

# New insights into the at-rest earth pressure coefficient of granular soils

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**ABSTRACT:** The earth pressure coefficient at-rest ( $K_0$ ) is a key parameter required for estimating the initial stress state in the ground in almost all geotechnical applications. Classical Jaky's equation has been widely used for this purpose. Although Jaky's equation can provide a good estimate of the at-rest earth pressure distribution in granular soil in some situations, the assumption behind the Jaky's model is not sound from a theoretical point of view, and the good prediction of Jaky's equation is somewhat coincidental. Moreover, it is noted that several most recent laboratory studies have reported results that are in contrast with the previous ones with regard to the influence of density on  $K_0$  values of granular soils. This paper presents a micromechanics-based analysis of the at-rest earth pressure coefficient in which the fabric anisotropy inherent in granular soils and the contact conditions at particle level are considered. Comparison between the predictions from the micromechanical model and from Jaky's equation is made, and the influence of anisotropy and contact conditions is discussed. Furthermore, the laboratory measured  $K_0$  values under various testing conditions are examined and compared with the predictions using the proposed model. The present study provides new insights into the fundamental aspect of the earth pressure coefficient at rest.

**KEYWORDS:** Earth pressure coefficient, granular soils, fabric anisotropy, micromechanics, laboratory tests.

## 1 INTRODUCTION

The initial distribution of earth pressure plays a pivotal role in both theoretical and practical geotechnical engineering, as it serves as the foundation for analyzing and predicting the mechanical behavior of the soil. The effective vertical stress within the ground can be calculated based on the soil's unit weight and the position of the groundwater table. The effective horizontal stress is typically estimated using the earth pressure coefficient at rest ( $K_0$ ), which is defined as the ratio of effective horizontal stress to effective vertical stress.

$$K_0 = \frac{\sigma'_h}{\sigma'_v} \quad (1)$$

where  $\sigma'_h$  is effective ground horizontal stress;  $\sigma'_v$  is effective ground vertical stress.

Accurately estimating the earth pressure coefficient at rest ( $K_0$ ) is critically important, yet its determination remains a significant challenge due to the complex behavior of granular materials. In laboratory settings, replicating field conditions is necessary to obtain a representative value of  $K_0$ . Two main types of laboratory tests are employed. The first method involves the use of an instrumented oedometer (Abdelhamid and Krizek, 1976), in which a cylindrical sample is placed within a rigid O-ring to maintain zero lateral strain. Vertical axial loading is applied, and the induced horizontal pressure is measured to determine  $K_0$ . The second method relies on a triaxial test, where the sample is enclosed by a membrane and surrounded by water to uniformly apply lateral pressure. Axial loading is applied vertically, and the lateral confining pressure is adjusted to maintain a constant ratio between the volume strain and axial strain, allowing for the determination of  $K_0$ . However, both methods have inherent limitations that prevent them from fully replicating field conditions. In the instrumented oedometer, ring friction during the test can affect the results, while in the triaxial test, it is difficult to completely eliminate lateral strain (Okochi and Tatsuoka, 1984), as minor lateral strains inevitably occur. Additionally, errors may arise from soil particles penetrating the membrane or from the use of bedding plates, further complicating the accuracy of laboratory measurements. Null gauges are diaphragm transducers operated in zero-deflection mode that provide precise, low-drift point measurements of the static normal stress, making them well suited to quantifying undisturbed earth pressures in model and centrifuge tests; their drawbacks are limited spatial coverage, the need for careful flush mounting and pneumatic control (Li et al., 2014). Tactile sensors are thin, flexible sensor arrays that

map high-resolution contact pressure fields—revealing lateral variability and localized peaks important for assessing at-rest conditions—but they are sensitive to shear, prone to creep and nonlinearity, and demand rigorous in-situ calibration and protection from damage (Palmer et al., 2009; Tessari et al., 2010).

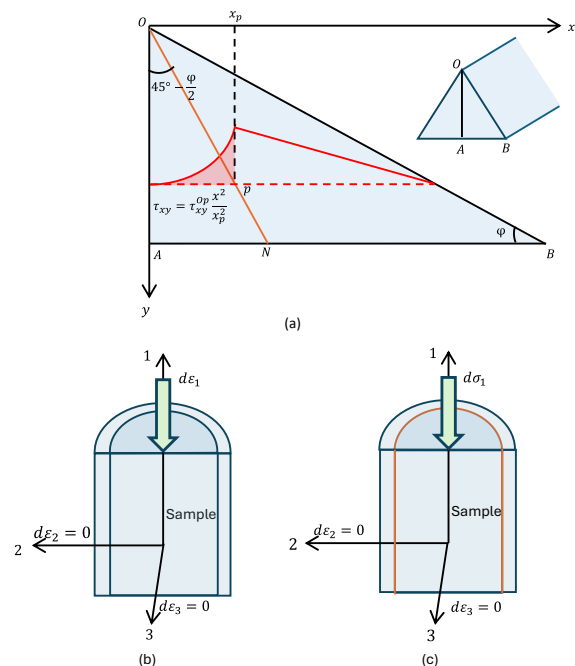


Figure 1. Wedge shaped sand prism for derivation of Jaky's equation.

