

Static and seismic earth pressures from Boussinesq's equations

Sébastien Burlon

Cerema, Paris, France, sebastien.burlon@cerema.fr

Youssef Abboud, Fahd Cuirra

Terrasol, SETEC Group, Paris, France.

ABSTRACT: The calculation of the active and passive earth pressure coefficients is still a major issue for the design of gravity or embedded walls both for static and seismic conditions. For static conditions, one of the conventional approaches to determine these coefficients in weighted ground conditions is to use the tables from Caquot, Kerisel and Absi (Caquot and Kerisel, 1948, Kerisel and Absi, 1990) which are based on the Boussinesq's theory. In order to illustrate how these differential equations are solved in practice, this paper includes an example dealing with their numerical integration by explaining the different steps that have been followed. The results are compared to those obtained by Caquot, Kerisel and Absi on the one hand and Sokolowski (1965) on the other hand. For seismic conditions, a solving procedure is also proposed and compared to the solutions obtained by applying rotations principles. For both static and seismic conditions, failure mechanisms are determined and compared to the solutions obtained through the kinematical approach of the yield analysis considering log-spirals and rotational mechanisms. The aim of this paper is to revisit a complex problem for which the original formulation is increasingly less highlighted.

KEYWORDS: Active earth pressure coefficient, passive earth pressure coefficient, weighted ground conditions, numerical integration, slip line, seismic design

1 INTRODUCTION

The calculation of the active and passive earth pressure coefficients is still a major issue for the design of gravity or embedded walls both for static and seismic conditions.

Conventionally, the active and passive earth pressures are calculated by applying a principle of superposition which leads to the consideration of three types of coefficients, by means of the corresponding states theorem (Caquot, 1934): an active or passive earth pressure coefficient $K_{a\gamma}$ or $K_{p\gamma}$ for weighted ground conditions, an active or passive earth pressure coefficient K_{aq} or K_{pq} for weightless ground conditions, an active or passive earth pressure coefficient K_{ac} or K_{pc} for cohesive soils.

For a sake of simplification for reading and understanding, the notations from the tables established by Kerisel and Absi in 1990 (Kerisel and Absi, 1990) are used (Figure 1): the angles δ , β and λ designate respectively the stress inclination of the active or passive earth pressures on the wall p or b , the slope of the ground surface and the wall inclination. The load applied on the free ground surface is q and its inclination from the normal axis is α (this angle designates more generally the inclination of the stress for any radial straight line starting from the origin O). The ground properties are the weight density γ , its friction angle φ and its cohesion c .

The active and passive earth pressures can be determined for any length l along the wall by the following equations:

$$p = K_{a\gamma}\gamma l + K_{aq}q - K_{ac}c \quad (1)$$

$$b = K_{p\gamma}\gamma l + K_{pq}q + K_{pc}c \quad (2)$$

The coefficients K_{aq} ou K_{pq} are used to calculate the active and passive earth pressures induced by a load q located on the free ground surface and can be determined by a closed-form solution established by Absi and L'Herminier (1962) or Lancelotta (2002). The use of the corresponding states theorem by assuming that the cohesion c is similar to a confining pressure $c/\tan\varphi$ allows to determine the coefficients K_{ac} and K_{pc} (see Annex A).

In static conditions, the coefficients $K_{a\gamma}$ and $K_{p\gamma}$ depend only on the friction angle φ and can be determined through the equations proposed by Boussinesq (1876). Caquot and Kerisel have given a first method to solve these equations leading to the publication of tables in 1948 (Caquot and Kerisel, 1948)

updated in 1966, 1972 and 1990. Other approaches had been developed by Résal (1903) or Ravizé (1945) notably. Some differences, which are negligible in practice, can be identified in the various tables reflecting the uncertainties in the solving process of the differential equations: for example, regarding the passive earth pressure, for a vertical wall ($\lambda=0$), a horizontal ground surface ($\beta=0$), a stress inclination δ equal to $-\varphi$ and a friction angle φ equal to 30° , the following values can be read: 6.42 in 1948, 6.56 in 1966 and 6.50 in 1990. Moreover, these values can be compared to those proposed by Sokolowski (1965) obtained by an application of the slip line field theory (Salençon, 1974, Absi, 1984) and using a finite difference method: for this example, the value is equal to 6.55.

