Proceedings of the 21st ICSMGE, Vienna, Austria, 14 – 19 June 2026. Pistrol, Adam & Schweiger (eds.)
Published by: ÖGG, Austrian Society for Geomechanics, Salzburg, Austria, ISBN 978-3-9503898-4-5
https://doi.org/10.53243/ICSMGE2026-873

Damping ratio determination for prescribed cycles from the free vibration decay (FVD) curve by the example of RCA–RTW mixtures

Katarzyna Gabryś, Wojciech Sas
Department of Geotechnics, Institute of Civil Engineering, Warsaw University of Life Sciences, Warsaw, Poland
katarzyna_gabrys@sggw.edu.pl

ABSTRACT: This study investigates the damping properties of recycled concrete aggregate (RCA) mixed with recycled tire waste (RTW) using the Free-Vibration Decay (FVD) method. The research focuses on understanding how varying rubber waste content (5%–20%) influences the material damping ratio (D) of these composite mixtures under dynamic conditions. The materials were tested in a resonant column apparatus under different isotropic effective stresses (90, 180, and 270 kPa). The authors focused on the correct number of consecutive cycles to determine the damping in FVD (two, three, five, ten, or average value up to 30 cycles). The results indicate that increasing rubber content leads to a higher damping ratio, reaching up to 6% for mixtures with 20% rubber content. The number of cycles has a more significant impact on the damping ratio for rubber-rich mixtures, with a slight decrease in damping observed after multiple cycles. When compared to natural soils like sand, the RCA–RTW mixtures exhibit significantly higher damping, primarily due to the viscoelastic properties of the rubber, which enable more efficient energy dissipation. These findings highlight the potential of RCA–RTW mixtures for vibration mitigation, seismic isolation, and dynamic load reduction in geotechnical applications, offering both enhanced damping performance and environmental benefits through the recycling of waste materials.

KEYWORDS: Resonant column, damping ratio, free-vibration decay, alternative materials, RCA–RTW mixture.

1 INTRODUCTION

In pursuit of sustainability and efficiency in the construction industry, and as part of efforts to streamline processes and innovate through new technologies, many alternative materials have emerged to replace or complement traditional materials. Alternative materials are defined as materials that have been historically viewed as low-value wastes but are increasingly recognized for their technical value and potential in various applications, particularly when combined as composite materials to optimize desired properties (Dawson, 2012).

In the context of geotechnical engineering applications, alternative materials include the following elements (Jastrzębska, 2025):

- Byproducts generated during minerals extraction, such as coal, ores, salt, and aggregates.
- Waste from industrial production: thermal processes in power plants and steelworks; mineral binding plants.
- Post-consumer, post-renovation, or dismantling products from private farms or construction, including road construction, such as concrete, aggregate, construction rubble, ceramic, or glass cullet.
- Used rubber materials, mainly car tires.
- Natural products from agriculture, breeding, and food production, such as: plant and animal fibers; coffee grounds; eggshells; ash from burning biomass, etc.
- Chemical waste includes fibers made from biopolymers (e.g., cellulose) or synthetic polymers (e.g., polypropylene, polyester) and synthetic fibers (e.g., polyethylene), or composite materials.
- Other materials that are no longer considered waste.

As with any other material, some alternative materials are better suited to certain applications than others. In the past, alternative materials were often seen as waste products of little value and were used rather indiscriminately because of their low cost. However, as their use has increased, the technical value of many alternative materials has become much better recognized (Netinger Grubeša et al. 2016). Using alternative materials conserves natural resources and energy, and avoids costly and potentially harmful waste disposal methods. Other benefits include decreased energy usage through the utilization of existing materials, reduced environmental contamination and global warming, and decreased landfill waste. However, investing in these materials is not always cost-effective, as creating new recycling facilities can be expensive (Dias et al. 2024).

It should be noted that different alternative materials are characterized by high variability in geotechnical parameters. Therefore, prior to their reuse, it is essential to carry out basic laboratory tests to determine the following properties: grain composition and size, compaction parameters, permeability conditions, swelling, and strength parameters, frost and abrasion resistance, California bearing ratio, etc. (Al-Naje et al. 2020).

