

Small-strain shear modulus of carbonate sand: Importance assessment of key parameters

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ABSTRACT: The small-strain shear modulus (G_{\max}) is a key mechanical parameter used to assess the dynamic behaviour of sandy soils. Although carbonate sand is a prevalent material in offshore geotechnical infrastructure projects, its G_{\max} has received relatively limited attention compared to the extensive research conducted on siliceous sand. This study evaluates the performance of several established G_{\max} models previously developed for calcareous granular soils, based on a dataset comprising around 370 experimental measurements derived from tests conducted on nine pure carbonate sands. It also investigates the contributions of key parameters potentially affecting G_{\max} of clean carbonate sand under isotropic loading conditions, using Shapley Additive exPlanations (SHAP) values applied to an XGBoost regression model. The results confirm that the mean effective confining pressure (σ'_m) and void ratio (e) are the most influential factors. Incorporating the minimum void ratio (e_{\min}) and specific gravity of solids (G_s), along with granulometric and morphological descriptors, is likely necessary to account for the mineralogical diversity of calcareous sandy soils in G_{\max} models.

KEYWORDS: Carbonate sand, small-strain shear modulus, mineralogy, feature importance ranking.

1 INTRODUCTION

The small-strain shear modulus (G_{\max}) is an essential soil characteristic widely employed for analysing and designing geotechnical structures under both static and dynamic loading conditions. Throughout the past sixty years, extensive research has been conducted to evaluate the key parameters affecting G_{\max} of sand (Hardin and Richart Jr, 1963, Hardin and Black, 1966, Seed and Idriss, 1970, Hardin and Drnevich, 1972, Tatsuoka et al., 1979, Seed et al., 1986, Saxena and Reddy, 1989, Fioravante, 2000, Menq, 2003, Ezaoui and Benedetto, 2009, Wichtmann and Triantafyllidis, 2009, Senetakis et al., 2012, Payan et al., 2016, Payan, 2017, He et al., 2019, Payan and Chenari, 2019, Kaviani-Hamedani et al., 2021, Fakharian et al., 2022, He et al., 2022, Fakharian et al., 2023, Liang et al., 2024, Chen and Yang, 2025, Said et al., 2025, Ahmadinezhad et al., 2026), reinforcing the importance of this parameter in geotechnical engineering research and practice.

Hardin and Richart Jr (1963) and Hardin and Black (1966) were the first to propose one of the most widely adopted empirical formulations for estimating G_{\max} of granular materials under isotropic loading conditions:

$$G_{\max} = Af(e)(\sigma'_m/p_a)^n \quad (1)$$

where $f(e)$ is a void ratio function, σ'_m is the mean effective confining pressure, p_a is the reference atmospheric pressure, and A and n are the model parameters determined experimentally. Over the years, several additional influencing factors have been incorporated into G_{\max} models elaborated for sandy soils, including the mean grain size (D_{50}) (Menq, 2003, He et al., 2022), coefficient of uniformity (C_u) (Menq, 2003, Wichtmann and Triantafyllidis, 2009, Senetakis et al., 2012, Payan et al., 2016, Giang et al., 2017), particle morphology (Payan et al., 2016, Payan, 2017), fines content (Payan et al., 2017, Akhyani and Ghalandarzadeh, 2022, Hang et al., 2024), and degree of saturation (S_r) (Ahmadinezhad et al., 2019, Karimzadeh et al., 2020, Jafarzadeh et al., 2021, Ahmadinezhad et al., 2023), among others. Nevertheless, most research to date has focused on siliceous sand, which substantially differs from carbonate materials in terms of mineralogical composition, particle microstructure, and mechanical response (He et al., 2020, Lv et al., 2021, Ma et al., 2023). Given the widespread occurrence of calcareous soils in nearshore and offshore infrastructure projects (Jafarian et al., 2018, He et al., 2020, Jafarian and Javdanian, 2020, He et al., 2022, Ma et al., 2023, Cao et al., 2024), it becomes essential to investigate the key parameters governing their small-strain shear stiffness.

