

# Comparative study of effects particle morphology and size distribution on data from triaxial tests on synthetic abrasive granular media

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**ABSTRACT:** Granular materials are widely used in geotechnical engineering. Previously published studies indicated that the mechanical behavior of natural granular soils varies with the microstructure of these soils affected mainly by their particle morphology (shape and surface texture), and particle size distributions. However, the significant variability observed in some natural soils makes it challenging to address the effects of different particle characteristics on the strength and deformation behavior of granular materials. To address this challenge, this research describes and analyzes an experimental laboratory study of the mechanical behavior of synthetic abrasive grains (SAG) in spherical and pyramidal shapes of varied sizes of grains (1 mm, 4 mm, and 10 mm). The objective of the paper is to identify and quantify the main physical particle characteristics that influence the shear strength of the tested materials. Physical characterization tests of the synthetic media (particle size, maximum and minimum void ratios,  $e_{\max}$  and  $e_{\min}$ ) and shear tests (consolidated drained triaxial compression tests) are performed to compare the experimental results. The article also presents and discusses the variation of  $e_{\max}$  and  $e_{\min}$  with the morphology and size distributions of the investigated materials. The study indicate that the shape of the abrasive particles influences the shear strength of the synthetic media due to the better interlocking and rearrangement of the angular particles, and that particle size interferes more with the mechanical behavior of pyramidal particle than spherical particle when varying the consolidation pressures of the triaxial tests.

**KEYWORDS:** Synthetic abrasive grains, grain size and shape, maximum and minimum void ratios, triaxial test results.

## 1 INTRODUCTION

Granular materials with an extensive range of grain sizes and shapes are commonly used in geotechnical engineering. Their mechanical properties are primarily governed by the particle-to-particle contacts, particle characteristics and particle rearrangement (Sarkar et al. 2019). The mechanical response of granular soils with specific test conditions and relative density varies with the microstructure of these soils affected by their particle characteristics (i.e., morphology and size). Morphology (i.e., shape and surface texture) and size distributions of soil grains have an important role in the characterization of granular soil behavior and a significant impact on all mechanical properties of granular soils (Santamarina & Cho 2004). However, the large variability observed in some natural soils makes it challenging to address independently the effects of different particle characteristics on the strength and deformation behavior of granular materials.

Malkorra et al. (2021) conducted triaxial tests on synthetic abrasive grains of spherical and pyramidal shapes to model the polishing process in metallic parts. Their results highlight the importance of grain shape and size in the mechanical response of granular soils. This article provides complimentary information, with measurements of the mechanical behavior of artificial granular media termed synthetic abrasive grains (SAG) in spherical and pyramidal shapes of varied sizes of grains (1 mm, 4 mm, and 10 mm). The objective of the paper is

to identify and quantify the main physical particle characteristics that influence the shear strength of the tested SAG materials. Physical characterization tests of the synthetic media (particle size, maximum and minimum void ratios,  $e_{\max}$  and  $e_{\min}$ ), and shear tests (consolidated drained triaxial compression tests) are performed to compare the experimental results. Additionally, the article presents and discusses the variation of  $e_{\max}$  and  $e_{\min}$  with the morphology and size distributions of the investigated SAG materials.

## 2 MATERIALS AND METHODS

To conduct this research, Synthetic Abrasive Grains (SAGs) with different shapes and sizes were used to simulate the behavior of natural granular soils. The SAGs produced by ABC Swisstech are typically designed for use in metal polishing processes but were selected here for their controlled geometry and surface properties. Two distinct grain shapes were chosen for the study: spherical grains (with a height-to-width ratio equal to 1) and pyramidal grains (with a regular triangular base and equal edge lengths). The grain shape selections represent common geometries found in natural soils, while allowing precise control over shape effects having the same material composition. The grain sizes were selected to replicate typical dimensions of natural granular soils. Spherical grains were produced in diameters of 1 mm, 4 mm, and 10 mm, whereas

pyramidal grains had base edge lengths of 4 mm and 10 mm, as shown in Table 1.