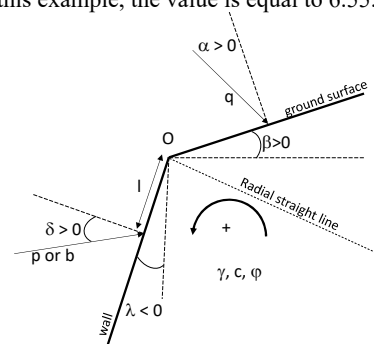


Figure 1. Notations

In seismic conditions, the rotation principles (Seed and Whitman, 1970) are usually used to determine these coefficients $K_{aE\gamma}$ and $K_{pE\gamma}$. Intermediate parameters are thus defined:

$$\theta_{eq} = \arctan\left(\frac{a_h}{1 \pm a_v}\right) \quad (3)$$

$$\lambda^* = \lambda \pm \theta_{eq} \text{ and } \beta^* = \beta \pm \theta_{eq} \quad (4)$$

$$\gamma^* = \gamma \frac{(1 \pm a_v)}{\cos\theta_{eq}} \text{ and } q^* = q \frac{(1 \pm a_v)}{\cos\theta_{eq}} \quad (5)$$

$$K_{aE\gamma} = K_{a\gamma}^* \frac{1 \pm a_v}{\cos\theta_{eq}} \text{ with } K_{a\gamma}^* = K_{a\gamma}(\lambda^*, \beta^*) \quad (6a)$$

$$K_{pE\gamma} = K_{p\gamma}^* \frac{1 \pm a_v}{\cos\theta_{eq}} \text{ with } K_{p\gamma}^* = K_{p\gamma}(\lambda^*, \beta^*) \quad (6b)$$

where a_h and a_v are the horizontal and vertical seismic acceleration components.

The aim of this paper is to present an original method to solve Boussinesq's equations for both static and seismic

conditions. In static condition, the results are compared to those obtained by Caquot, Kerisel and Absi on the one hand and Sokolowski (1965) on the other hand. For seismic conditions, the results of the solving process are compared to the solutions obtained by applying rotations principles (Seed and Whitman, 1970, Lancelotta, 2007, Mylonakis and al., 2007). For both static and seismic conditions, failure mechanisms are determined and compared to the solutions obtained through the kinematical approach of the yield analysis considering log-spirals and rotational mechanisms.

2 BOUSSINESQ'S EQUATIONS – STATIC CONDITIONS

2.1 Fundamental equations

Boussinesq's equilibrium leads to consider the static equilibrium of an elementary volume OMN behind a retaining structure with a weight W per unit length (Figure 2). The equilibrium in terms of rotation around the point O gives the equation 7 and the equilibrium in terms of forces along [OM] gives the equation 8 (see, Caquot and Kerisel, 1949 for more details). The following system of differential equations is thus obtained:

$$\frac{dn}{d\omega} = 3t - \sin\omega \quad (7)$$

$$\text{obtained from: } \frac{n}{3} - \frac{n+dn}{3} + t d\omega = \frac{2}{3} W \sin\omega$$

$$\frac{dt}{d\omega} = nm - \cos\omega \quad (8)$$

$$\text{obtained from: } \frac{nd\omega}{2} + \frac{dt}{2} - kn d\omega + W \cos\omega = 0$$

$$\text{with: } m = 2k - 1, W = \frac{d\omega}{2} \text{ and}$$

$$m = 1 + 4 \tan^2 \varphi \pm \frac{4}{\cos \varphi} \sqrt{\tan^2 \varphi - \tan^2 \alpha}$$

where ω is the angle between the fictitious vertical plane and any radial plane and α is the inclination of the stress on the normal line of radial plane with n the normal component and t the tangential component: $\tan \alpha = t/n$

The parameter m ensures that the Mohr-Coulomb failure criterion is respected. It describes an ellipse depending on the variable $\tan \alpha$: the upper part of the ellipse (with the sign $+$) is linked to the active earth pressure while the lower part of the ellipse (with the sign $-$) is linked to the passive earth pressure.

Boussinesq shows that this differential equation system cannot be solved using closed-form solutions. Consequently, numerical integration is necessary: solving procedures have been largely developed in the framework of the slip line theory after 1950 (see for example, Absi, 1984) and are based on the principles of lower bounds regarding the yield analysis (see for example, Salençon, 1974).

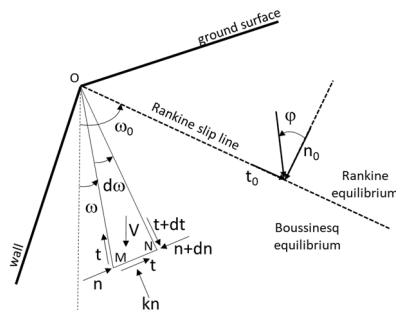


Figure 2. Equilibrium for an elementary volume (from Caquot and Kerisel, 1949)

2.2 Solving procedure

The solving procedure starts with the determination of two domains (Figure 2): the Rankine domain and the Boussinesq domain. For the Rankine domain, the solution is defined by

straight slip lines and it has been shown by Ravizé (1945 from Caquot and Kerisel, 1949) that it is the only solution. The slip line at the end of the Rankine domain defines a line of singularities, used as an initial boundary condition to solve the differential equations (7) and (8) in the Boussinesq domain.

