

## Extent of liquefiable layers: Simplified vs. Numerical methods

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**ABSTRACT:** An increasing number of offshore wind farms are being constructed in seismic regions over liquefaction susceptible soils. Identification of soil layers that liquefy during earthquake loading is required for the foundation design of wind turbines. Liquefaction assessment is usually undertaken using empirical or numerical methods. Empirical methods rely on correlations and assume each layer acts independently and cannot account for soil softening due to pore-pressure development. To overcome these challenges, a more realistic prediction of liquefaction using an effective stress-based approach with appropriate constitutive models is recommended. In this paper, liquefaction assessment was undertaken for a given site using 3 methods, Method A: simplified method (empirical assessment using Boulanger and Idriss, 2014), Method B: numerical method using 1D nonlinear site-specific response analyses (using DEEPSOIL) and Method C: Finite Element Analysis (FEA) method (two-dimensional fully coupled, nonlinear dynamic FE analysis using Plaxis 2D with PM4Sand constitutive model). Comparison of the results from all three methods for a given soil condition highlights that liquefaction assessment using Method A and Method B is appropriate as an initial assessment to estimate the liquefaction depths, as they provide conservative liquefaction depths. However, more rigorous analysis such as Method C can provide more accurate insight into liquefaction. Thus, Method C is recommended for use in the detailed design of offshore wind farms, as accurate and in-depth details of liquefaction would lead both safe and cost-effective foundation designs. This could lead to millions of cost savings on offshore wind projects.

**KEYWORDS:** Liquefaction, Site Response 1-D SSRA, Offshore Seismic Design, PM4Sand, Plaxis.

### 1 INTRODUCTION

Seismically induced liquefaction with associated densification and/or lateral movement of sediments is a well-documented factor in relatively small-scale subsidence not involving depths greater than 30 meters. However, soil liquefaction can occur at depths in excess of 100 meters under selected conditions (Stewart and Knox, 1995) affecting the foundations of offshore wind turbines. An increasing number of offshore wind farms are being constructed in seismic regions over liquefaction susceptible soils, thus identification of liquefiable soil layers is required during the design. Over the past 50 years, a methodology termed the “simplified procedure” has evolved as a standard practice for evaluating the liquefaction resistance of soils. The simplified procedure was developed from empirical evaluations of field observations and field and laboratory test data (Youd et al., 2001).

Liquefaction potential at a particular site can be assessed either by numerical or empirical methods. In numerical methods, the soil is modelled with an appropriate constitutive model which is able to predict the soil behaviour under seismic loading during a site response analysis. The accuracy of the results depends on the suitability of the constitutive model and the accuracy of input parameters for the soil model. The main benefit of this approach is that it can trace pore pressure generation during shaking. On the other hand, the model input parameters require substantial laboratory and field testing, thus it is mainly used for research and large projects (Bán et al., 2016).

Empirical methods are based on correlations between seismic loading and soil resistance to liquefaction. The seismic loading on a soil layer, is usually expressed in terms of equivalent uniform Cyclic Stress Ratio (CSR) shown in Equation (1); while liquefaction resistance expressed in terms of Cyclic Resistance Ratio (CRR) can be determined using case histories to characterize resistance in the form of measured in situ test parameters, most commonly Standard Penetration Test (SPT) blow-count, Cone Penetration Test (CPT) tip resistance or propagation velocities from low-strain geophysical tests  $V_s$  (Boulanger and Idriss, 2014; Youd et al., 2001). CSR is then

compared to the CRR, which separates liquefaction and non-liquefaction case histories and can be determined from the relevant in situ index, to obtain the Factor of Safety (FoS). This type of assessment has much wider usage than explicit models, primarily because it is easier and less expensive to implement.

This study aims to estimate the liquefaction potential using 3 methods as illustrated in Figure 1; Method A: simplified method i.e. empirical assessment using Boulanger and Idriss (2014), Method B: numerical method with 1D nonlinear site-specific response analyses (1-D SSRA) carried out using DEEPSOIL, and Method C: Finite Element Analysis (FEA) method in which the pore water pressure ratio ( $r_u$ ) was estimated via fully coupled, nonlinear FE analysis using Plaxis 2D with a PM4Sand constitutive model for the liquefiable sand layer. The maximum horizontal acceleration at the ground surface  $a_{max}$  was computed by 1-D SSRA. A typical offshore CPT profile and two earthquake time history records at bedrock outcrop with a magnitude of  $M=7.7$  were used. It is shown that empirical methods A and B cannot capture the holistic behaviour of a potentially liquefiable soil deposit, and it may be appropriate for regional liquefaction assessments or assessment of assets of low importance or value. However, more advanced techniques with appropriate constitutive model (method C) may be more appropriate for structural optimizations and cost reductions in offshore projects.