This study aims to evaluate the predictive capability of several empirical G_{\max} expressions previously developed for calcareous granular soils. It also systematically reassesses the relative importance of key parameters potentially contributing to G_{\max} of these materials under isotropic loading conditions. A comprehensive experimental dataset, compiled from previously published studies and earlier research conducted at the University of New South Wales, is analysed using SHapley Additive exPlanations (SHAP) values applied to an XGBoost regression model. This data-driven approach provides a quantitative evaluation of the most influential features governing G_{\max} of clean carbonate sand, thereby laying the groundwork for future experimental and theoretical developments in this field.

2 DATASET COLLECTION

A test dataset comprising around 370 experimental measurements of G_{\max} of nine distinct pure carbonate sands was employed in this study. The data points were compiled from previously published works (Fioravante, 2000, Jafarian et al., 2018, Jafarian and Javdanian, 2020, Shi et al., 2021, Hang et al., 2024) as well as earlier experimental investigations conducted at the University of New South Wales (He et al., 2019, Said, 2024, Said et al., 2024). Each collected G_{\max} measurement is coupled with its associated soil characteristics and testing conditions.

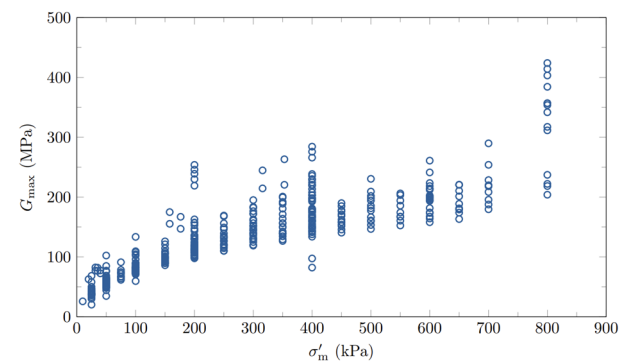


Figure 1. Variation of G_{\max} with σ'_m .

These G_{\max} measurements were obtained from resonant column (RC) and bender element (BE) tests, both of which are widely recognised for capturing G_{\max} at small strain amplitudes. Prior to analysis, anomalous data points were identified and filtered to ensure the consistency and reliability of the dataset. Figure 1

Table 1. Some established models for the prediction of G_{\max} of carbonate sand.

Model	A	$f(e)$	n
Fioravante (2000)	106.00	$e^{-0.8}$	0.516
Giang et al. (2017)	$115.371 \exp(-0.107C_u)$	$e^{-4.416 \exp(-0.294C_u)}$	$0.421C_u^{0.125}$
Jafarian et al. (2018)	92.00	$(2.17 - e)^2 / (1 + e)$	0.400
He et al. (2019) – PH Sand	79.34	$e^{-1.3}$	0.480
He et al. (2019) – WA Sand	122.89	$e^{-1.3}$	0.410
Jafarian and Javdanian (2020)	96.63	$(2.17 - e)^2 / (1 + e)$	0.453
He et al. (2022)	$1075.50C_u^{-0.332}D_{50}^{0.0952}$	$1/(1+e)^3$	$0.356C_u^{0.140}D_{50}^{-0.075}$

presents the variation of G_{\max} with σ'_m , a conventionally used parameter in empirical G_{\max} expressions.

3 PREDICTABILITY OF PUBLISHED MODELS

Table 1 presents a compilation of established empirical G_{\max} models developed for calcareous biogenic sandy soils, each formulated following the general form outlined in Equation (1). Using the collected experimental dataset, the predictive performance of these expressions in estimating G_{\max} is evaluated, and the corresponding Empirical Cumulative Distribution Functions (ECDFs) of the relative prediction errors are plotted in Figure 2. An ECDF is a step function which represents the cumulative percentage of data points with prediction errors less than or equal to the corresponding error value shown on the x-axis.