The Rankine slip line corresponds to an angle ω_0 calculated from the vertical fictitious plane: the stress inclination is equal to φ for active state and $-\varphi$ for passive state as it ensures Rankine equilibrium. The normal and tangential components n_0 and t_0 are determined by considering the equilibrium of the wedge OMN (as the stress inclination is equal to $\pm\varphi$ the slip line inclination is also inclined to $\pm\varphi$ from the radial plane defined by the angle ω_0 , see Figure 3):

$$p_0 = 2P_0 = 2(W_T \pm W_N \tan \varphi) = \frac{\cos(\omega_0 - \beta) \sin \omega_0}{\sin(\omega_0 - \beta \pm \varphi)} \quad (9)$$

$$n_0 = \cos \varphi \frac{\cos(\omega_0 - \beta) \sin \omega_0}{\sin(\omega_0 - \beta \pm \varphi)} \text{ and } t_0 = \pm \sin \varphi \frac{\cos(\omega_0 - \beta) \sin \omega_0}{\sin(\omega_0 - \beta \pm \varphi)} \quad (10)$$

$$\text{with: } \omega_0 = \frac{\pi}{4} \pm \frac{\varphi}{2} \pm \frac{(\omega_{\beta \pm \beta})}{2} \text{ and } \omega_{\beta} = \arcsin\left(\frac{\sin \beta}{\sin \varphi}\right)$$

with $(-)$ for active state and $(+)$ for passive state. The angle ω_0 is positive from the vertical fictitious plane.

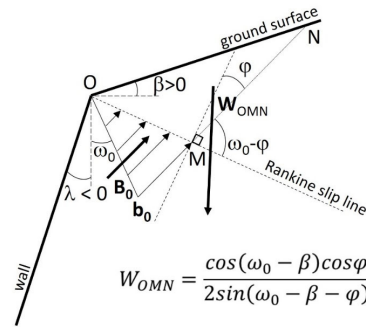


Figure 3. Calculation of the terms W_{OMN} , B_0 and b_0

As shown by Salençon (1974), the system of differential equations (7) and (8) with the boundary conditions on the Rankine slip line is undetermined. It is possible to solve these equations from the Rankine slip line to the wall but the stress inclination on the wall cannot be chosen.

Figure 4 shows the calculations for the case study: $\varphi = 30^\circ$, $\beta = 0$, $\lambda = 0$, with $\delta/\varphi = -1$ for the passive state. The first solution leads to path 1, starting from the Rankine slip line toward the wall. To compute the earth pressure for any stress inclination, a second solution (path 2) is calculated in the opposite direction, from the wall toward the Rankine slip line. The passive pressure at the wall is adjusted iteratively (using, for example, a dichotomy method) to ensure continuity at the Rankine slip line between paths 1 and 2. For both paths 1 and 2, the solutions are obtained by explicit numerical integration of the governing equations (Equations 7 and 8) using a 4th order Runge-Kutta procedure (RK4) (see Burlon, 2023 for more details). The resulting coefficients closely match those of Caquot, Kerisel, Absi, and Sokolowski.

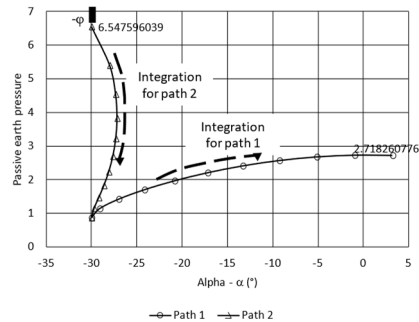


Figure 4. Passive earth pressure coefficient variation according to α

3 BOUSSINESQ'S EQUATIONS – SEISMIC CONDITIONS

In seismic conditions, the equations (7) and (8) are modified to account for the inertial forces into the ground where a_h and a_v are respectively the horizontal and vertical acceleration components.

$$\frac{dn}{d\omega} = 3t - ((1 \pm a_v)\sin\omega \pm a_h\cos\omega) \quad (11)$$

$$\frac{dt}{d\omega} = nm - ((1 \pm a_v)\cos\omega \pm a_h\sin\omega) \quad (12)$$

The initial boundary conditions on the Rankine slip line are also modified. The angle ω_0 is modified to ω_0^* in order to integrate the equation (11) and (12):

$$\omega_0^* = \frac{\pi}{4} \pm \frac{\varphi}{2} \pm \frac{(\omega_{\beta^*} \pm \beta^*)}{2} \pm \theta_{eq} \quad (13)$$

with $\omega_{\beta^*} = \arcsin\left(\frac{\sin\beta^*}{\sin\varphi}\right)$

with (-) for active state and (+) for passive state.

The values n_0 and t_0 are also modified according to the notations of Figure 3. The solving steps remain the same as those performed for static conditions.

