel threads.

Once the training and validation databases are established the surrogate ANN model can be learned. A relatively large network 11-30-18-1 has been selected for training, where eleven network inputs are for z coordinate and ten components of R_i vector, and a single output is for the temperature. Figure 4 shows the progress of the BFGS algorithm during training. The final root mean squared error for both, training and validation datasets, achieves a low value around 0.1 °K. It has been verified with some additional tests, that smaller network architectures resulted in higher prediction errors, while larger networks did not yield significantly better accuracy but incurred substantially greater computational costs. Thus, the trained model has been adopted for further computations. Training, using the *spinn* software and the multistart method with five optimization runs, takes approximately 325 seconds in total (65 seconds per run). The process was carried out on a mobile Nvidia RTX A2000 GPU operating in single precision.

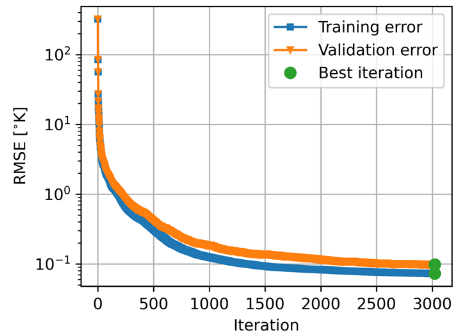

Figure 4. Training of the ANN surrogate model: graphs show decreasing root mean squared error (MSE root) for training and validation datasets.

The final step of the identification procedure involves determining the unknown shape of the column. Since no real field measurements were available for testing, the reference temperature data were generated through an additional FEM simulation. For this purpose, a reference set of radii $R_i^\star$ was randomly selected, and the corresponding temperatures $T_p^\star$ were computed at depths $z_p^\star$ spaced every 0.30 m along the column axis, at time $t = 36$ h. This resulted in $P = 21$ temperature measurements, interpreted as mean values of random variables. Next, a standard error of $\sigma_T = 1$ °K was assumed for these means, and the optimization problem (9) was solved for $Q = 300$ different realizations of the noisy temperature data. The same procedure was repeated for higher standard error of $\sigma_T = 2$ °K. Figure 5 presents the identified column shapes for both noise levels. It can be observed that the average radii $\bar{R}_i^{\text{opt}}$ are closer to the reference radii $R_i^\star$ when the noise is lower. The standard errors σ_{R_i}, shown as grey bands in Figure 5, are also smaller in this case. However, the root mean square error for the shape identified using $\sigma_T = 1$ °K is 0.021 m, while for $\sigma_T = 2$ °K it is 0.047 m. Thus, even with increased measurement uncertainty, the identification process yields reasonable results. From a geotechnical perspective, both results are sufficiently accurate to support a reliable assessment of the jet-grouted column's quality.

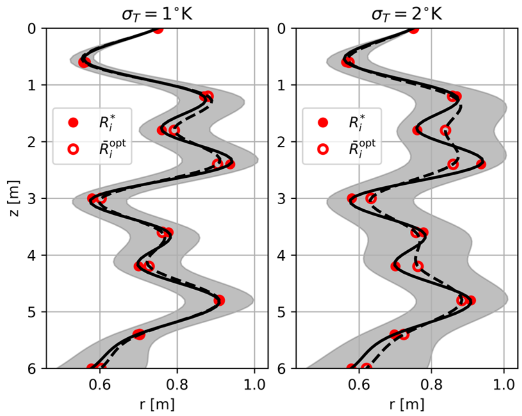

Figure 5. Column shape identification results: solid lines show reference shape constructed with radii $R_i^\star$, dashed lines show identified shapes constructed with radii $\bar{R}_i^{\mathrm{opt}}$ for two noise levels σ_T; grey bands indicate standard errors for identified shapes.

As a final remark, it is worth noting that the computational cost of performing 300 runs of the optimization problem (9) is very low – approximately 1.8 seconds in total, or 0.006 seconds per run. This represents a significant advantage over the FEM-based approach, where a single identification can take between 1.5 and 3 hours on specialized hardware, as reported by Wojciechowski (2023). The total identification time reported in this section is approximately 6 minutes on standard laptop hardware, including database creation and ANN training. Thus, the ANN surrogate model approach remains advantageous even when the identification process is treated as deterministic and is executed only once.

7 CONCLUSIONS

This study demonstrates the successful application of artificial neural networks (ANNs) to the inverse problem of identifying the shape of jet-grouted columns based on thermal measurements. The ANN surrogate model, trained on data generated through finite element method (FEM) simulations, provides accurate and instantaneous predictions of temperature profiles, enabling efficient shape recognition. Differentiability of ANN approximation allows the use of gradient-based optimization techniques, which are significantly faster and more effective than metaheuristic methods used in FEM-based approach. Monte Carlo simulations confirm the robustness of the approach under relatively high levels of measurement noise, yielding reliable estimates of column geometry and associated uncertainties. The total computational time for the entire identification process is reduced from hours to minutes, making the method suitable for practical engineering applications. For such applications, however, realistic thermal properties of the soil and grouted column, as well as accurate initial and boundary conditions, must be incorporated into the simulations.

8 ACKNOWLEDGEMENTS

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