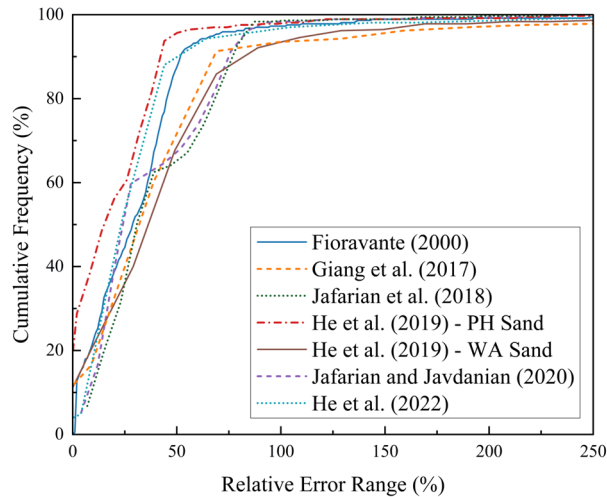


Figure 2. ECDFs of the relative prediction errors for G_{\max} models elaborated for carbonate sandy soils.

Among all the models evaluated in Figure 2, the one proposed by He et al. (2019) for the Philippines (PH) Sand exhibits the best predictive performance, as evidenced by its steepest ECDF slope, indicating a higher cumulative frequency of predictions with lower relative errors. For instance, around 50% of the G_{\max} values predicted using this model fall within a relative error range of approximately 15%. The expressions developed by Fioravante (2000) and He et al. (2022) also demonstrate better predictive accuracy than the remaining models. However, none of the examined frameworks reliably predicts G_{\max} of all nine carbonate sands included in this study, reflecting the complex dynamic behaviour of these soils. This limitation may be attributed to the irregular morphology of carbonate grains (He et al., 2022) or their intricate intra-particle porosity (He et al., 2019, Lv et al., 2021, Ma et al., 2023), two features that remain unaccounted for in existing models.

4 POTENTIAL CONTRIBUTING PARAMETERS

All the data points considered in this study are derived from dry and fully saturated tests conducted under isotropic loading conditions. As a result, the mean effective confining pressure (σ'_m) is the sole representative of the stress state, and the degree of saturation (S_r) can be excluded from the analysis, since G_{\max} of pure sand remains nearly unchanged under both fully dry and fully saturated conditions (Oh and Vanapalli, 2014, Payan, 2017, Said, 2024).

Due to the mineralogical diversity of carbonate soils, they do not exhibit the same void ratio ranges. The void ratio (e) is therefore considered alongside the extreme void ratios (e_{\min} and e_{\max}) to provide a more accurate representation of the density state. The specific gravity of solids (G_s) is also included as an indicator of soil mineralogy, since it reflects changes in the chemical composition of sand particles.

The mean grain size (D_{50}), coefficient of uniformity (C_u), and coefficient of curvature (C_c) are incorporated to capture key aspects of the gradation curve. Particle roundness (R) and sphericity (S) are used to quantitatively describe the general grain morphology (Santamarina and Cho, 2004, Cho et al., 2006, Payan et al., 2016, Payan, 2017).

Table 2. Statistical ranges of the parameters potentially affecting G_{\max} , based on the experimental dataset.

Parameter	Min	Max	Mean Value	SD
σ'_m	10	1600	248	196
e	0.39	1.60	0.72	0.21
e_{\min}	0.36	1.28	0.61	0.18
e_{\max}	0.52	1.78	0.96	0.23
G_s	2.36	2.89	2.66	0.06
D_{50}	0.10	3.50	0.74	0.68
C_u	1.19	8.12	2.65	1.66
C_c	0.66	1.50	0.92	0.13
R	0.23	0.80	0.42	0.11
S	0.35	0.90	0.58	0.08

In total, ten variables (σ'_m , e , e_{\min} , e_{\max} , G_s , D_{50} , C_u , C_c , R , and S) are selected for the analysis. These features, along with their statistical ranges derived from the collected dataset, are summarised in Table 2, where σ'_m is expressed in kPa, D_{50} in mm, and all remaining variables are unitless. Standard scaling is applied to all these parameters to account for differences in their orders of magnitude.