Considering the following case ($\varphi=30^\circ$, $\beta=0$, $\lambda=0$ with $\delta/\varphi=1$ for the active state and $\delta/\varphi=-1$ for the passive state), the acceleration components a_h and a_v respectively equal to 0.2 and -0.1. The stress variations in function of α and ω can be determined to obtain the seismic coefficients $K_{aE\gamma}$ and $K_{pE\gamma}$: the active earth pressure coefficient is equal to 0.449 (in comparison to 0.307 in static conditions) and the passive earth pressure coefficient is equal to 4.90 (in comparison to 6.55 in static conditions).

Figures 5 and 6 shows the comparison between the seismic coefficients $K_{aE\gamma}$ and $K_{pE\gamma}$ and the coefficients $K_{a\gamma}^*$ and $K_{p\gamma}^*$ obtained by the rotation principles considering a modified geometry from the initial geometry. The difference can be interpreted by the rotating term: $(1 \pm a_0)/\cos\theta_{eq}$. The active and passive earth pressures $K_{aE\gamma}$ and $K_{pE\gamma}$ calculated by Boussinesq's equations in seismic conditions correspond to the active and passive earth pressures obtained by applying the rotation principles:

$$K_{aE\gamma} = K_{a\gamma}^* \frac{1 \pm a_v}{\cos\theta_{eq}} \quad \text{and} \quad K_{pE\gamma} = K_{p\gamma}^* \frac{1 \pm a_v}{\cos\theta_{eq}} \quad (17)$$

4 AN EXAMPLE IN STATIC CONDITIONS

4.1 Studied case

An simple example is used: $\varphi=30^\circ$, $\beta=0$, $\lambda=0$. Various solutions are presented in terms of comparisons: the values from Kerisel and Absi (1990), the values from Coulomb (only for the active pressure cases) and the values considering a closed-form solution used for weightless conditions (see L'Herminier and Absi, 1962 or Lancelotta, 2002) where the load at the free surface q is replaced by the ground weight $\gamma.l.\cos(\beta-\lambda)$.

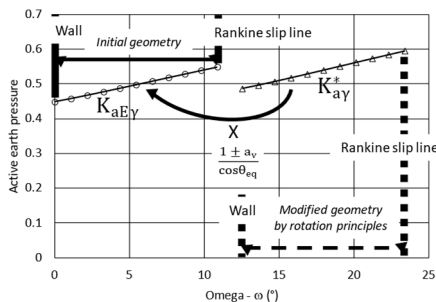


Figure 5. Comparison between $K_{a\gamma}^*$ and $K_{aE\gamma}$

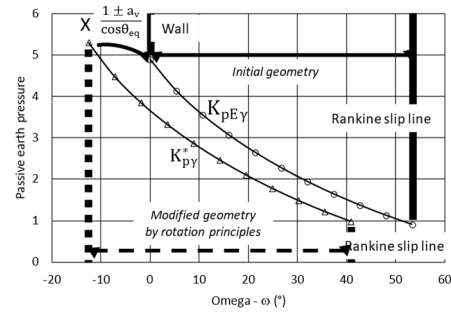


Figure 6. Comparison between $K_{p\gamma}^*$ and $K_{pE\gamma}$

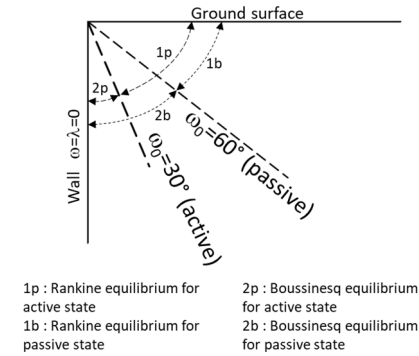


Figure 7. First Rankine slip line for the active and passive states for the three cases under consideration

4.2 Results

The results are presented in table 1 for the present example. The results obtained by the proposed method and Caquot Kerisel method are quite similar, which shows that the solving procedure developed in this paper is quite appropriate. Coulomb method as it derives from a kinematical approach in the framework of the yield analysis gives lower values for active states (for passive states, the values have not been calculated as it is known that the assumption of straight failure lines is not valid). The closed-form solution derived from weightless conditions provides results that are very close to the proposed method or Caquot Kerisel method. They are also on the safe side with higher values for active states and lower values for passive states. When the inclination of the slope or the wall is high, the obtained values can be considered as too cautious.

4.2.1 Comparisons with log-spiral method

Comparisons with log-spirals may be also presented in terms of active and passive earth pressure coefficients and failure mechanisms. Log-spiral methods (Rendulic, 1935 from Brinch Hansen, 1953) can be seen as an application of the kinematical approach in the framework of the yield analysis. Hence, the values of active earth pressures are expected to be slightly lower than those calculated by means of Boussinesq's equations and the values of passive earth pressures are expected to be slightly higher (Soubra, 2000; Patki, 2015). Log-spiral calculations are performed using the software Talren (Talren 2022): the most critical log-spirals used for the calculation of the active or passive earth pressure coefficient is determined by investigating various locations of the log spiral centres, of the curvature angles, and positive or negative concavities. Table 2 provides some values obtained by the two calculation methods (proposed solution for Boussinesq equations and log-spirals method). The results are in line with the expectations stated earlier and the two values are always very close.