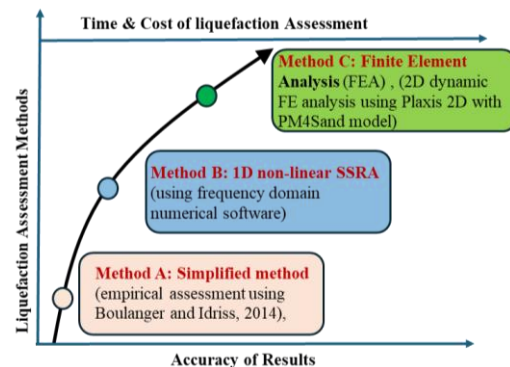


Figure 1. Summary of liquefaction assessment methods

## 2 SOIL PROFILE

The soil profile used for this study was based on one cone penetration testing with pore water pressure measurement (CPTu) up to the depth of 60m bsl overlaying bedrock. Seismic cone penetration test was also carried out in the same exploratory hole and the shear wave velocity ( $V_s$ ) was estimated. Cone resistance ( $q_c$ ) and sleeve friction resistance ( $f_s$ ) are shown in Figure 2.

The determination of the soil stratigraphy and the identification of the soil type was based on the CPT-based charts proposed by Robertson (2010) and Tumay et al. (2008). The soil behaviour type zones are colour based to aid visual representation (Figure 2). Based on the normalised CPT soil classification, up to approximately 22m bsl the soil can be classified as loose sand to silty sand. Between 22m and 33m bsl the soil is a mixture of very loose to loose sand and silt mixtures and between 33m and 38m bsl clay, while below 38m bsl the soil is mainly classified as loose to medium dense sand and silt mixtures, with thin layers of clay and clean sand to silty sand. Table 1 summarises the soil profile identified based on the CPTu data. A unique soil ID is assigned to each layer for reference purposes. The soil type for stratified 03 and 05 has been considered as mixtures of sand and clay soils.

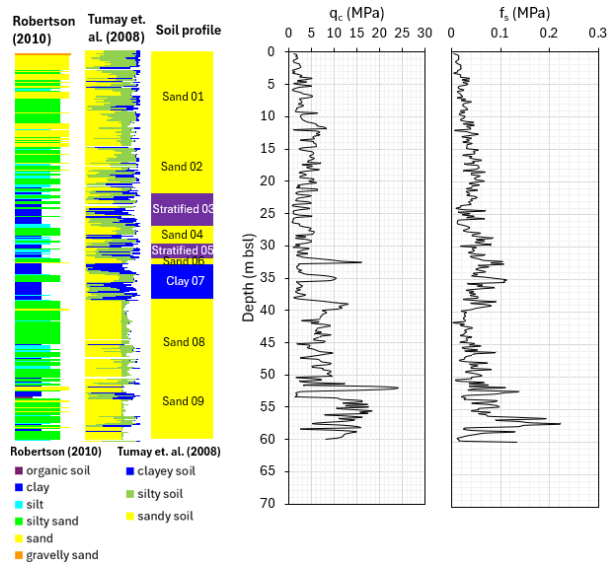


Figure 2. Soil type, cone resistance  $q_c$  and sleeve friction resistance  $f_s$ .

Table 1. Soil profile.

Soil layer ID	Depth below Seabed Level (m bsl)		Thickness (m)
	Top	Bottom	
Sand 01	0.0	11.0	11.0
Sand 02	11.0	22.0	11.0
Stratified 03	22.0	27.0	5.0
Sand 04	27.0	29.8	2.8
Stratified 05	29.8	32.0	2.2
Sand 06	32.0	33.0	1.0
Clay 07	33.0	38.3	5.3
Sand 08	38.3	50.0	11.7
Sand 09	50.0	60.0	10.0

## 3 LIQUEFACTION ASSESSMENT

### 3.1 Simplified method (Method A)

This study aims to estimate the liquefaction potential using the CPT-based empirical method proposed by Boulanger and Idriss (2014). Cyclic Stress Ratio (CSR) was computed empirically as shown in Equation (1) using the stress reduction coefficient  $r_d$  assessed via simplified equation proposed by Idriss and Boulanger (2006) as shown in Equation (2) and Equation (3).

