

Bayesian Inference of Grouted Anchors for Reliability and Pull-Out Capacity Estimation

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ABSTRACT: Bayesian analysis is a statistical method for uncertainty quantification. The aim is the estimation of a posterior-distribution of uncertain parameters or data, taking prior knowledge and observed data into account. In geotechnical engineering, data of higher quality is scarce. Therefore, this method is of special interest because the estimated posterior-distribution is an approximation of a distribution which comes about for a large number of observations. Grouted anchors are structural elements commonly used in larger geotechnical engineering projects. Nevertheless, there are few methods in research or practice which enable a precise estimation of grouted anchor bearing capacity. Existing semi-empirical procedures do not yield sufficient accuracy, which is why according to the Eurocode every anchor needs to be tested on site to verify its pull-out resistance. If too many anchors fail, the design is generally adapted without taking into account the additional informational value which the load tests provide apart from verification. In this publication, a data-driven Bayesian framework is presented, which uses anchor suitability test data to make predictions about the pull-out capacity and the probability of failure in an individual model (single anchors) and a hierarchical model (anchor groups). The statistical model consists of multivariate-normal likelihood function, weak prior-distribution and random variable covariance formulation. The results indicate plausible probabilities of failure, in line with the data distribution, providing an added value in grout anchor performance assessment. Major influence on the quality of the inference can be appointed to data quality and covariance modelling choices. By quantifying the pull-out capacity distribution at limit state, the results can form the foundation for reliability analysis of anchored retaining structures.

KEYWORDS: Bayesian, anchor, reliability, resistance, bearing capacity.

1 INTRODUCTION

Grouted anchors are a heavily monitored structural element in geotechnical engineering, used in many applications to tie back forces into soil or rock. Even though their bearing behavior was intensively researched ever since their introduction (Littlejohn 1970, Ostermayer 1975) their pull-out capacity remains difficult to estimate due to various sources of uncertainty. Thus, from early on, national and international guidelines (DIN 4125, EN ISO 22477-5) recommended to conduct elaborate full-scale load tests to explore and verify the boundaries of these elements for the conditions at the respective site. Three types of tests exist, investigation tests, suitability tests and acceptance tests, where each test is less elaborate than the previous one in the mentioned order. For tests conducted according to EN ISO 22477-5 testing procedure 1, the load is applied in steps, and being held for specific time periods, while the anchor head displacement over time is measured. For the performance assessment, these non-linear displacement measurements are linearized by projecting them on a logarithmic time axis. The gradient of this linearization is known as the creep coefficient (Eq. 1).

$$k_s = \frac{s_b - s_a}{\log\left(\frac{t_b}{t_a}\right)} \quad (1)$$

The points a and b are not fixed values, but are supposed to be chosen in a way, that they represent the gradient of the curve as a whole. As a limit state criterion in Germany, a limiting creep coefficient of 2.0 mm prevailed for investigation and suitability tests (DIN 1054), after which the creep displacements usually become too large for permanently anchored systems. By collecting these measurements for each load step, the idealized creep coefficient evolution for increased load can be illustrated, which is supposed to follow an exponential trend (Ostermayer 1975).

After completion, the data gathered is seldomly used for optimization or reliability analysis, also because it is not available in sufficient quantities to perform conventional