This paper focuses on the dynamic properties of selected alternative aggregates: compositions of materials consisting of construction and demolition waste (in this case, RCA was selected) mixed with end-of-life materials (in this case, RTW was selected). The study particularly addresses the evaluation of the damping ratio in a resonant column, so-called materials damping in shear (Ds). In geotechnical engineering, the damping ratio of soil is a crucial parameter that quantifies how effectively the soil dissipates energy during dynamic or cyclic loading, such as that experienced during earthquakes or machinery vibrations (Soból et al. 2019). It is a dimensionless value representing the ratio of actual damping to critical damping. A higher damping ratio dissipates energy, reducing vibrations and potentially improving structural stability, while a lower ratio may lead to greater vibrations and potential damage.

The material damping ratio is traditionally determined from laboratory tests since accurate measurements over a range in strain levels can be performed using the half-power bandwidth (HPB) and free-vibration decay (FVD) methods (Hwang & Stokoe 2024). Resonant column testing typically involves, in addition to the FVD procedure, the steady-state vibration (SSV) method (Ahmad & Ray 2023). However, it should be emphasized here that HPB and SSV are the same research method, when the sample is vibrating during its first mode and the damping is calculated by measuring the width of peaks in its frequency response function (Ahmad & Ray 2023). Inversely, during the free-vibration decay, the specimen is allowed to free-vibrate whilst the decayed strain amplitude during this vibration is measured. Several studies have compared the SSV and FVD techniques, providing suggestions on their scopes and limitations (Senetakis et al. 2015; Mog & Anbazhagan 2022). Stokoe et al. (1999) or Menq (2003) have suggested using the SSV method for measuring small deformations and the FVD method for measuring medium deformations. It should be noted, however, that the ASTM specification (1992), detailing both the SSV and FVD techniques, does not mention a preferred method concerning the level of deformation.

The presented article deals only with the FVD procedure. The authors focused on the correct number of consecutive cycles to determine the damping in FVD (D_{FVD}) based on data of alternative material mixtures. Previous studies have principally examined material damping derivations of various types of natural soils, such as sands and/or clayey soils. However, the scientific literature does not contain many reports on the dynamic properties of alternative materials.

2 BRIEF DESCRIPTION OF MATERIALS AND METHODS

2.1 *Materials*

In this study, four different RCA–RTW compacted mixtures were tested in a resonant column apparatus, in the torsional mode of vibration, in a saturated state at variable levels of isotropic effective stress (p' = 90/180/270 kPa). Shredded car tires (0.5 mm ≤ d ≤ 2.0 mm), a by-product of the scrap tire shredding process, were mixed with other waste material, i.e., a by-product of construction and demolition (0.015 mm ≤ d ≤ 2.0 mm). The maximum proportion of rubber waste by mass (χ%), in the RCA–RTW mixtures considered in this study, was 20%. Table 1 provides details on the composition of the samples.

Table 1. Composition of mixtures employed in the study.

Designation	% RCA	% fines of RCA	% RTW P [a]	% RTW G [b]
M1_R	80	0	10	10
M2_R	75	10	15	0
M3_R	70	20	10	0
M4_R	65	30	5	0

[a] tire waste powder 0.5-1.0 mm

[b] tire waste granulate 1.0–2.0mm

The information regarding the physical characteristics of recycled rubber waste and recycled concrete aggregate is described in Gabryś (2023a) and Gabryś & Sas (2024). The details of the blinds' preparation are included, e.g., in Gabryś (2023). Previously, the authors investigated the compaction properties (Gabryś et al. 2023), compressibility (Gabryś 2023b), and shear strength (Gabryś et al. 2021) of RCA–RTW composites. The authors' latest, unpublished study concerns the permeability of mixtures. Since 2023, the authors have been continuously conducting research on the dynamic and cyclic response of RCA–RTW mixes for the design of many geotechnical structures, such as retaining walls, foundations, and slopes subjected to cyclic shear loading (like from earthquake or traffic loads). The primary aim of this research is to ascertain the potential of soil–rubber mixtures in reducing vibrations. The ambit of this article also aligns with this research topic.

The resonant column (RC) used is of the Drnevich type and follows the fixed-free configuration. It was employed as the most reliable tool for the determination of the damping ratio of soil specimens in the shear strain range of 10^{-4}%–10^{-2}%. A more detailed description with technical features and the blueprint of the devices can be found in Sas et al. 2017.