$$CSR_{M,\sigma'_v} = \frac{\tau_{cyc}}{\sigma'_v} = \frac{0.65\tau_{max,z}}{\sigma'_v} = 0.65 \frac{\sigma_v a_{max}}{\sigma'_v g} r_d \quad (1)$$

where  $\tau_{cyc}$  is the cyclic shear stress assumed to have an amplitude of 65% of the peak cyclic shear stress;  $a_{max}$  is the maximum horizontal acceleration at the ground surface (PGA);  $g$  is the gravitational acceleration;  $r_d$  is a stress reduction coefficient that accounts for the flexibility of the soil column;  $\sigma'_v$  the effective vertical stress,  $\sigma_v$  is the total vertical stress at depth  $z$ , and  $\tau_{max,z}$  is the computed peak cyclic shear stress at depth  $z$ .

For  $z \leq 34m$

$$r_d = \exp[a(z) + \beta(z) \cdot M] \quad (2)$$

$$a(z) = -1.012 - 1.126 \sin\left(\frac{z}{11.73} + 5.133\right)$$

$$\beta(z) = 0.106 + 0.118 \sin\left(\frac{z}{11.28} + 5.142\right)$$

For  $z > 34m$

$$r_d = 0.12 \exp[0.22 \cdot M] \quad (3)$$

where  $M$  is the earthquake magnitude,  $z$  is the depth below the ground surface in meters and the arguments inside the sin terms are in radians.

Factor  $r_d$  is a nonlinear function of soil stratigraphy, dynamic soil properties (e.g. low strain shear modulus and soil damping), and input earthquake ground motion. The uncertainty in  $r_d$  increases with increasing depth such that Equation (2) should only be applied for depths less than about 20 m. Although, the stress reduction coefficient  $r_d$  can be assessed in practice via simplified equations in a form of depth-dependent, earthquake magnitude- and depth- dependent and more complex forms (Cetin et al., 2004), remarkable scattering in the values of  $r_d$  are observed in literature (Bán et al., 2016). Because of this high level of uncertainty, factor  $r_d$  and hence CSR, can be estimated most accurately with a detailed site-specific response analysis (SSRA). Liquefaction evaluations for depths greater than about 20m can often benefit from site response analyses to estimate the earthquake-induced cyclic stress ratio because the uncertainty in  $r_d$  becomes large at these depths (Boulanger and Idriss, 2014).

The factor of safety against cyclic liquefaction is determined by comparing the cyclic resistance ratio (CRR) to the cyclic stress ratio (CSR). Liquefaction is expected to be induced under a specific ground motion if the CSR exceeds the CRR, i.e. soils tend to liquefy if FoS is less than 1.0 defined as shown in Equation (4):

$$FoS = \frac{CRR_{7.5}}{CSR} \times MSF \times K_\sigma \quad (4)$$

where  $CRR_{7.5}$  is the cyclic resistance ratio and  $MSF$  is the magnitude scaling factor and  $K_{\sigma}$  is the overburden correction factor as defined by Boulanger and Idriss (2014).

### 3.2 Numerical method (Method B)

DEEPSOIL v7.0 was used for the One-Dimensional Site Response Analysis (1-D SSRA). The pressure dependent hyperbolic MKZ model was used to simulate the non-linear behaviour of the soil under earthquake shaking. The results from SSRA were then used for liquefaction assessment. The bedrock was assumed at 60m depth below seabed. Input soil parameters for the soil layers are the shear wave velocity and unit weight, while the soil dynamic properties include the modulus reduction and the damping ratio.

Two strong earthquake recordings (EQ #1 and EQ #2) of a moment magnitude  $M=7.7$  derived from Probabilistic Seismic Hazard Analysis (PSHA) were used as the outcrop motion applied at the bottom of the model in DEEPSOIL. The acceleration time histories and the corresponding acceleration response spectra are shown in Figure 3. The maximum acceleration and duration of EQ #1 are 0.38g and 67sec, while for EQ #2 are 0.43g and 30sec, respectively. The fundamental frequency of EQ #1 and #2 is 6.5Hz and 6.1Hz, respectively.