5 KEY PARAMETER IMPORTANCE RANKING

To evaluate the influence of each input variable on the model's output, SHAP importance values (Lundberg, 2017) are employed in conjunction with the XGBoost algorithm (Chen and Guestrin, 2016). Based on cooperative game theory, the SHAP technique ensures a fair allocation of feature importance to each parameter by accounting for all possible interactions among variables.

Figure 3 illustrates the SHAP-based ranking of the selected variables potentially contributing to G_{\max} of pure carbonate sand under isotropic loading conditions. Among all the assessed parameters, σ'_m is identified as the most dominant factor, followed by e which ranks second. Figure 3 also highlights the prominent influence of e_{\min} .

The fourth most influential variable is G_s , reflecting that mineralogical variations may be particularly relevant in carbonate soils due to their diverse biogenic origins. The higher SHAP values attributed to e_{\min} and G_s , in comparison with C_u , R , and S , suggest that, beyond the effects of the gradation curve (Menq, 2003, Wichtmann and Triantafyllidis, 2009, Senetakis et al., 2012, Giang et al., 2017, He et al., 2022) and particle morphology (Payan et al., 2016, Payan, 2017), mineralogy is an important contributor to G_{\max} of non-conventional carbonate sandy soils.

Calcareous soils consist of porous biogenic particles, leading to high apparent void ratios (Senetakis et al., 2012, He et al., 2019, Said et al., 2024). However, intra-particle voids may not significantly influence the soil mechanical response. Thus, the intra-particle porosity should be accounted for when estimating G_{\max} of carbonate sandy materials.

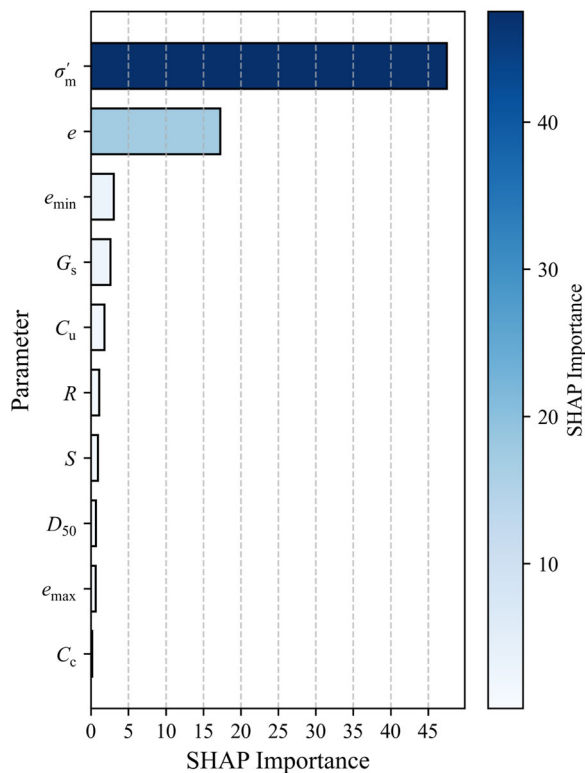


Figure 3. Ranking of the selected parameters affecting G_{\max} of pure carbonate sand under isotropic loading conditions, as determined by the SHAP feature importance method applied to an XGBoost model.

6 CONCLUSIONS

The small-strain shear modulus (G_{\max}) is a fundamental soil property in geotechnical engineering. This study investigates the factors influencing G_{\max} of clean carbonate sand under isotropic loading conditions, after outlining the limited predictive performance of existing G_{\max} models. An XGBoost regression, interpreted through SHAP values, is employed to quantify the relative importance of key parameters potentially affecting G_{\max} . While the results reaffirm that the mean effective confining pressure (σ'_m) and void ratio (e) are the most influential factors, the minimum void ratio (e_{\min}) and specific

gravity of solids (G_s) rank third and fourth, respectively. Beyond the conventionally considered granulometric and morphological descriptors, the findings of this study reveal that the soil mineralogical origin and particle chemical composition also play a significant role in controlling G_{\max} of calcareous sandy materials. Based on the SHAP ranking, e_{\min} and G_s may serve as effective proxies for capturing this mineralogical influence.

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