Table 1. Comparison of active and passive earth pressure coefficients for the case 1

Active state					
Inclination	$\delta/\varphi=-1$	$\delta/\varphi=-2/3$	$\delta/\varphi=0$	$\delta/\varphi=2/3$	$\delta/\varphi=1$
Proposed method	0.886	0.477	0.333	0.301	0.307
Caquot and Kerisel	0.981	0.476	0.333	0.300	0.308
Coulomb	0.866	0.469	0.333	0.297	0.297
Closed-form solution	1.006	0.484	0.333	0.301	0.315
Passive state					
Inclination	$\delta/\varphi=-1$	$\delta/\varphi=-2/3$	$\delta/\varphi=0$	$\delta/\varphi=2/3$	$\delta/\varphi=1$
Proposed method	6.55	5.26	3.0	1.46	---
Caquot and Kerisel	6.50	5.30	3.0	1.46	---
Closed-form solution	5.80	5.26	3.0	1.38	---

Table 2. Comparisons with log-spiral method

Active state					
Inclination	$\delta/\varphi=-1$	$\delta/\varphi=-2/3$	$\delta/\varphi=0$	$\delta/\varphi=2/3$	$\delta/\varphi=1$
Proposed method	0.886	0.477	0.333	0.301	0.307
Log-spiral	0.817	0.467	0.333	0.297	0.297
Passive state					
Inclination	$\delta/\varphi=-1$	$\delta/\varphi=-2/3$	$\delta/\varphi=0$	$\delta/\varphi=2/3$	$\delta/\varphi=1$
Proposed method	6.55	5.26	3.0	1.46	---
Log-spiral	6.931	5.341	3.0	1.650	---

5 AN EXAMPLE IN SEISMIC CONDITIONS

5.1 Case under consideration

The example is now considered for various acceleration conditions: $a_h=0.1$ (a) ; 0.2 (b) and 0.3 (c). The vertical acceleration component can be positive (+) or negative (-) and is equal to the half of the horizontal acceleration component. Considering the rotation principles with the angle θ_{eq} , the closed-form solutions (Lancelotta, 2007) presented previously provide the values $K'_{aE\gamma}$ and $K'_{pE\gamma}$:

$$K'_{aE\gamma} = K'_{aq} \cos(\beta - \lambda) \quad (14)$$

$$K'_{pE\gamma} = K'_{pq} \cos(\beta - \lambda) \quad (15)$$

5.2 Results

For a stress inclination ratio δ/φ equal to 1.0 for the active state and -1.0 for the passive state, the results are provided in Table 3 for the proposed method based on Boussinesq's equations, the log-spiral method and the closed-form solution derived from Lancelotta (2007). They give very close and consistent results: when the vertical acceleration is negative (upwards), the earth pressures are reduced in comparison to the case where the vertical acceleration is positive (downwards) for both the active and passive states. The results from the closed-form solution are still on the safe side. The log-spiral method gives lower values for active states and higher values for passive states, which is consistent for a kinematical approach in the framework of the yield analysis. For the three methods, the variations are higher for the active states than for the passive states.

Table 3. Seismic earth pressures

	$K_{aE\gamma}$	$K'_{aE\gamma}$	$K_{aE\gamma}$ (log-spiral)	$K_{pE\gamma}$	$K'_{pE\gamma}$	$K_{pE\gamma}$ (log-spiral)
Static	0.307	0.315	0.297	6.548	5.804	6.931
(a+)	0.392	0.396	0.387	6.403	5.740	6.845
(a-)	0.362	0.366	0.358	5.747	5.157	6.148
(b+)	0.498	0.500	0.496	6.223	5.637	6.712
(b-)	0.449	0.449	0.448	4.895	4.456	5.301
(c+)	0.632	0.632	0.632	6.004	5.493	6.527
(c-)	0.600	0.600	0.600	3.957	3.667	4.340

6 FAILURE MECHANISMS

6.1 Main results

In addition to the active and passive earth pressure coefficients, the failure mechanisms can be plotted: the first part in Rankine domain is a straight line as assumed by this theory while the second part in Boussinesq domain is close to a log-spiral curve (without being one exactly). This curve can be determined using the Mohr circle and the slip line field theory (Absi, 1984) by the following equation considering a radius r equal to 1.0 at the location of the wall:

$$r = e^{-\int \tan \xi(\omega) d\omega}$$

$$\xi(\omega) = \frac{\pi}{4} - \pm \frac{\varphi}{2} \pm \frac{(\omega_\alpha \pm \alpha)}{2} \text{ and } \omega_\alpha = \arcsin\left(\frac{\sin \alpha}{\sin \varphi}\right) \quad (16)$$

with (-) for active state and (+) for passive state.