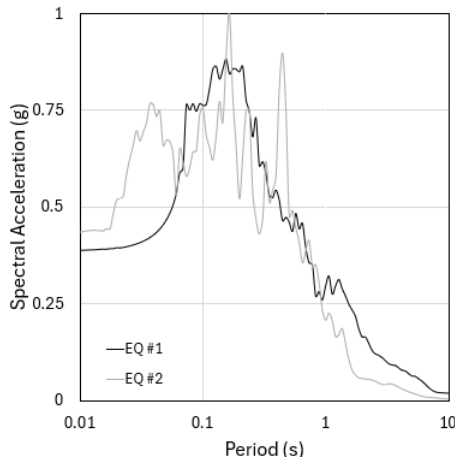


Figure 3. Acceleration-time histories and spectral acceleration of input earthquakes

The soil layers presented in Table 1 were divided into layers with smaller thicknesses, in such a way that the maximum frequency of each layer is not less than 50Hz (Figure 4). The shear wave velocity and unit weight curve used in DEEPSOIL is compared with the empirically derived CPT correlation from literature (Robertson and Cabal, 2022) and available laboratory tests (i.e. resonant column and bender element), while the relative density ( $D_r$ ) of each layer is estimated using the CPT-based empirical correlation proposed by Jamiolkowski et al. (2003), as shown in Figure 4. The modulus reduction and damping curves implemented in DEEPSOIL were selected to closely match those of sand and clay, based on available laboratory data and published literature (Darendeli, 2001; Vucetic and Dobry, 1991), as illustrated in Figure 5. For those in the transition zone, each layer has been categorized according to its main soil type. For the purposes of this study, both Stratified 03 and Stratified 05 are analyzed as sand layers. It is noted that cyclic resistance ratio (CRR) for Method B is calculated similarly to Method A, i.e. using the CPT-based empirical method proposed by Boulanger and Idriss (2014).

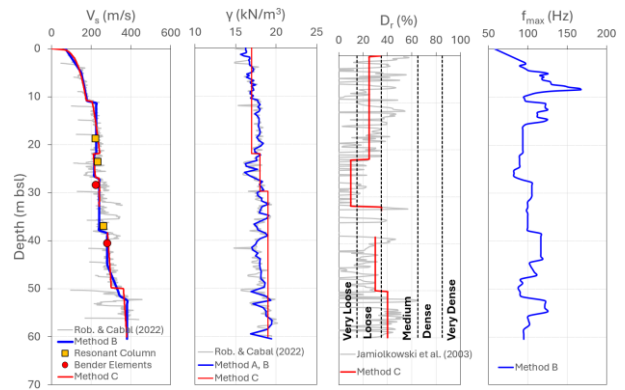


Figure 4. Shear wave velocity, unit weight, relative density and maximum frequency

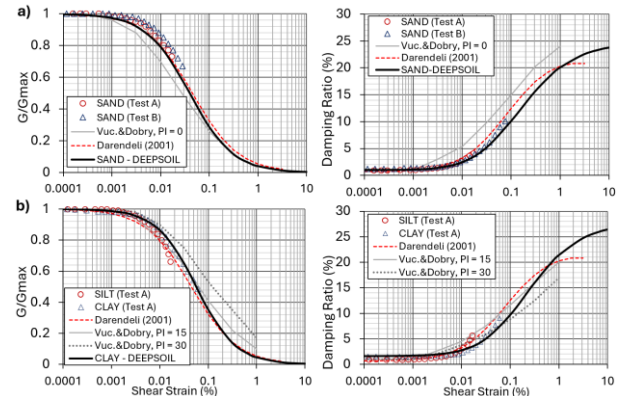


Figure 5. Shear modulus reduction and damping ratio curves for a) Sand and b) Clay

### 3.3 Finite Element Analysis (Method C)

A two-dimensional fully coupled, nonlinear dynamic finite element (FE) analysis was carried out to estimate the liquefaction potential. Plaxis 2D Ultimate software and the PM4Sand constitutive model were used to model the dynamic behaviour of the soil under the earthquake loading. PM4Sand is a stress-ratio controlled, critical state compatible, bounding surface plasticity model for sands (Boulanger and Ziotopoulou, 2017).

The site-specific response analysis was performed in free-field conditions and the earthquake records were applied at the bedrock. Dynamic boundary conditions were modelled with tied degrees of freedom. The mesh density was determined using Equation (5):

$$\text{Average Element Size} = V_{s,min} / 8 f_{max} \quad (5)$$

where  $V_{s,min}$  is the lowest shear wave velocity and  $f_{max}$  is the highest frequency of the input earthquake.