2.2 *Material damping ratio from the FVD procedure*

The resonant column test begins with the application of a torsional oscillation with an increasing amplitude sine wave to the top of the specimen. The frequency is progressively increased for each tested strain amplitude to obtain the specimen's dynamic response. The frequency at which maximum response amplitude is observed is the resonance frequency (f_r) of the sample. The dynamic shear modulus (G_S) is a function of the fundamental frequency, based on wave propagation and elasticity theories. Once the f_r frequency has been found, the power is cut off, the coils are switched off, and the specimen is left to vibrate freely. The damping of the soil sample movement is recorded by an accelerometer and displayed on the screen as a free vibration decay curve.

Damping is determined from the logarithmic decrement (δ) equation (Soból et al. 2019):

$$\delta = \frac{1}{N} ln \frac{Z_1}{Z_{1+N}} \quad (1)$$

where N is the number of successive damping cycles, Z_1 is the first amplitude, and Z_{1+N} is the amplitude after N cycles (Figure 1). Material damping is subsequently determined from the following equation (Stokoe et al. 1999):

$$D_{FVD} = \sqrt{\frac{\delta^2}{\delta^2 + 4 \cdot \pi^2}} \quad (2)$$

Material damping derived from Eq. (2) corresponds to the free-vibration method, hence the symbol for the parameter D_{FVD}.

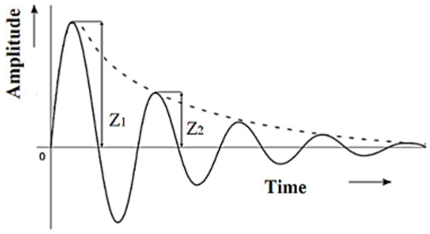


Figure 1. An example of a free vibration decay curve.

N denotes the number of successive cycles after the free vibration of the sample is initiated. The number of cycles should be selected appropriately, paying attention to when the sample's movements are utterly damped. The ASTM specification (1992) says that the N number should be 10 or less. Bolton & Wilson (1990) suggested using the last five powered cycles, Stokoe et al. (1999) three successive cycles, and Senetakis et al. (2015) two successive cycles when the FVD method is applied for damping derivation. Ray (1984) proposed using a small number of cycles when driving at high amplitude. A larger N would cause a reduction in strain amplitude by a factor of about 3 over the measurement interval. Soból et al. (2015) observed that 10 cycles of free vibration are sufficient to dampen vibrations in fine-grained soils. In some cases, between 10 and 20 cycles are needed, but no more than 20. According to the RC manual (GDS Resonant Column 2015), between 10 and 50 cycles are commonly used in the calculation. In line with the above discussion, the authors of this study investigated the possible effect of the number of prescribed cycles (N) used in measuring the D–ratio on the damping of alternative material mixtures.

3 RESULTS

The selected test results from the RC device from the FVD method are presented and discussed in this section. The standard approaches of two/three/five/ten successive cycles were implemented. Additionally, for each damping test, the decay was checked carefully, and the number of cycles needed for the whole decay was determined. The authors have already employed this non-standard procedure, but for cohesive soils (sasiCl, Cl, and clSa) (Figure 2, Gabryś et al. 2018). For the studied RCA–RTW mixtures, the N number ranged from a few to several tens, but less than 30, depending on the rubber waste additive, effective stress, and shear strain amplitude. The principle is straightforward: the lower the rubber insertion, the higher the pressure and the higher the level of deformation, the more cycles are required to dampen the vibrations. Moreover, every damping test was repeated ten times at each shear strain level. Therefore, in the presented results, there is an average value from a selected number of cycles (in the figures shown as "up to 30 cycles").