Table 1. Shape, sizes, surface area, volume, and mass of SAG's

Shape	Size (mm)		Area (mm <sup>2</sup> )	Volume (mm <sup>3</sup> )	Mass (g)
	a	b			
Spheric	1	-	3.14	0.52	0.001
	4	-	50.24	33.49	0.090
	10	-	314.00	523.33	1.690
Pyramid	4	4	61.86	27.72	
	10	10	386.60	480.30	1.516

### 2.1 Determination of $e_{min}$ and $e_{max}$ values

The experimental study followed current international standards for determining the  $e_{min}$  and  $e_{max}$  of granular materials. Specifically, the  $e_{min}$  was determined in accordance with ASTM D4254-16, standard test methods for determining minimum index density and unit weight of cohesionless, free-draining soils, and the  $e_{max}$  was determined in accordance with ASTM D4253-16, standard test methods for determining maximum index density and unit weight of cohesionless, free-draining soils.

Sample preparation was carried out by calculating the relative density based on the determined  $e_{min}$  and  $e_{max}$  in accordance with ASTM standards using a vibratory table. The procedures were adapted to the available equipment at the École Nationale d'Ingénieurs de Saint-Étienne (ENISE) Soil Laboratory, which includes a vibratory table and a CBR mold (Figure 1).



Figure 1. Mold with the grains and weight of 20 kg on the vibratory table to perform the measurements of  $e_{min}$ .

### 2.2 Triaxial compression tests

The triaxial compression tests were conducted under consolidated drained (CD) conditions to evaluate the effective stress behavior of the particulate materials, independent of porosity influences. The CD triaxial tests involve three steps: a) consolidation under a low hydraulic gradient (20 kPa), b) further consolidation under mean effective strains of 100 and 200 kPa, c) shearing using a shear rate of 0.05 mm/s. The tests were carried out until the maximum axial strain (limited to 20%) which was sufficient to induce breakage in the tested artificial grains.

The triaxial testing device can accommodate cylindrical particulate material samples with diameter of 100 mm and heights of 165 mm and 200 mm at the final step of the test. Each sample was prepared to achieve a relative density of 50%, calculated based on the previously determined  $e_{max}$  and  $e_{min}$  values. The corresponding void ratio ( $e$ ) was determined to maintain the target relative density throughout testing.

## 3 RESULTS AND DISCUSSION

Granular soils' stress-strain and strength responses are affected by the grain size and shape. The limiting void ratios and their range (i.e.,  $e_{max} - e_{min}$ ) are mostly governed by their grain size, distribution, grain shape and fabric for cohesionless natural soils. Estimating in-situ void ratio and its limits and range, can help to understand granular soils' stress-strain responses.

### 3.1 Variation of $e_{max}$ and $e_{min}$ for the tested SAG's

The results of the  $e_{max}$  and  $e_{min}$  and the void ratio ( $e$ ) of the tested specimen determined for the specific relative density (50%) and grain particle shape in the experimental study for the spherical grains are presented in Table 2. The  $e_{max}$  values for spherical grains ranged from 0.80 to 0.74, with minimal variation between grain sizes. Specifically, grains with diameters of 10 mm and 4 mm showed similar  $e_{max}$  values of 0.75 and 0.74, respectively, while the smallest grains (1 mm) exhibited the highest  $e_{max}$  value of 0.80. The  $e_{min}$  values for spherical grains ranged from 0.61 to 0.68. In the case of spherical artificial grains, compaction was more difficult due to their tendency to roll and the lack of interlocking between particles. This effect was more pronounced in the larger grains (10 mm), which exhibited a lower number of contact points and, consequently, greater mobility during compaction.

Table 2. Values of  $e_{max}$  and  $e_{min}$  and the void ratio ( $e$ ) of the tested SAG's for the specific relative density (50%) and grain particle shape.

Particle Shape	Size (mm)	$e_{max}$	$e_{min}$	( $e_{max} - e_{min}$ )	( $e$ )
Sphere	10	0.75	0.68	0.07	0.71
	4	0.74	0.61	0.13	0.68
	1	0.80	0.66	0.14	0.73
Pyramid	10	0.85	0.67	0.18	0.76
	4	0.78	0.60	0.18	0.69

The results indicate that the  $e_{max}$  and  $e_{min}$  values are generally influenced by grain shape and size values. The effects are more pronounced for grains with a size of 10 mm and pyramidal shape such as the maximum void ratio is 13% higher than that of the spherical shape. Similarly, for grains of 4 mm, the pyramidal shape resulted in a maximum void ratio 5% greater than its spherical counterpart. In contrast, the minimum void ratio appeared to be less affected by particle size.

No differences in the extreme void ratios ( $e_{max} - e_{min}$ ) are observed for the pyramidal grains with sizes of grains of 4 mm and 10 mm (Table 2). Cetin & Ilgac (2021) observed that gravelly soils tend to have a smaller variation between their maximum and minimum void ratios. Both maximum and minimum void ratios, as well as the void ratio range, increase with greater particle angularity and lower roundness.