PM4Sand model has advantages and limitations related to their use and applicability (Tolozza, 2018). Although PM4Sand model is adequate to capture liquefaction behaviour of sand layers, it does not accurately capture the initial stress conditions (Subasi et al., 2021; Vilhar et al., 2018). Therefore, the Hardening Soil model with small-strain stiffness (HSsmall) was used to accurately model the initial stress conditions.

The numerical analyses were carried out in two stages: 1) first stage, the initial stress field was established using  $K_0$  procedure and 2) second stage, the dynamic analysis was performed by using PM4Sand for the sand layers and HSsmall for the clay layer. The drainage type was chosen as Undrained A to be able to generate excess pore pressures.

### 3.3.1 PM4Sand calibration

The primary input parameters of the PM4Sand model are the apparent relative density  $D_r$ , the shear modulus coefficient  $G_{0,PM4Sand}$  and the contraction rate parameter  $h_{p0}$ . The detailed descriptions and equations of the PM4Sand model are documented and explained in Boulanger and Ziotopoulou (2017) as suggested by PLAXIS (2024). These parameters were calibrated using two stress-controlled cyclic direct simple shear (CDSS) tests carried out in soil samples retrieved from depths 9.1m and 44.6m bsl. The primary input parameters for the PM4Sand model for sand layers are summarised in Table 2. All secondary input parameters were set to their default values.

Table 2. PM4Sand constitutive model primary parameters.

Layer	$\gamma$ (kN/m <sup>3</sup> )	$D_r$ (%)	$G_{0,PM4Sand}$ (-)	$h_{p0}$
Sand 01	17	25	700	0.7
Sand 02	17	25	950	0.7
Stratified 03	17	10	700	0.7
Sand 04	18	10	800	0.7
Stratified 05	18	10	770	0.7
Sand 06	19	35	740	0.5
Sand 08	19	30	950	0.5
Sand 09	19	40	1370	0.5

For the calibration of primary input parameters of the PM4Sand model, the following steps were followed:

- **Step 1:** Select a value for the relative density  $D_r$ . The relative density  $D_r$  was estimated using the CPT-based empirical correlation proposed by Jamiolkowski et al. (2003), as shown in Figure 4. It should be noted that  $D_r$  is considered an "apparent relative density", hence  $D_r$  can be adjusted to improve the calibration to some other relationship or data.
- **Step 2:** Select a value for the shear modulus coefficient  $G_{0,PM4Sand}$ . The  $G_{0,PM4Sand}$  parameter was calibrated to fit shear wave velocity  $V_s$  using Equation (6), as shown in Figure 4.

$$G_{0,PM4Sand} = \frac{\left(\frac{\gamma}{g} \times V_s^2\right)}{P_A} \sqrt{\frac{P_A}{\frac{1+K_0}{2} \times \sigma'_v}} \quad (6)$$

$$K_0 = 1 - \sin \phi'$$

where,  $\gamma$  is the soil unit weight,  $g = 9.81 \text{ m/s}^2$ ,  $P_A$  is the atmospheric pressure (101.325 kN/m<sup>2</sup>),  $V_s$  is the shear wave velocity,  $\sigma'_v$  is the vertical effective stress,  $K_0$  is the normally consolidated coefficient of lateral earth pressure,  $\phi$  is the friction angle.

- **Step 3:** Calibration of contraction rate parameter  $h_{p0}$ . During model calibration, this variable was adjusted to obtain a target cyclic resistance ratio (liquefaction resistance) with respect to cyclic laboratory tests. Generally, decreasing the value of  $h_{p0}$  gives more contractant soil behaviour and consequently a lower CRR (Plaxis, 2024; Toloza, 2018). Undrained cyclic direct simple shear (CDSS) stress controlled single element tests, using SoilTest option in Plaxis software, were performed to calibrate the parameter  $h_{p0}$  and achieve a certain cyclic resistance ( $CRR = \tau_{cyc}/\sigma'_v$ ). Contraction rate parameter  $h_{p0}$ , can be defined/calibrated based on the pore pressure ratio ( $r_u \approx 1$ ) or based on a specific shear strain. In this study, the onset of liquefaction was defined as 3% single amplitude shear strain (Boulanger and Ziotopoulou, 2017). Comparison between PM4Sand model and laboratory