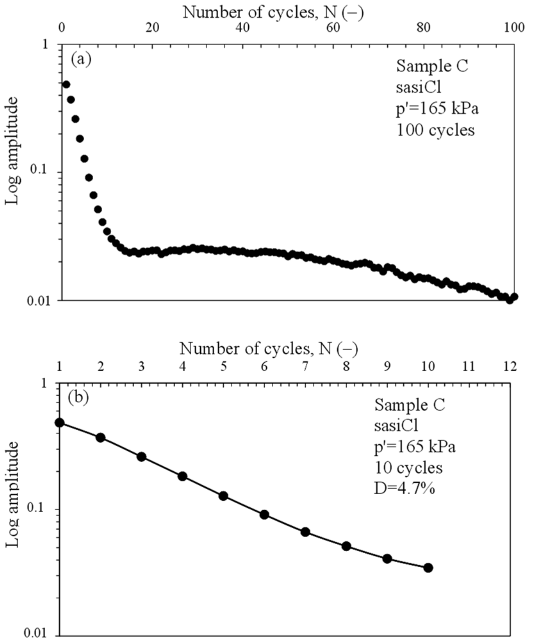
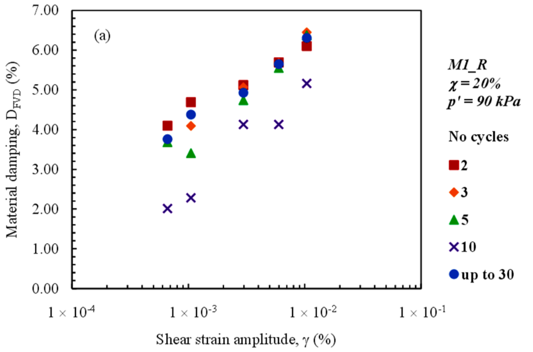


Figure 2. Log (peak amplitude) against number of cycles: a–total recorded data; b–a line of best fit (Gabryś et al. 2018).

In Figures 3 and 4, the examples of the variation in the damping ratio with the shear strain amplitude of two mixtures, M1_R with $\chi = 20\%$ and M4_R with $\chi = 5\%$, considering all the variables, are shown. These results, especially presented in Figure 4, are typical for natural aggregates. They correspond to the damping reported in sands by Senetakis et al. (2015). Regardless of the composition investigated, the following can be observed: an increase in materials' damping in shear with increasing shear strain. The lower the rubber additive content, the smaller the spread of results (Figure 4). Moreover, the variation in $D = f(\gamma)$ is approaching exponential. In the case of the M1_R mix, the distribution of the results is greater, and the increase in D_{FVD} with γ is rather linear. Next, considering the mean effective stress, there is a difference in the damping values, i.e., a decrease in D_{FVD} values by an average of ~20% with an increase in p'. The lowest damping (mostly < 1%) was obtained for the highest tested $p' = 270$ kPa (Figure 4c). Subsequently, there is a systematic trend of increasing damping in RCA–RTW mixtures irrespective of the number of successive cycles. The number of cycles is important in the case of a higher rubber additive (Figure 3). For the $\chi = 20\%$, the percentage variation in the damping results of the FVD method is clear and rather significant for, e.g., 2 and 10 cycles. In general, from Figure 3, it can be seen that the damping calculated after application of 10 prescribed cycles differs significantly from the rest of the results. Interestingly, in the case of remaining compositions with $\chi < 20\%$, the effect of the N cycles on the damping results is less noticeable. Contrary to the outcomes reported in literature (see e.g., Mog & Anbazhagan 2021), damping ratios obtained at the 2nd, 3rd, 5th, 10th, etc. cycles are close to each other, independent of the strain level. This important observation may facilitate damping tests in alternative material mixtures. The analysis of the number of cycles may not be required for the damping calculation. However, it is important to remember that the strain level for this research is small to medium, and the damping ratio exhibits a greater degree of change, particularly under high strain (Ram & Mohanty 2023). For the tested alternative material mixtures, damping could not be obtained for very small strains, despite material damping normally being constant and independent of shearing strain amplitude (i.e., soil exhibits linear behavior).

A clarification of the damping behavior for the mixture with the highest rubber content ($\chi = 20\%$) is warranted, particularly regarding the difference between damping ratios calculated using 10 cycles and those obtained when the full decay (up to ~30 cycles) is considered. For this composition, the free-vibration decay does not follow a strictly linear logarithmic trend but instead exhibits a two-stage attenuation mechanism. The first stage, occurring within approximately the initial 5–10 cycles, is characterized by rapid decay governed mainly by frictional interactions and particle rearrangement within the RCA skeleton. These mechanisms tend to stabilize quickly, resulting in relatively low damping ratios when only the early part of the decay is used in the logarithmic decrement. In contrast, the subsequent stage of attenuation is dominated by the time-dependent viscoelastic response of the rubber inclusions, which dissipate energy over a substantially longer duration. As a result, rubber-rich mixtures retain noticeable oscillation amplitudes over many additional cycles, and including this later portion of the decay in the damping calculation leads to an increase in the resulting D_{FVD} value. This behavior is consistent with previously reported observations for rubberized soils, where prolonged free-vibration tails are attributed to viscoelastic relaxation effects. Importantly, such discrepancies are not observed in mixtures with lower rubber contents ($\chi \leq 15\%$), because their decay is more rapid, more linear, and completed within the first several cycles. Therefore, the apparent inconsistency in Fig. 3a reflects the intrinsic two-stage decay mechanism of rubber-rich composites rather than an experimental artifact, and emphasizes the need to consider an appropriately large number of cycles when evaluating the damping of materials with high RTW content.