Figure 2 suggests that these findings are consistent with previous experimental observations (Cubrinovski & Ishihara 2002; Santos Neto et al. 2024):  $e_{max}$  and  $e_{min}$  values tend to increase as mean grain size decreases in granular natural soils. Although the limiting void ratios (and dry unit weights) measurements of granular soils are not unique, they depend on the method used for their determination (Lade et al. 1998; Lunne et al. 2019).

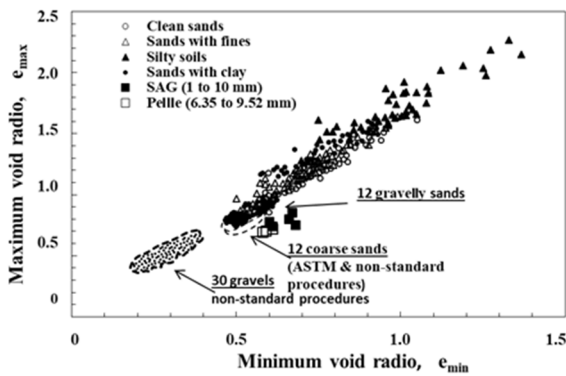


Figure 2. Variation of  $e_{max}$  and  $e_{min}$  for natural deposits (adapted from Cubrinovski & Ishihara 2002) and the tested SAG's.

### 3.2 Results of the CD triaxial tests

As shown in the stress-strain curves in Figure 3 (i.e., spherical grains produced in diameters of 1 mm, 4 mm, and 10 mm) and Figure 4 (i.e., pyramidal grains produced in base edge lengths of 4 mm and 10 mm) the deviatoric stress ( $q$ ) increases obviously with the increase of confining pressure for the tested SAG's granular materials.

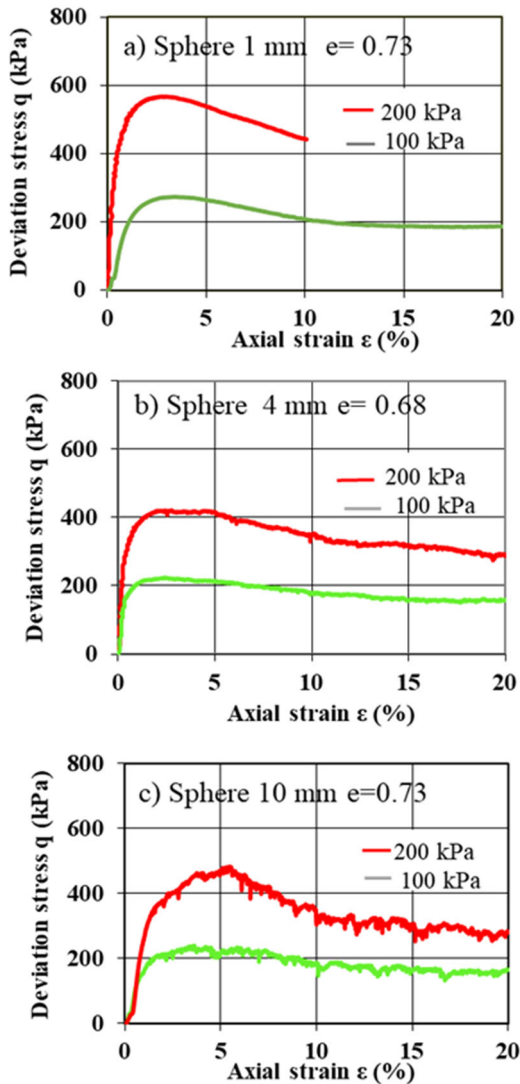


Figure 3. Stress-strain curves of the tested SAG's a) Sphere 1mm, b) Sphere 4 mm and c) Sphere 10mm.

The results of the CD triaxial tests for spherical particles indicate similar mechanical behavior across different

consolidation stress levels (Figure 3). However, the grain size had a noticeable impact on shear strength. Specimens composed of 1 mm grains (Figure 3a) exhibited higher shear resistance compared to those with 4 mm (Figure 3b) and 10 mm (Figure 3c) grain sizes (diameter) for specimens with relative density of 50%. Analysis of the stress-strain curves reveals a higher uniformity coefficient in samples with smaller grains, characterized by the absence of sudden reductions in deviator stress. In contrast, specimens with larger grains (10 mm) showed abrupt stress drops during shearing, likely due to reduced particle interlocking and rearrangement capacity. The internal friction angles varied between 30 and 35°, with the highest values observed in the specimens with 1 mm grains and the lowest in those with 4 mm grains.