Sample 9.1m bsb:  $\sigma'_v = 45 \text{ kPa}$ ,  $K_0 = 0.5$ ,  $CRR = \tau_{cyc}/\sigma'_v = 0.2$ ,  $D_r = 0.25$ ,  $G_0 = 700$ ,  $h_{p0} = 0.7$

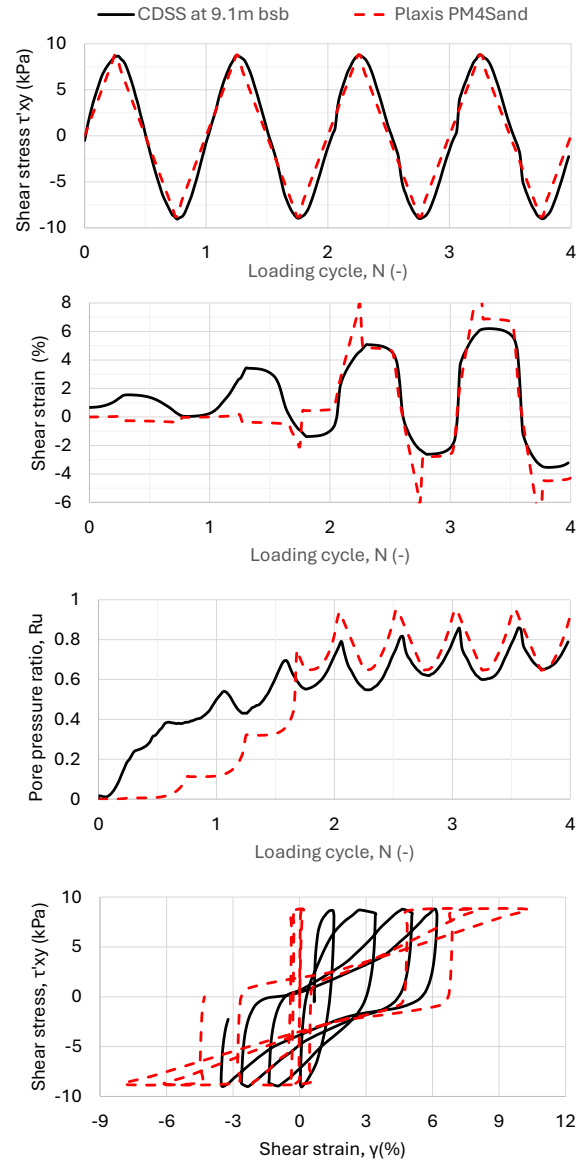


Figure 6. Comparison between Plaxis PM4Sand model and laboratory CDSS test results for soil sample at 9.1m bsl.

CDSS test results for soil sample at 9.1m bsl (Sand 01) for cyclic resistance  $CRR = \tau_{cyc}/\sigma'_v$  of 0.2, is shown in Figure 6.

## 4 RESULTS & DISCUSSION

The PGA vs depth profile and a comparison between magnitude-dependent stress reduction coefficient  $r_d$  (I&B14) given in Equation (2) & (3) and DEEPSOIL (i.e.  $r_d = \frac{\tau_{max,z}}{\sigma'_v(a_{max}/g)}$ ) for the two earthquakes are shown in Figure 7. Remarkable scattering in the values of  $r_d$  are observed in agreement with literature (Bán et al., 2016).

Liquefaction triggering analyses results using Method A, B and C are shown in Figure 8. Computed CSR based on Method A for EQ #2 is considerably higher than the corresponding CSR computed by Method B. On the contrary, for EQ #1, the CSR based on Method A and B are in better agreement. The reason is that the CSR estimated based on

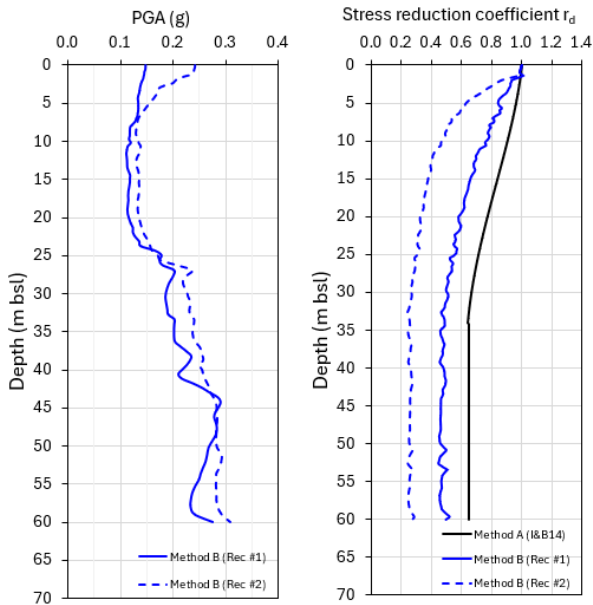


Figure 7. PGA vs depth profile (left) and stress reduction coefficient  $r_d$  vs depth profile (right) derived using Method B for motions EQ #1 and EQ #2 and Method A