$\xi(\omega)$ represents the angle between the slip line and the normal line to any radial plane. At the location of the wall, for a stress inclination equal to zero ($\alpha=0$ and $\omega_\alpha=0$), the slip line inclinations are equal to $\pi/4 \pm \varphi/2$ for active and passive state. For a stress inclination α on the wall corresponding to a ratio δ/φ respectively equal to 1.0 for active state and -1.0 for passive state, the slip line inclination is equal to φ or $-\varphi$ on the wall ($\alpha=\pm\varphi$ and $\omega_\alpha=\pm\pi/2$). Figure 8 shows the failure mechanisms determined for various stress inclinations. The ability to represent the failure mechanisms for active and passive states is useful to see the involved ground volume, especially for slopes of cuttings or embankments, and thus check if the calculation of the active or passive earth pressure coefficient is acceptable.

For the active states, the slip lines are straight, which explains the small differences between Coulomb and Boussinesq theories. Nevertheless, for the passive states, it is not possible to assume straight lines (except when the stress inclination on the wall is equal to zero). Moreover, it can be noticed that the concavity of the slip lines can be positive or negative according to the inclination on the wall. The failure mechanisms are quite similar to those obtained by Sokolowski (1965).

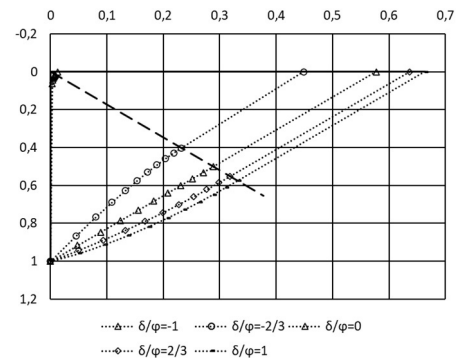


Figure 8. Envelope of slip lines for the active state

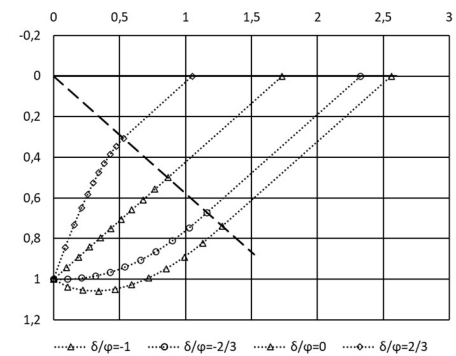


Figure 9. Envelope of slip lines for the passive state

For static conditions, Figures 10 and 11 show the failure mechanisms using Boussinesq's equations and log-spirals only regarding the active and passive earth pressure mechanisms. The two failure mechanisms are close, which is in accordance with the low differences that have been obtained between the values of earth pressures. The concavity of the failure mechanisms is also the same, which means that both negative and positive values of concavity should be investigated with log-spiral methods.

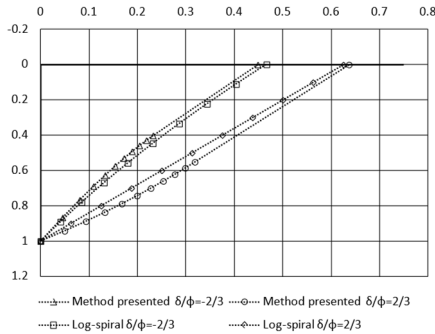


Figure 10. Failure mechanisms-comparison with log-spirals (active)

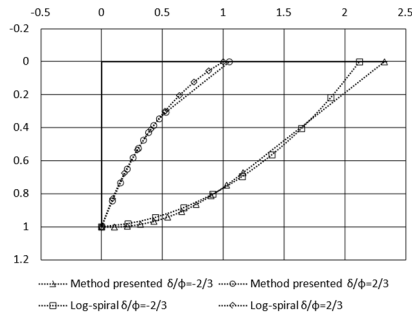


Figure 11. Failure mechanisms-comparison with log-spirals (passive)

For seismic conditions, Figures 12 and 13 give an example for a horizontal acceleration equal to 0.3 both for active and passive earth pressures with positive and negative vertical accelerations. In comparison to static conditions, the extension of the failure mechanism is increased in seismic conditions: this observation is especially important for the design of retaining structures interacting with each other. For the active state, the extension modification in seismic conditions is larger than for the passive state, which is consistent with the stronger variation of the active earth pressure coefficient as mentioned previously.