Figure 4a presents the stress-strain curves for samples composed of 4 mm pyramidal artificial grains. A comparison between specimens consolidated at 100 kPa and 200 kPa shows that the latter exhibited more than twice the deviator stress, despite displaying similar axial deformation characteristics. This increase in strength is attributed to the greater resistance of the pyramidal grains to axial deformation under higher confinement, which enhances intergranular contact pressures and, consequently, increases deviator stress. Figure 4b illustrates the stress-strain curves for samples of 10 mm pyramidal grains. Both curves lack a distinct peak strength. Instead, the samples exhibit oscillations in deviator stress throughout the test. These fluctuations may be associated with intermittent particle rearrangement and loss of interlocking, which are characteristic behaviors of angular particles with larger dimensions. However, in the later stages of axial deformation, both samples tend to stabilize, showing a more uniform stress response as the grains reestablish contact networks under shear.

The artificial grains consolidated at 200 kPa demonstrated a notable increase in deviator stress ( $q$ ), stabilizing after 10% axial strain, whereas the sample consolidated at 100 kPa began to exhibit stability after approximately 5% axial strain. For the 10 mm pyramidal grains, the specimen consolidated at 200 kPa exhibited a deviator stress approximately twice as high as that of the specimen consolidated at 100 kPa. This highlights the significant influence of confining stress to the strength of angular particles, where grain interlocking plays a critical role in mobilizing shear resistance.

The difference in strength between spherical and pyramidal artificial soils is also remarkable. In the specimens composed of pyramidal artificial grains, the internal friction angle is 43° with 10 mm grains and 44° in those with 4 mm grains. Considering the same grain size (diameter for the sphere shape and base edge length for the pyramidal shape) the ratios between the friction angles of the pyramidal shape in relation to the spherical shape are 1.33 for grain size 10 mm and 1.44 for grain size 4 mm. For these materials the shear strength is more influenced by the shape than by the size. For the same size, pyramidal grains have a greater point of contact between particles than spherical grains, which leads to greater energy dissipation, increasing the angle of internal friction.

In the spherical grains, a peak deviator stress is observed at the beginning of the stress-strain curve, which is attributed to the weak interlocking capacity of the grains. After the initial shearing, these particles are unable to reestablish their contact network, resulting in a loss of strength and the absence of mechanisms capable of restraining the internal movement of the grains. In contrast, the 10 mm pyramidal grains, after shearing, present residual strength and tend to stabilize the deviator stress due to their ability to regain interlocking through rearrangement of the angular grains. For the 4 mm pyramidal grains, the smaller size and reduced surface areas limit the capacity for

interlocking reformation, resulting in behavior characterized by a distinct peak resistance followed by gradual stress reduction.

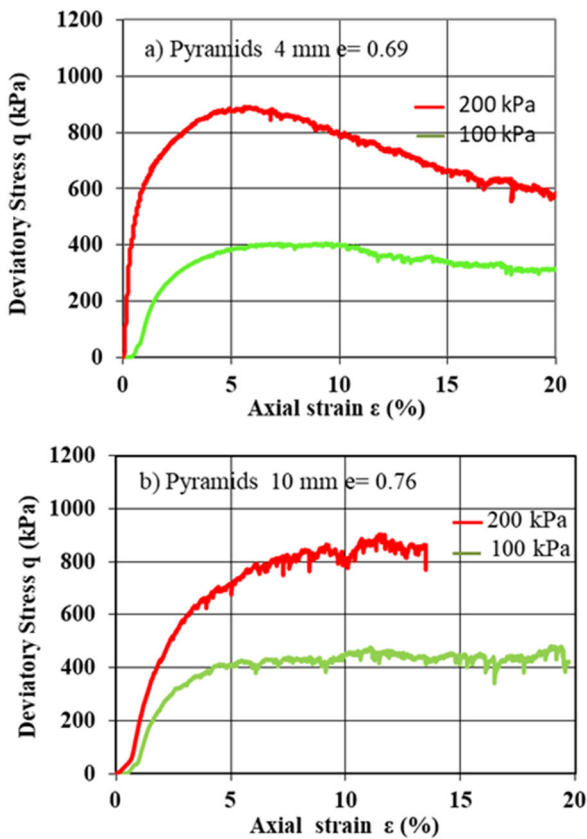


Figure 4. Stress-strain curves of the tested SAG's a) Pyramids 4 mm and b) Pyramids 10mm.