Method A is strongly affected by the horizontal maximum acceleration  $a_{max}$  at the ground surface, as  $r_d$  (assessed via Method A) is affected only by the earthquake magnitude  $M$ , hence remains the same for the two examined earthquake motions. Thus, for Method A, a high variability of the CSR (therefore also in FoS) is observed between the two earthquake recordings, while this is not observed using Method B.

A high variability of the FoS is observed between Method A, B and C. Based on Method A, FoS is lower than 0.75 along the whole depth (~55m) for both EQ #1 and EQ #2. Thus, all the non-cohesive soil layers are liquefiable. On the contrary, FoS determined using Method B is lower than 0.75 up to

approximately 15m bsl for both earthquakes and marginally below 1.0 for depths up to 25m bsl. Method B indicates that critical liquefiable layer thickness is ~42m (i.e. 24% reduction compared to Method A) as the soil layers encountered at depths between 25m - 33m bsl and 52m - 57m bsl are not liquefiable (FoS  $\approx$  1.0).

For Method C, a threshold of excess pore pressure ratio  $r_u \geq 0.8$  is used in literature to define the limit for liquefaction potential (Olson et al., 2020). The maximum pore pressure ratio ( $r_{u,max}$ ) computed from Method C is generally less than 0.8, with thin soil sections assessed to be liquefied (i.e.  $r_{u,max} \geq 0.8$ ). Therefore, potential liquefiable zones for both EQ #1 and EQ #2 are between 2.3m - 5.5m bsl, 12.5m - 15.4m bsl, 27m - 28m bsl, 39.5m - 42m bsl, 47m - 48m bsl, 49m - 50m bsl, and 57.8m - 60m bsl. More specifically, results for EQ #1 indicate liquefaction between 2.3m - 5.5m bsl, 27m - 28m bsl, 47m - 48m bsl, and 58.5m - 60m bsl (i.e. total liquefaction thickness of 6.7m), while EQ #2 indicates liquefaction between 4.9m - 5.5m bsl, 12.5m - 15.4m bsl, 27m - 28m bsl, 39.5m - 42m bsl, 49m - 50m bsl, and 57.8m - 60m bsl (i.e. total liquefaction thickness of 10.2m). Thus, for Method C, the maximum critical liquefiable layer thickness is ~10.5m, approximately, indicating 81% and 75% reduction compared to Method A and Method B, respectively.

The acceptable value of FoS for liquefaction occurrence site depends on several factors such as site-specific conditions, and the type of the structure and its importance, and various guidelines have been proposed. Eurocode 8 proposes that soils can be considered susceptible to liquefaction when FoS is less than 1.25. According to USNRC Regulatory Guide 1.198, liquefaction triggering is achieved for soil with FoS  $\leq$  1.1 while for  $1.1 < \text{FoS} < 1.4$  significant soil softening may occur. Due to the lack of a standardised approach, Method A is considered appropriate for preliminary assessment. However, a more rigorous and advanced site-specific numerical analysis with appropriate constitutive model (e.g. PM4sand) is required for detailed design.