statistical analysis. In a reliability analysis one calculates with the statistical distributions of uncertain parameters instead of a deterministic estimate, combined with a partial safety factor. Thereby uncertainties are propagated through the entire model and can influence the decision-making (Baecher and Christian 2011). Nevertheless, knowledge about parameter distributions is necessary to perform such an analysis, which is due to the discussed data scarcity difficult to establish. Bayesian analysis on the other hand, is a tool which provides the means for uncertainty quantification, able to handle scarce data, by combining data with prior knowledge (Gelman et al. 2013). Several applications of Bayesian analysis in geotechnical engineering already exist, mainly due to the characteristic scarce data suitability. Effenberger and Moormann (2025a) treats Bayesian modelling of grouted anchors too, whereas the covariance modelling choices are investigated more thoroughly with the aim to separate observation error from model error. Effenberger and Moormann (2025b) focusses on pull-out capacity estimation, illustrating how predictions from suitability test alone are inline with measured capacities from investigation tests of grouted anchors. Van der Zon et al. (2024) introduce a simple phenomenological model to estimate the creep evolution for increased load to make an updated estimate of the bearing behavior of grouted anchors. Other applications are updating pile bearing capacity (Huang 2016) or the estimation of soil parameters directly from laboratory data (Zhang 2022). Depending on the structure of the data, the appropriate Bayesian model might be a hierarchical one, where the uncertain parameters are modeled as distributions, with their hyperparameters as random variables. Underlying assumption is that the parameters in question stem from the same group, can be modeled by the same distribution therefore also be sampled from the same distribution (Gelman et al. 2013). Bozorgzadeh and Bathurst (2020) provide informative foundations regarding hierarchical modelling and illustrate this procedure with a case study of reinforced soil retaining walls.

2 METHODOLOGY

Bayesian analysis is a powerful tool for uncertainty quantification. By combining prior knowledge with data, one

can estimate a posterior distribution (Eq. 1), which serves as an estimate for a distribution as if a lot of observations would be at hand. Since data is generally scarce in geotechnical engineering, this technique has promising potential. The posterior distribution according to Bayes Theorem is given by,

$$P(\theta|D) = \frac{P(D|\theta) \cdot P_0(\theta)}{\int P(D|\theta) \cdot P_0(\theta)} \quad (2)$$

Key components are the likelihood function $P(D|\theta)$ and the prior distribution P_0 . The likelihood function is defined as a multivariate normal distribution consisting of a mean, position and covariance.

$$P(D|\theta) = \frac{e^{\frac{1}{2}(\mu-D)^T \Sigma (\mu-D)}}{\sqrt{(2\pi)^k |\Sigma|}} \quad (3)$$

$$P(D|\theta) = \prod_{j=1}^D \frac{e^{\frac{1}{2}(\mu-D_j)^T \Sigma (\mu-D_j)}}{\sqrt{(2\pi)^k |\Sigma|}} \quad (4)$$

The mean value for this application is the data vector, anchor head displacement, measured over time for constant load. These data are obtained by following national and international guidelines for construction and design of grouted anchors (EN ISO 22477-5), which prescribe the execution of suitability tests on min. 3 anchors on site due to German national annex to EN 1537. Here the procedure is illustrated using suitability test data of permanent anchors, tying back a retaining wall of a tunneling portal, where the grout body is located in clay stone.

As the measured anchor head displacement is supposed to be related to parametric variance, an analytical or other comparably low-cost calculational model is necessary, able to reproduce the measurement. Here, the rheological displacement model by Montero-Cubillo et al. (2020) is used. Thus, the rheological soil parameters, which govern the time-dependent anchor head response are treated as the random variables.

Since there is basically no knowledge and also no physical indication about the rheological soil parameters on site at the time of the analysis, no prior knowledge is introduced via the prior distributions of these random variables. They were chosen as uniform distributions with broad bounds not to constrain the posterior distribution, making the entire framework a data-driven approach. This prior distribution modeling choice implies that larger and smaller displacement measurements compared to the ones observed are conceivable.

The last component of the likelihood function is the covariance matrix, determining directly the variance of the likelihood and thereby of the posterior distribution. As no prior knowledge on how large the uncertainty of this modelling problem is, was assumed, and because this uncertainty quantification is a result of major interest, the covariance matrix with its terms were modeled as random variables. Regarding the matrix structure, it was assumed, that the variance itself must deviate over the timeframe, while the covariance from one timestep to the next should be negligible. Therefore, a diagonal matrix was chosen, where each term on the diagonal is modelled as a separate random variable for the individual model. In the hierarchical model, the hyperparameters of a lognormal distribution were treated as the random variables respectively.