Figures 14 and 15 show another example considering stabilising (stb) and destabilising (dst) horizontal accelerations and illustrate the modifications of the failure mechanisms with the direction of the acceleration. The active and passive earth pressure coefficients are the following ($|a_h|=0.2$ and $a_v=-0.1$):

$$\begin{aligned} K_{aE\gamma}(dst) &= 0.449 & K_{aE\gamma} &= 0.307 & K_{aE\gamma}(stb) &= 0.185 \\ K_{pE\gamma}(dst) &= 4.895 & K_{pE\gamma} &= 6.548 & K_{pE\gamma}(stb) &= 6.732 \end{aligned}$$

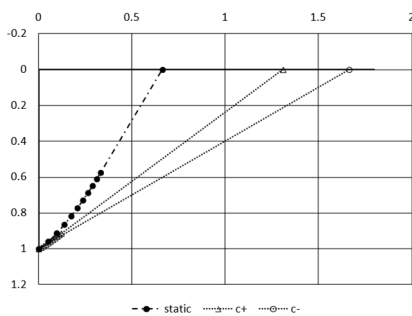


Figure 12. Slip line envelopes for the active state ($a_h=0.3$ and $|a_v|=0.15$)

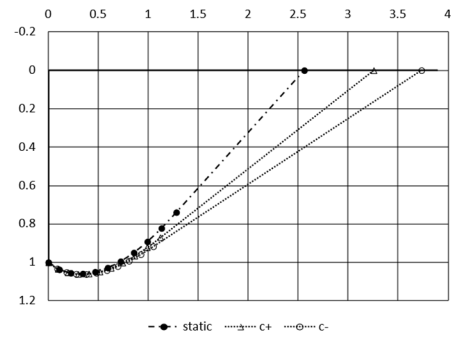


Figure 13. Slip line envelopes for the passive state ($a_h=0.3$ and $|a_v|=0.15$)

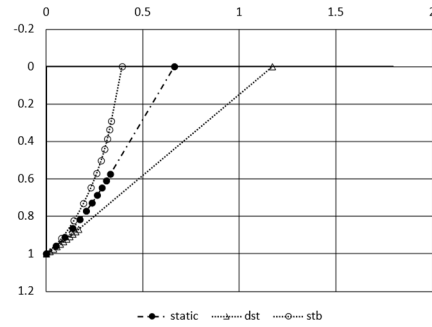


Figure 14. Envelope of slip lines for the active state ($|a_h|=0.2$ and $a_v=-0.1$)

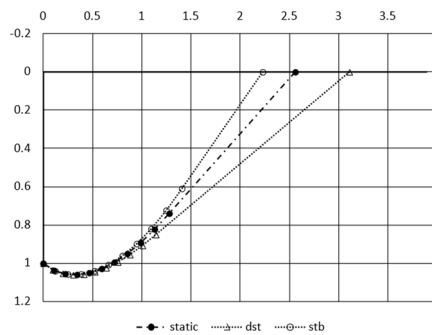


Figure 15. Slip line envelope for the passive state Figure 16. ($|a_h|=0.2$ and $a_v=-0.1$)

CONCLUSIONS

This paper presents an original numerical procedure to solve Boussinesq's equations to calculate earth pressure coefficients for both static and seismic conditions. In static conditions, the obtained results have been compared to the values from the tables established by Caquot, Kerisel and Absi on the one hand and by the log-spiral approach on the other hand. The results are quite similar, which proves the relevance and the consistency of the developed numerical procedures. The closed-form solutions proposed by both L'Herminier and Absi (1962) and Lancelotta (2002) are also very close and can be considered as cautious estimate for both active and passive earth pressures. The failure mechanisms have also been plotted in order to appreciate the ground volume involved in the active and passive states. In seismic conditions, the same results have been established, and similar conclusions are drawn. The failure mechanisms provide some insights about the extent of the zone of influence in both static and seismic conditions. Rotation principles can be applied to any calculation approach that has been validated in static conditions.

ACKNOWLEDGMENTS

A part of this work has been initiated during the elaboration of the second generation of Eurocode 7 and Eurocode 8 regarding the design of retaining structures for both static and seismic conditions. The comparison of national approaches is always an interesting and motivating task. The authors would like to thank their European colleagues, especially from France, Italy and United-Kingdom for the open-minded and fruitful discussions.