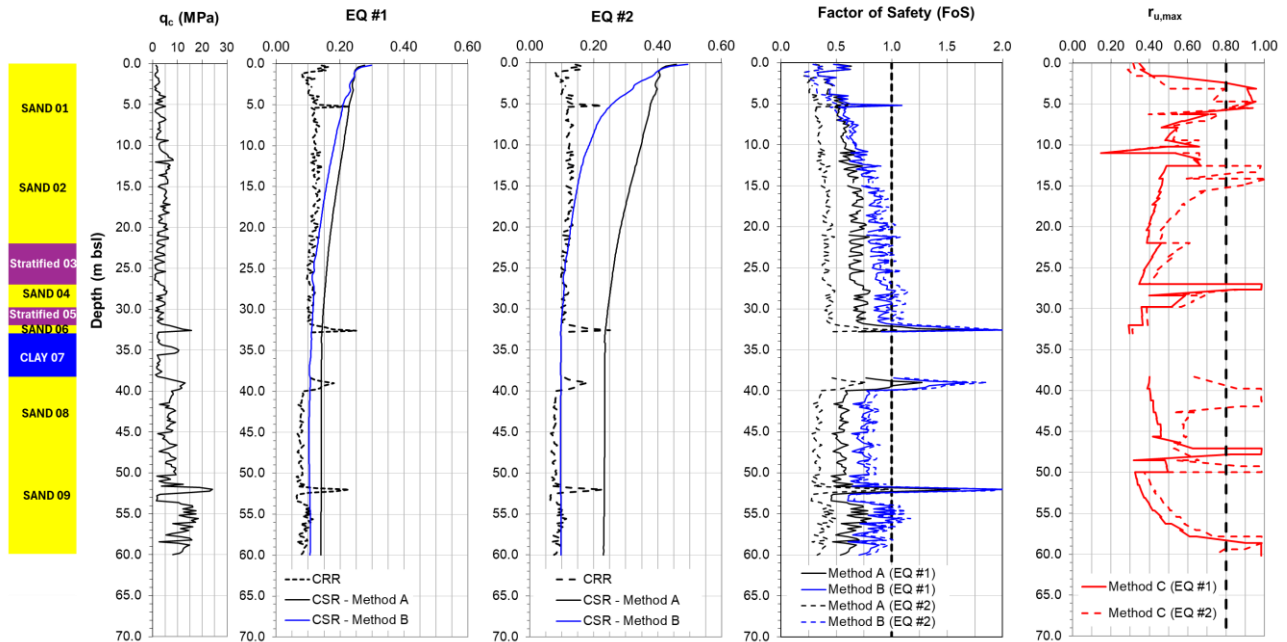


Figure 8. Liquefaction triggering analysis using Method A, B and C.

## 5 CONCLUSIONS

In this paper, the liquefaction assessment for typical offshore soil profile was carried out using three methods (a) simplified method (Method A), (b) numerical method (Method B), and (c) FE analysis (Method C). Based on the results presented in this study, the following conclusions can be drawn:

- Method A cannot capture the holistic behaviour of a potentially liquefiable soil deposit; it is appropriate as an initial assessment to understand the response of individual layers but is mostly very conservative. Thus, it is appropriate as screening liquefaction assessments but not suitable for detailed designs.
- For Method A, CSR is strongly affected by  $a_{max}$  at ground surface. For Method B, when CSR has been computed based on 1D site response analysis, the variation of FoS with depth is low and approximately independent of the earthquake motions.
- For the case presented in this study, Method A and B indicated that the thickness of liquefiable soil is 55m, and 42m, respectively. However, Method C indicated that full liquefaction only occurs in limited layers of a few meters thickness. Thus, use of Method C would lead to safe and cost-effective offshore foundation design. Therefore, Method C is more appropriate for structural and cost optimisation during offshore detailed design.
- Liquefaction evaluation at depths greater than 20m is not recommended using Method A. For deeper depths. Methods B and C are recommended.
- Method C allows to obtain pore pressure generation during earthquake loading and therefore provides insight into real soil behaviour and ability to differential full liquefaction and partial liquefaction.

In conclusion, Method A is overconservative and only appropriate as an initial assessment but not suitable for detailed designs. Method B is best for conceptual stage and FEED designs. It is acceptable for detailed design but will not lead to optimised foundation design. Method C would lead to safe and cost-effective offshore foundation design (Figure 9). Therefore, Method C is recommended for cost optimisation during offshore detailed design of foundations.

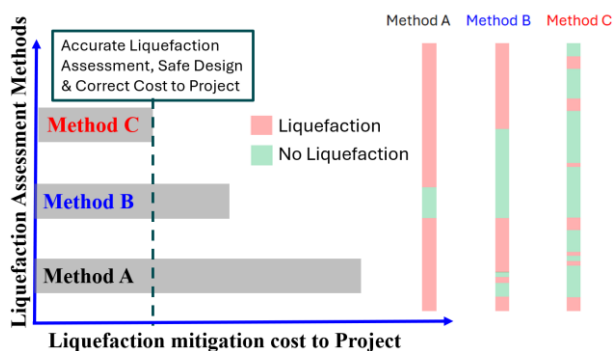


Figure 9. Summary of liquefaction mitigation cost to project

## 6 ACKNOWLEDGEMENTS

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