Obtaining the posterior distribution (Eq. (2)) is a computational challenge, due to the integral over the entire parameter space in the denominator. Apart from conjugate distribution combinations, usually no analytical solutions are available, which is why numerical solvers need to be employed. In this study the posterior is evaluated using the No-U-Turn-

Sampler (Hoffman and Gelman 2014), implemented in the python library Jax, where GPU computations are an option, increasing the speed drastically compared to the CPU. The posterior result consists of the marginal distributions of each random variable. However, it is difficult to judge anchor bearing behavior based on parameter distributions one had little knowledge about in the first place. Therefore, the posterior-predictive distribution is evaluated in a forward analysis.

$$P(D'|D) = \int P(D'|\theta) P(\theta|D) d\theta \quad (5)$$

Based on this, the creep coefficient distribution is estimated, using the posterior-predictive samples, which is the foundation for the reliability estimate. As indicated by code, an anchor is regarded to be at geotechnical limit state once the creep coefficient becomes larger than 2.0 mm.

$$g(k_s(D')) = k_{s,lim} - k_s(D') \quad (6)$$

$$P_f = \int_{\{D': g(k_s(D')) \leq 0\}} P(D'|D) dD' \quad (7)$$

Since this probability of failure estimate is based on samples of the posterior distribution, uncertainty is involved in such a Monte-Carlo approximation, i.e. if no sample falls into the limit state domain, denoting failure. Thus, the conjugate beta-binominal model as proposed by Betz et al. (2022) is used to quantify the uncertainty revolving around the reliability estimate. The more samples fall in to the limit state domain compared to the total amount of samples, the larger the confidence in the probability of failure estimate.

Apart from reliability estimates, the framework can deliver pull-out capacity distribution estimates. As early grouted anchor studies (Ostermayer 1975) already indicated, the creep coefficient evolution for increasing load follows an exponential trend.

This key characteristic is leveraged in combination with the posterior-predictive creep coefficient distribution of each load step for a probabilistic estimate of the load at limit state. A non-physical exponential function is fitted to the posterior-predictive distributions and extrapolated accordingly until the limit state is reached.

$$k_s(P) = (e^{P \cdot a} - 1) \cdot b \quad (8)$$

The resulting distribution is regarded as the pull-out capacity estimate.

3 RESULTS

Only the results of the posterior-predictive distributions and the probabilistic extrapolations, not the posterior distributions are reported, since they are more illustrative. The individual model aims to quantify the total uncertainty, consisting mainly of observation uncertainty and model uncertainty (Figure 1). The hierarchical model (Figure 2) considers the data of the entire group tested on site, and therefore captures a wider spectrum of uncertainty.