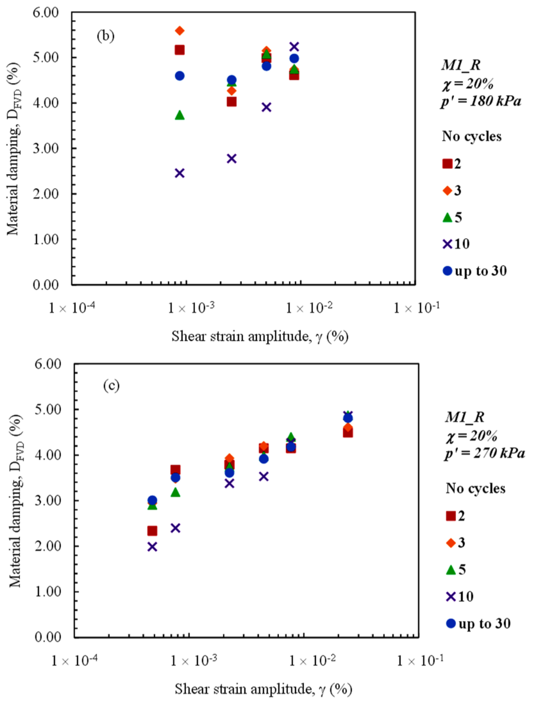


Figure 3. Graphical representation of the damping ratio of the M1_R mixture for *p'* equal to: a–90 kPa; b–180 kPa; c–270 kPa.

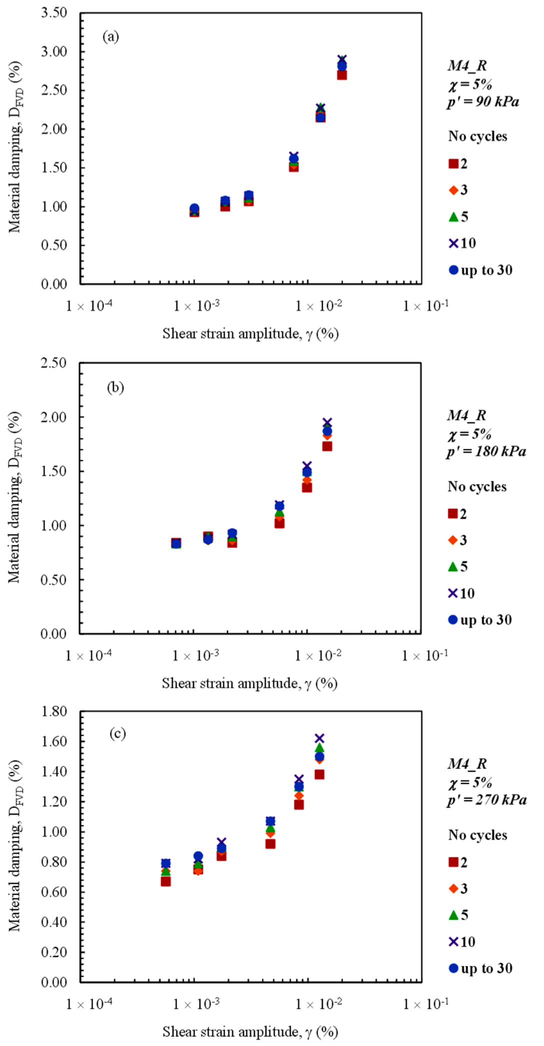


Figure 4. Graphical representation of the damping ratio of the M4_R mixture for *p'* equal to: a–90 kPa; b–180 kPa; c–270 kPa.