BIBLIOGRAPHY

- Absi, E. 1984. La théorie de la plasticité et l'équilibre limite en mécanique des sols. *Annales de l'ITBTP, Sols et Fondations*.
- Boussinesq, J. 1876. *Essai théorique sur l'équilibre des massifs pulvérulents comparé à celui des massifs solides et sur la poussée des terres sans cohésion*. Hayez, Bruxelles.
- Brinch Hansen, J. 1953. *Earth pressure calculation*. The Danish Technical Press, Copenhagen.
- Burlon, S. 2023. Un exemple d'intégration numérique des coefficients de poussée et de butée en milieux pesants. *Revue française de Géotechnique*, 174, 3.
- Caquot, A. 1934. *Équilibre des massifs à frottement interne – stabilité des terres pulvérulentes ou cohérentes*. Gauthier-Villars, Paris.
- Caquot, A., and Kerisel, J. 1948. *Tables de butée, de poussée et de force portante des fondations*. Gauthier-Villars, Paris.
- Caquot, A., and Kerisel, J. 1949. *Traité de mécanique des sols* (2nd ed.). Gauthier-Villars, Paris.
- L'Herminier, R., and Absi, E. 1962. *Équilibre limite d'un coin dans un milieu non pesant*. Cahiers de la recherche théorique et expérimentale sur les matériaux et les structures, Eyrolles, Paris.
- Kerisel, J., and Absi, E. 1990. *Tables de poussée et de butée des terres*. Presses de l'École Nationale des Ponts et Chaussées, Paris.
- Lancellotta, R. 2002. Analytical solution of passive earth pressure. *Géotechnique* 52(8), 617–619.
- Lancellotta, R. 2007. Lower-bound approach for seismic passive earth resistance. *Géotechnique* 57(3), 319–321.
- Mylonakis, G., Kloukinas, P., and Papantonopoulos, C. 2007. An alternative to the Mononobe–Okabe equations for seismic pressures. *Soil Dynamics and Earthquake Engineering* 27, 957–969.
- Patki, M.A., Mandal, J.N., and Dewaikar, D.M. 2015. Computation of passive earth pressure coefficients for a vertical retaining wall with inclined cohesionless backfill. *International Journal of Geo-Engineering* 6(4).
- Ravizé, H. 1945. *Poussée des terres – équations de l'équilibre limite – nouvelle méthode de détermination des coefficients de poussée et de butée*. Dunod, Paris.
- Rendulic, L. 1935. Ein Beitrag zur Bestimmung der Gleitsicherheit. *Der Bauingenieur*, 19/20.
- Résal, J. 1903. *Poussée des terres – stabilité des murs de soutènement*. Librairie Polytechnique Béranger, Paris.
- Seed, H.B., and Whitman, R.V. 1970. Design of earth retaining structures for dynamic loads. *Proceedings of the Specialty Conference on Lateral Stresses in the Ground and Design of Earth Retaining Structures*, ASCE, Ithaca, New York, 103–147.
- Sokolowski, V.V. 1965. *Statics of granular media*. Pergamon Press, Oxford.
- Soubra, A.-H. 2000. Static and seismic passive earth pressure coefficients on rigid retaining structures. *Canadian Geotechnical Journal* 37(2), 463–478.
- Soubra, A.-H., and Macuh, B. 2002. Active and passive earth pressure coefficients by a kinematical approach. *Proceedings of the ICE – Geotechnical Engineering* 155(2), 119–131.
- Talren. 2022. *User manual*. Terrasol, Paris.

ANNEX A: COEFFICIENTS K_{AQ} , K_{PQ} , K_{AC} AND K_{PC}

This annex gives the equations proposed by L'Herminier and Absi (1962) or Lancellotta (2002) to calculate active and passive earth pressure coefficients K_{aq} and K_{pq} in weightless conditions. Only the fundamental equation is given when the log-spiral is between to Rankine mechanisms. When the two Rankine mechanisms are in interaction (ψ_a or $\psi_b < 0$), other equations

should be used (see L'Herminier and Absi, 1962, for more details).

The coefficients K_{aq} and K_{pq} are determined according to the following equations:

$$K_{aq} = \frac{\cos\delta - \sin\varphi\cos\omega_\delta}{\cos\alpha + \sin\varphi\cos\omega_\alpha} e^{-2\psi_a\tan\varphi}$$

$$K_{pq} = \frac{\cos\delta + \sin\varphi\cos\omega_\delta}{\cos\alpha - \sin\varphi\cos\omega_\alpha} e^{2\psi_p\tan\varphi}$$

$$\sin\omega_\delta = \frac{\sin\delta}{\sin\varphi}$$

$$\sin\omega_\alpha = \frac{\sin\alpha}{\sin\varphi}$$

$$\psi_a = \frac{\omega_\alpha + \alpha}{2} + \frac{\omega_\delta - \delta}{2} + \beta - \lambda$$

$$\psi_p = \frac{-\omega_\alpha + \alpha}{2} - \frac{\omega_\delta + \delta}{2} + \beta - \lambda$$

For a vertical loading, the inclination α is chosen as following: $\alpha = -\beta$.

The coefficients K_{ac} and K_{pc} used to account for the cohesion can be determined by the following equations for their implementation by means of the corresponding state theorem:

$$K_{ac} = \frac{1}{\tan\varphi} \left(\frac{1}{\cos\delta} - K_{aq,\alpha=0} \right) \text{ with } K_{aq,\alpha=0} \text{ the value } K_{aq} \text{ calculated for } \alpha = 0$$

$$K_{pc} = \frac{1}{\tan\varphi} \left(K_{pq,\alpha=0} - \frac{1}{\cos\delta} \right) \text{ with } K_{pq,\alpha=0} \text{ the value } K_{pq} \text{ calculated pour } \alpha = 0$$

These relationships are based on the fact that the adhesion between the wall and the ground is equal to:

$$\frac{a}{c} = \frac{\tan\delta}{\tan\varphi}$$