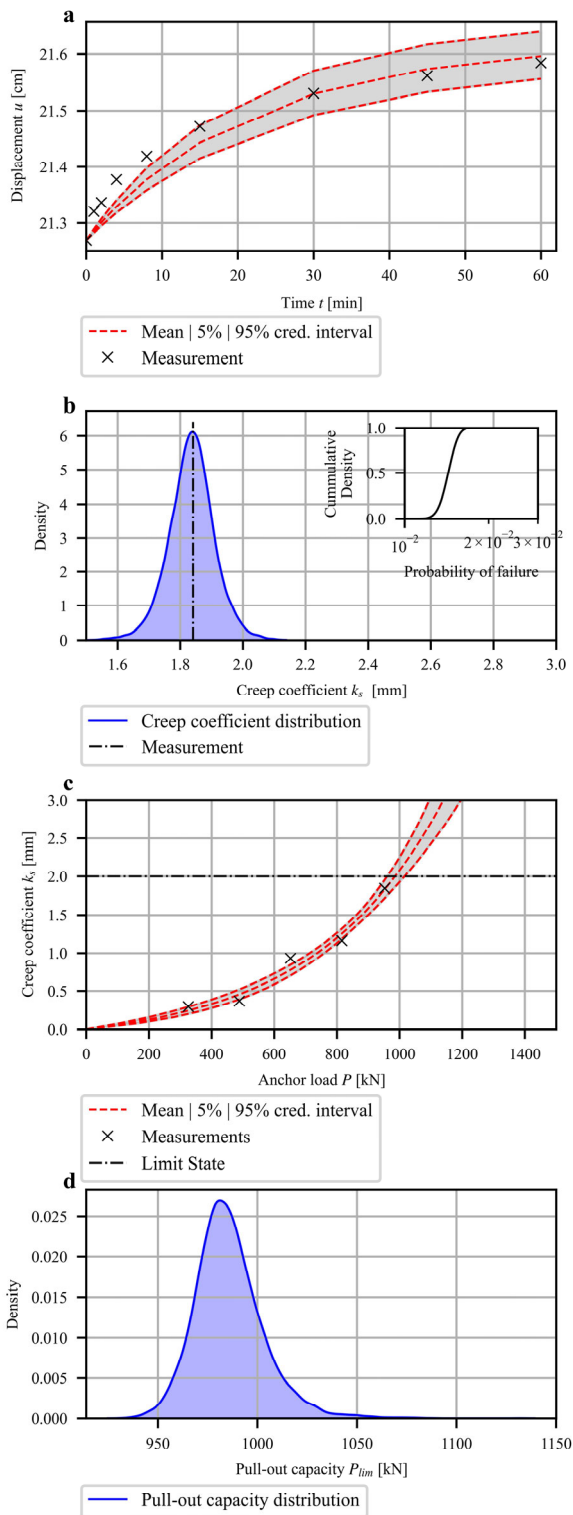


Figure 1. Individual model, (a) posterior-predictive distribution, (b) posterior-predictive creep coefficient distribution, (c) probabilistic extrapolation, (d) pull-out capacity distribution.

As Figure 1a illustrates, the posterior-predictive intervals are wrapped around the measurement vector. This uncertainty, propagates to the posterior-predictive creep coefficient distribution (Figure 1b), which here resembles a normal distribution for most anchors. The probabilistic extrapolation (Figure 1c) shows very dense credible intervals for this specific

anchor, which results in a dense pull-out capacity distribution (Figure 1d) respectively.

Figure 2 illustrates the corresponding results for the hierarchical model, where all observations of the group are combined to determine hyperparameters of the soil parameter distributions.