Additional work is necessary to investigate the main physical particle characteristics that influence the shear strength of the tested materials. Some preliminary results on the influence of non-plastic fines on granular soil behavior show that the mixtures containing pyramidal grains with 85% of 10 mm and 15% of 4 mm particles exhibited higher deviator stresses compared to the corresponding mixtures of spherical grains with the same proportions. This is explained by the greater number of inter-particle contact points and the increased energy dissipation caused by the interlocking and rolling resistance of the pyramidal grains compared to spherical grains.

#### 4 CONCLUSIONS

This study described and analyzed the results of consolidated drained triaxial tests performed on spherical artificial grains with sizes of 1 mm, 4 mm, and 10 mm, as well as on pyramidal artificial grains of 4 mm and 10 mm. The investigation included the determination of maximum and minimum void ratios ( $e_{max}$  and  $e_{min}$ ) and the calculation of the void ratio ( $e$ ) values used for sample preparation in the triaxial tests at the relative density of 50%. Based on the experimental results, the following conclusions were drawn with a focus on grain size and shape effect on the strength of the investigated granular materials:

The CD triaxial tests results (i.e., stress-strain curves) of the tested SAG's show that spherical and pyramidal media exhibit different mechanical behavior.

Among the spherical artificial grains, the specimens composed of 1 mm grain size exhibited the highest deviator stress. The 10 mm spheres presented the second highest deviator stress, which is associated with their larger contact

areas, promoting localized resistance despite fewer contact points overall.

For the pyramidal grains, increasing the particle size from 4 mm to 10 mm resulted in higher residual deviator stress under a confining pressure of 200 kPa. This behavior is attributed to the enhanced capacity for interlocking and rearrangement of the larger angular grains during shearing.

The shape of synthetic abrasive grains (SAGs) has a greater influence on the material's internal friction angle than the grain size for the same consolidated stress and dimensions (diameter for the spherical shape and base edge length for the pyramidal shape).

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#### 6 REFERENCES

- American Society for Testing and Materials. ASTM-D4254-16: Standard Test Methods for Minimum Index Density and Unit Weight of Soils and Calculation of Relative Density. 2016.
- American Society for Testing and Materials. ASTM-D4253-16: Standard Test Methods for Maximum Index Density and Unit Weight of Soils Using a Vibratory Table. 2016.
- Cetin, K. O., and Ilgac, M. 2021. Probabilistic assessments of void ratio limits and their range of cohesionless soils. *Soil Dynamics and Earthquake Engineering* 142(3), 365-381.
- Cubrinovski M., and Ishihara K. 2002. Maximum and minimum void ratio characteristics of sands. *Soils and Foundations* 42(6), 65-78.
- Lade, P. V., Liggio, C. D., and Yamamoto, J. A. 1998. Effects of non-plastic fines on minimum and maximum void ratios of sand. *Geotechnical testing journal* 21 (4), 336-347.
- Lunne, T., Knudsen, S., Blaker, Vestgården, T., Powell, J. J. M., Wallace, C. F., ... and Ghanekar, R. K. 2019. Methods used to determine maximum and minimum dry unit weights of sand: Is there a need for a new standard? *Canadian Geotechnical Journal* 56(4), 536-553.
- Malkorra, I., Souli, H., Claudin, C., Salvatore, F., Arrazola, P., Rech, J., ... and Rolet, J. 2021. Identification of interaction mechanisms during drag finishing by means of an original macroscopic numerical model. *International Journal of Machine Tools and Manufacture*, 168, 103779.
- Santos Neto, H.V., Burgos, R.D.F., Bicalho, K.V., Souli, H., and Ferreira, S. R. M. 2024. Influência das Características Morfológicas na Resistência ao Cisalhamento de um Solo Natural Granular. *Anais XXI Congresso Brasileiro de Mecânica dos Solos e Engenharia Geotécnica*, Santa Catarina.
- Santamarina, J.C., and Cho, G.C. 2004. Soil Behavior: The Role of Particle Shape. *Proc. Conference on Advances in Geotechnical Engineering*, 604-617.
- Sarkar, D., Goudarzi, M., and König, D. 2019. An interpretation of the influence of particle shape on the mechanical behavior of granular material. *Granular Matter* 21 (53).1-53.24.