In Figure 5, a comparison between the damping values from the FVD method derived for all studied mixtures, for the desired number of cycles, is shown. $N = 2$ (Figure 5a) and $N =$ average from up to 30 (Figure 5b) were chosen for the example. Firstly, the impact of rubber on the damping properties of the composition can be noticed here. Generally, as rubber content increases, the damping ratio increases as well. The M1_R mix, with the highest inclusion of rubber waste, exhibits the highest damping values, increasing with strain amplitude, reaching around 6%. The M1_R blend, however, shows a clear separation in damping behavior. The effect of the rubber inclusion is less pronounced for D_S when $\chi \leq 15\%$. The M2_R mix is characterized already by moderate damping and a slightly increasing trend. The lowest damping is shown by the last two mixtures, regardless of the number of applied cycles. The *D*–ratio values in Figure 5a are slightly higher in comparison with those in Figure 5b. The M1_R mix consistently has the highest damping, making it potentially more effective for applications requiring high energy dissipation. Another observation based on the analysis of Figure 5 concerns the number of cycles. The variation in damping is the same for all tested alternative materials and for both the two and the average number of cycles. Although a minor drop, almost imperceptible, in D_{FVD} can be detected for N up to 30. Probably, if the damping was calculated using an even larger number of cycles, e.g., 50 or 100 cycles, as Mog & Anbazhagan (2021) investigated, a greater reduction in damping would be obtained. This would most likely be caused by material adaptation or rearrangement, by reduction in internal friction, or structural changes, or possible breakdown or settling of the rubber-fiber matrix interface.

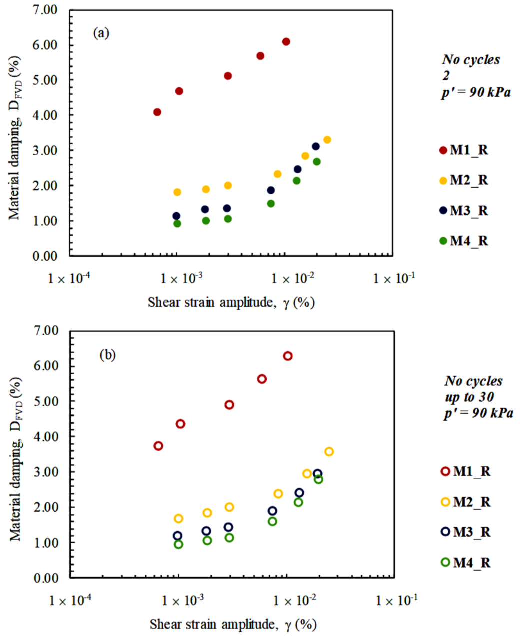


Figure 5. Graphical representation of the damping ratio of all tested RCA–RTW mixtures at *p'* = 90 kPa, considering in the calculations: a–2 cycles; b–average from up to 30 cycles.

Comparing the obtained damping characteristics for waste materials with the damping of, for example, sand, the RCA–RTW mixtures display higher damping across all shear strain levels. The composition with 20% of rubber inclusion is characterized by damping up to ~6%. The typical *D* values for the clean sands are in the range from 0.5 to 2%, whereas for the soft clays, from ~2 to 5% (Kokusho et al. 1982; Cui et al. 2023). For natural granular soil, damping increases with strain, but rarely exceeds 3% (Senetakis et al. 2015). Mixtures with the rubber maintain better damping. They have viscoelastic behavior due to rubber, which means that they absorb and dissipate more energy than frictional materials like sand. In natural sand, energy dissipation comes mostly from frictional sliding and particle rearrangement (Hardin, 1965).

4 CONCLUSIONS

This study investigates the damping properties of recycled concrete aggregate (RCA) mixed with recycled tire waste (RTW), focusing on the Free-Vibration Decay (FVD) method to determine the damping ratio of these composite materials under varying conditions. The research aimed to assess the influence of rubber waste content on the dynamic behavior of these mixtures and explore how the number of cycles impacts the damping results. The mixtures, which varied in rubber waste content from 5% to 20%, were subjected to dynamic testing in a resonant column apparatus under isotropic effective stresses of 90/180/270 kPa.

The *D*–ratio was determined by measuring the free vibration decay after the specimen vibrated at its resonance frequency. The study examined different numbers of successive cycles (2, 3, 5, 10, and up to 30) to understand their effect on the damping ratio and how it changed with shear strain amplitude. The results revealed that higher rubber content (e.g., $\chi = 20\%$) led to higher damping ratios, which increased with shear strain. In addition, the number of cycles had a noticeable effect on the damping, particularly for higher rubber content mixtures. However, the influence of the number of cycles was less significant for mixtures with lower rubber content.