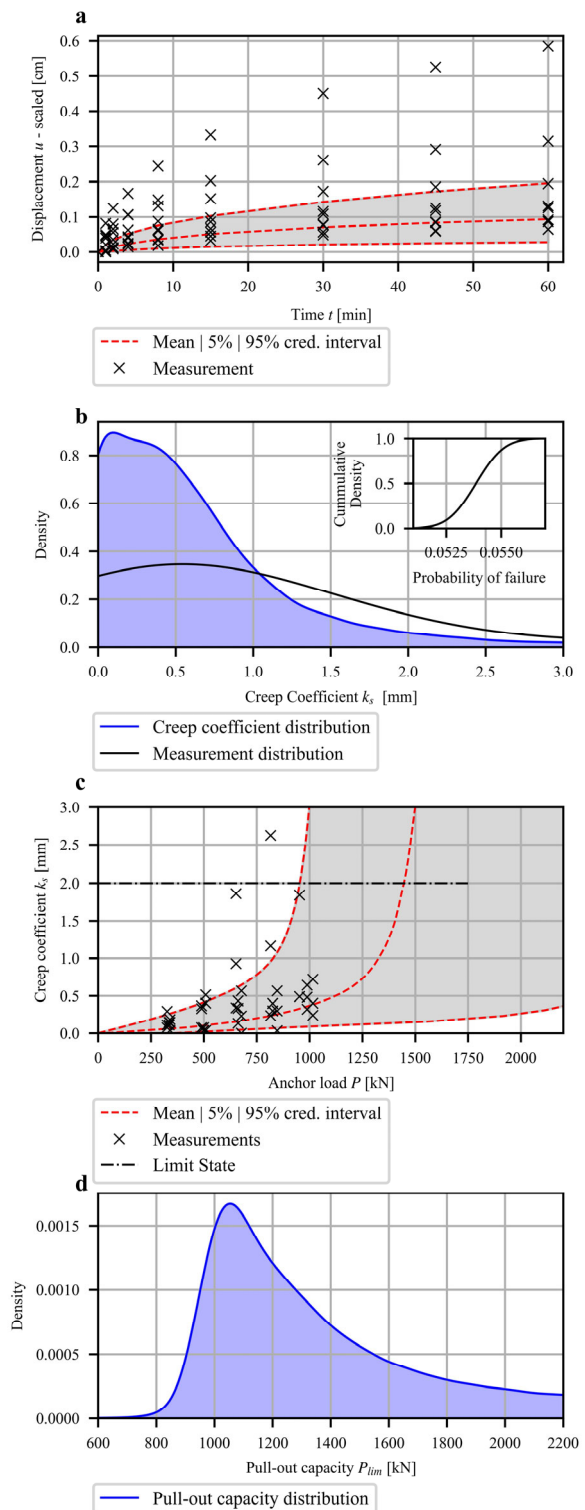


Figure 2. Hierarchical model, (a) posterior-predictive distribution, (b) posterior-predictive creep coefficient distribution, (c) probabilistic extrapolation, (d) pull-out capacity distribution.

The credible interval of the posterior-predictive distribution (Figure 2a) is much denser compared to the variance which the data indicates. Two measurements clearly lie outside these bounds, while one measurement is fairly close at the border. The displacements were scaled to zero, to only depict time-dependent behavior, since the initial displacement at $t = 0$ does not directly influence the limit state criterion which was considered in this study. Because the posterior-predictive is denser than the measurements, the respective creep coefficient distribution (Figure 2b) is denser as well compared to the data. The probabilistic extrapolation credible intervals (Figure 2c) are highly nonlinearly distributed within the highlighted domain of its credible intervals. The distributions shape is analogous to the pull-out capacity estimation distribution (Figure 2d), which was observed to be lognormal, where the strong tail indicates more skewedness.

4 DISCUSSION

The spread of the posterior-predictive and therefore all of the downstream variance observable in the results is mainly dependent on the covariance inherent to the data and thus the modelling choices respectively, for both, individual and hierarchical model. Variance of the posterior is directly influencing the variance of the posterior-predictive and posterior-predictive creep coefficient distribution, thus affecting all downstream models, like the probability of failure estimate.

The illustrated cumulative density of the probability of failure becomes wider for fewer samples in the limit state domain. For the illustrated anchors, this domain is sufficiently explored in both models, resulting in steep *cdf* curves and confidence in the corresponding estimate. The probability of failure was comparatively large for both models. For the individual model, the last creep measurement came to a value close to the limit state criterion, which naturally indicates a larger probability of exceeding said limit state. In the hierarchical model, the probability of failure of the anchors in the group was determined to be approx. 5.5%. However, since one of the nine considered anchors failed during testing, the data alone indicates a probability of failure of 1/9, approx. 11%, which means that because the procedure appoints less likelihood to the higher creep values to occur, their contribution is not as heavily weighted.

Regarding the probabilistic extrapolation, the procedure works well, if the data actually shows the trend imposed via the exponential function, which is often not the case if real data is involved. Especially for the individual model, if no exponential behaviour is at hand, the resulting pull-out capacity is wide with high variance, nevertheless appropriately reflecting the uncertainty revolving around the bearing behavior of such an anchor.

5 CONCLUSIONS

The presented framework enables the estimation of grouted anchor reliability and pull-out capacity. For the reliability assessment, the posterior-predictive creep coefficient distribution is determined through a Bayesian analysis. The limit state function is assessed for a limiting creep coefficient of 2.0 mm. For the pull-out capacity distribution, a non-physical exponential function is probabilistically extrapolated using the posterior-predictive distributions of each load step and the limit state criterion mentioned above.

While the resulting probabilities of failure are in line with the measurements, they are directly influenced by the variance of the posterior and posterior-predictive distributions. Therefore, appropriate covariance modelling choices are

important to ensure reasonable results. Furthermore, as the probabilities of failure can be large, these should not be interpreted as a catastrophic failure condition, but rather as unsatisfactory performance, not necessarily equivalent to the collapse of the entire anchored structure.

The probabilistic extrapolation yields appropriate results for both models, in line with the measurements, where the result is of lognormal shape. Especially the weak tails of the pull-out capacity distribution are of interest, since the strong tails can denote a geotechnical bearing capacity much larger than the structural bearing capacity of the steel tendon. Future analyses are supposed to determine the suitability of the estimated pull-out capacity distributions for the reliability analysis of an anchored structure.

6 REFERENCES

- Baecher, G.B. and Christian, J.T. 2011. *Reliability and Statistics in geotechnical engineering*. John Wiley & Sons.
- Betz, W. and Papaioannou, I. and Straub, D. 2022. Bayesian post-processing of Monte Carlo simulation in reliability analysis. *Reliability Engineering & System Safety* 227.
- Bozorgzadeh, N. and Bathurst, R. J. 2020. Hierarchical Bayesian approaches to statistical modelling of geotechnical data. *Georisk Assessment and Management of Risk for Engineered Systems and Geohazards* 16(3). 452-469.
- Deutsches Institut für Normung, 1976. DIN 4125 Verpreßanker für dauernde Verankerungen (Daueranker) im Lockergestein - Teil 2: Bemessung, Ausführung und Prüfung. Berlin. Beuth Verlag GmbH
- Deutsches Institut für Normung, 2021. DIN 1054 Subsoil—Verification of the safety of earthworks and foundations—Supplementary rules to DIN EN 1997-1. Berlin. Beuth Verlag GmbH
- Deutsches Institut für Normung, 2017. DIN SPEC 18537 Supplementary provisions to DIN EN 1537:2014-07. Execution of special geotechnical works – Ground anchors. Berlin. Beuth Verlag GmbH.
- Effenberger, M. J. P. and Moormann, C. 2025a. Bayesian Uncertainty Analysis of Grouted Anchors. In Proceedings of the *14th International Conference on Structural Safety and Reliability - ICOSSAR'25*. Los Angeles. Scipedia
- Effenberger, M. J. P. and Moormann, C. 2025b. Bayesian Inference of Grouted Anchors for Reliability Analysis. In Proceedings of the *9th International Symposium for Geotechnical Safety and Risk - ISGSR'25*. Oslo. (accepted)
- Gelman, A. and Carlin, J. B. and Stern, H. S. and Dunson, D. B. and Vehtari A. and Rubin, D. B. 2013. *Bayesian data analysis*. CRC Press Taylor & Francis Group
- Hoffman, M. D. and Gelman, A. 2014. The No-U-Turn Sampler: Adaptively setting Path Lengths in Hamiltonian Monte Carlo. *Journal of Machine Learning Research* 15. 1351-1381.
- Huang, J. and Kelly, R. and Li, D. and Zhou, C. and Sloan, S. 2016. Updating reliability of single piles and pile groups by load tests. *Computers and Geotechnics* 73. 221-230
- International Committee for Standardization, 2019. DIN EN ISO 22477 Geotechnical investigation and testing – Testing of geotechnical structures – Part 5: Testing of grouted anchors.
- Littlejohn, G. S. 1970. Soil anchors. *Ground Engineering*.
- Montero-Cubillo, N. S. and Galindo, R. and Serrano-González, A. and Olalla-Marañón, C. and Simic-Sureda, F. D. 2020. Analytical Model of an Anchored Wall in Creep Soils. *International Journal of Geomechanics* 20(4)
- Ostermayer, H. 1975. Construction, carrying behaviour and creep characteristics of ground anchors. *Diaphragm Walls and Anchorages*. 141-151.
- Van der Zon, J. and Flessati, L. and Mavritsakis, A. and Habets, C. and Schweckendiek, T. and Roubos, A. 2024. Identifying unaccounted capacity of ground anchors through Bayesian updating: a case study. *Georisk Assessment and Management of Risk for Engineered Systems and Geohazards*. 1-16.
- Zhang, J. and Yang, S. and Zhang, L. L. and Zhou, M. L. 2022. Bayesian estimation of soil-water characteristic curves. *Canadian Geotechnical Journal* 59(4). 569-582.