The main conclusions drawn from this study are highlighted below.

- *Rubber waste enhances damping.* The inclusion of rubber significantly improves the damping ratio of the RCA–RTW mixtures, making them more effective for vibration mitigation and seismic isolation applications. Mixtures with the highest rubber content ($\chi = 20\%$) show damping values up to ~6%, far exceeding those of typical natural soils.
- *Impact of cycles on damping.* The number of cycles used in the FVD calculation has a noticeable effect on the damping ratio, particularly for mixtures with higher rubber content. While mixtures with low rubber content show minimal variation across different prescribed cycle counts, rubber-rich mixtures exhibit more pronounced differences. For mixtures with modest rubber additions ($\chi \leq 15\%$), determining the exact number of cycles is generally unnecessary, as damping remains stable. This behavior indicates that rubber-rich mixtures may undergo structural adjustment or reduced internal friction during repeated loading, causing slight changes in damping with increasing cycle count.
- *Two-stage decay in rubber-rich mixtures*. For the mixture with 20% RTW, the free-vibration decay does not follow a single linear trend but instead exhibits a two-stage attenuation mechanism. The initial cycles are dominated by rapid decay associated with frictional particle rearrangement, while subsequent cycles are governed by the slower viscoelastic response of the rubber inclusions. As a result, damping values derived from the full decay (up to ~30 cycles) are higher than those calculated using only the first 10 cycles. This behavior reflects the intrinsic time-dependent energy dissipation capability of rubber-rich composites and explains the apparent discrepancy observed in Fig. 3.
- *Comparison with natural soils.* The damping performance of the RCA–RTW mixtures is significantly superior to that of natural granular soils such as sand and soft clays, which typically dissipate much less energy. The enhanced viscoelastic damping provided by the rubber component makes these mixtures promising for applications requiring high energy dissipation—such as earthquake-resistant foundations, pavement subgrades, and vibration-damping layers. Their use also contributes to sustainability by reducing waste and decreasing reliance on natural aggregate resources.

5 ACKNOWLEDGEMENTS

Research presented in this article was funded in part by the National Science Centre, Poland, research project No. 2024/53/B/ST8/02470, contract No. UMO-2024/53/B/ST8/02470. The NCN's support for this research included the preparation of research materials through the purchase of specialist equipment.

For the purpose of Open Access, the author has applied a CC-BY public copyright licence to any Author Accepted Manuscript (AAM) version arising from this submission.

The experimental data supporting the findings of this study are available in RepOD under the identifier DOI: 10.18150/IEJQOF.

6 REFERENCES

Ahmad, M., and Ray, R. Damping of Dry Sand in Resonant Column-Torsional Simple Shear Device. *Sustainability* 15, 11060.

Al-Naje, F.Q., Abed, A.H., and Al-Taie, A.J. A Review of Sustainable Materials to Improve Geotechnical Properties of Soils. *Al-Nahrain Journal for Engineering Sciences* 23(3), 289–305.

ASTM. 1992. Standard test method for modulus and damping of soils by the resonant column method: D4015-92. Annual book of ASTM standards. USA: ASTM International.

Bolton, M.D., and Wilson, J.M.R. 1990. Soil stiffness and damping. In *Structural Dynamics*. Editors Kraetzing, W.B. et al. Balkema, Rotterdam, 209–216.

Cui, Z.D., Zhang, L.J., and Zhan, Z.X. 2023. Dynamic shear modulus and damping ratio of saturated soft clay under the seismic loading. *Geomechanics and Engineering* 32(4), 411–426.

Dawson, A. 2012. 7 - Alternative and recycled materials for earth construction. In *Woodhead Publishing Series in Energy, Modern Earth Buildings*. Editor(s): Matthew R. Hall, Rick Lindsay, Meror Krayenhoff .Woodhead Publishing, 172–203.

Dias, S. Almeida, J., Tadeu A., and de Brito J. 2024. Alternative concrete aggregates - Review of physical and mechanical properties and successful applications. *Cement and Concrete Composites* 152, 105663.

Gabryś, K. 2023a. Geomechanical characterization of crushed concrete–rubber waste mixtures. *Sustainability* 15(19), 14446.

Gabryś, K. 2023b. Experimental research on compressibility characteristics of recycled concrete aggregate – recycled tire waste mixtures. *Journal of Material Cycles and Waste Management* 25(3), 1–12.

Gabryś, K., Radzevičius, A., Szymański, A., and Šadzevičius, R. 2021. Shear Strength Characteristics of Recycled Concrete Aggregate and 672 Recycled Tire Waste Mixtures from Monotonic Triaxial Tests. *Materials* 14(23), 7400.

Gabryś, K., and Sas, W. 2024. Dynamic shear modulus and damping ratio of recycled concrete aggregate-recycled tire waste mixture using resonant column apparatus. *E3S Web of Conferences 544*, IS-Porto 2023.

Gabryś, K., Soból, E., Sas, W., and Szymański, A. 2018. Material damping ratio from free-vibration method. *Annals of Warsaw University of Life Sciences – SGGW, Land Reclamation* 50(2), 83–97.

Gabryś, K., Zając, K., and Sas, W. 2023. Compaction characteristics of waste materials: recycled concrete aggregate, recycled rubber waste, and recycled concrete aggregate−recycled rubber waste mixtures. In *Proceedings of the 17th Danube European Conference on Geotechnical Engineering* (17DECGE), June 7-9, 2023, Bucharest, România.

GDS Resonant Column 2015. The GDS Resonant Column System Handbook.

Hardin, B.O. 1965. The Nature of Damping in Sands. *Journal of the Soil Mechanics and Foundations Division* 91(1), 63–97.

Hwang, S., and Stokoe, II, K.H. 2024. In-situ material damping measurements using the crosshole seismic method. *E3S Web of Conferences* 544, 02003, IS-Porto 2023.

Jastrzębska, M. 2025. Use of Alternative Materials in Sustainable Geotechnics: State of World Knowledge and Some Examples from Poland. *Applied Sciences* 15, 3352.

Kokusho, T., Yoshida, Y., and Esashi, Y. 1982. Dynamic Properties of Soft Clay for Wide Strain Range. *Soils and Foundations* 22(4), 1–18.

Menq, F.Y. 2003. Dynamic properties of sandy and gravelly soils. Ph. D. dissertation. USA: University of Texas at Austin.

Mog, K., and Anbazhagan, P. 2022. Evaluation of the damping ratio of soils in a resonant column using different methods. *Soils and Foundations* 62, 101091.

Netinger Grubeša, I., Barišić, I., Fucic, A., and Bansode, S.S. 2016. 8 - Recommendations for future research. In *Characteristics and Uses of Steel Slag in Building Construction.* Editor(s): Ivanka Netinger Grubeša, Ivana Barišić, Aleksandra Fucic, Samitinjay S. Bansode, Woodhead Publishing, 161–171.

Ram, A.K., and Mohanty, S. 2023. Laboratory investigation on damping characteristics of homogeneous and stratified soil-ash system. Journal of Rock Mechanics and Geotechnical Engineering 15, 2757–2777.

Ray, R.P. 1984. *Changes in Shear Modulus and Damping in Cohesionless Soils Due to Repeated Loading*. Ph.D. Thesis, University of Michigan, Ann Arbor, MI, USA.

Sas, W., Gabrys, K., and Szymanski, A. 2017. Experimental studies of dynamic properties of Quaternary clayey soils. Soil Dynamics and Earthquake Engineering 95, 29–39.

Senetakis, K., Anastasiadis, A., and Pitilakis, K. 2015. A comparison of material damping measurements in resonant column using the steady-state and free-vibration decay methods. Soil Dynamics and Earthquake Engineering

Soból, E., Głuchowski, A., Szymański, A., and Sas, W. 2019. The New Empirical Equation Describing Damping Phenomenon in Dynamically Loaded Subgrade Cohesive Soils. *Applied Sciences* 9(21), 4518.

Soból, E., Sas. W., and Szymański, A. 2015. Zastosowanie kolumny rezonansowej do określenia reakcji gruntów drobnoziarnistych obciążonych dynamicznie (The use of resonant column to determine the response of dynamically loaded fine grained soils). *Przegląd Naukowy Inżynieria i Kształtowanie Środowiska* 24(2), 133–144.

Stokoe, K., Darendeli, M., Andrus, R., and Brown, L.T. 1999. Dynamic soil properties: laboratory, field, and correlation studies. In *Proceedings of the 2nd International Conference on Earthquake Geotechnical Engineering*, Lisbon.