In geotechnical engineering practice, Jaky's equation (Jaky, 1944; 1948) is most widely adopted to estimate the value of earth pressure coefficient at-rest due to its simplicity:

$$K_0 = \left(1 + \frac{2}{3} \text{SIN} \phi\right) \left(\frac{1 - \text{SIN} \phi}{1 + \text{SIN} \phi}\right) \approx 1 - \text{SIN} \phi' \quad (2)$$

where  $\phi'$  is generally adopted as the effective friction angle of soil in practice.

Jaky's equation is derived through the stress analysis of a wedge-shaped sand prism, as depicted in Figure 1. The stress along the midline (OA) of the sand prism is assumed to represent the at-rest state. However, this assumption has been

found to be invalid, as the stress state along the midline can range from an active stress state to a passive stress state (Michalowski and Park, 2004; Michalowski, 2005). In addition to the assumptions regarding stress states, the earth pressure coefficient at rest,  $K_0$ , serves as an important parameter that characterizes the soil's stress state under its initial undisturbed condition, which is well below the yield threshold. The derivation of Jaky's equation incorporates the soil's friction angle, specifically the angle of repose, which is expected to correspond to the critical state friction angle.

The impact of void ratio on the earth pressure coefficient at rest ( $K_0$ ) is a subject of diverse perspectives within the geotechnical engineering community. One viewpoint suggests that  $K_0$  increases with an increase in void ratio, indicating that looser soils exhibit higher values of  $K_0$  (Chu and Gan, 2004; Gu et al., 2018). However, Wanatowski and Chu (2007) observed minimal variation in  $K_0$  for sand specimens prepared using the water sedimentation method. Talesnick (2012) found that denser granular materials exhibit larger values of  $K_0$ . This disparity necessitates a deeper understanding of the underlying mechanisms behind the void ratio's influence on  $K_0$ . Microscopic factors, such as particle shape and fabric anisotropy, also significantly affect  $K_0$ . This paper aims to investigate the mechanisms influencing  $K_0$  at the microscopic level using micromechanical models.

## 2 MICROMECHANICAL FRAMEWORK

The micromechanical model serves as a theoretical framework to establish a direct connection between the macroscopic mechanical behaviour of granular materials and the microscopic properties of their particle assembly. Through the application of specific homogenization techniques, the constitutive relationship at the inter-particle contact level is systematically linked to the stress-strain constitutive relationship at the macroscopic scale. This approach enables the modelling and prediction of the mechanical behaviour of granular materials using the micromechanical model.

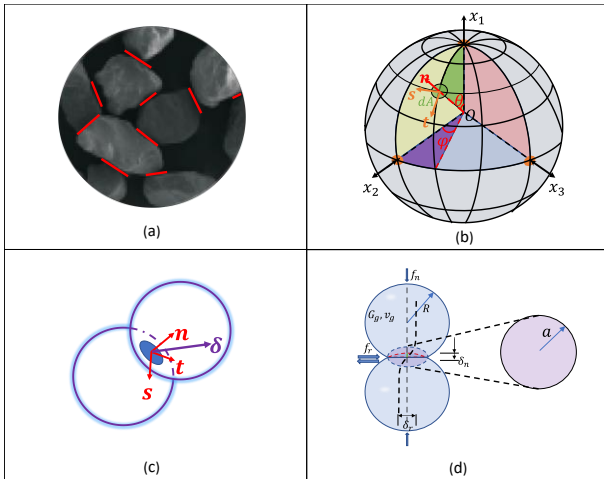


Figure 2. Micromechanical model diagram: (a) Inter-particle contact model; (b) spherical coordinate system; (c) contact vectors; (d) Hertz-Mindlin contact model.

### 2.1 Inter-particle contact

At the inter-particle contact level, the inter-particle contact force  $\mathbf{f}$  is obtained by the product of contact stiffness tensor  $\mathbf{K}$  and contact displacement  $\boldsymbol{\delta}$ .

$$\mathbf{f} = \mathbf{K}\boldsymbol{\delta} \quad (3)$$

The contact stiffness tensor is given by the normal contact stiffness  $k_n$  and tangential contact stiffness  $k_r$  with the contact vectors  $\mathbf{n}$ ,  $\mathbf{s}$  and  $\mathbf{t}$  under the spherical coordinate system given in Figure 2:

$$K_{ij} = k_n n_i n_j + k_r (s_i s_j + t_i t_j) \quad (4)$$

Normal contact stiffness can be determined by the Hertz contact model (Johnson, 1987):

$$k_n = \frac{\partial f_n}{\partial \delta_n} = \frac{2G_g a}{1 - \nu_g} \quad (5)$$

where  $G_g$  and  $\nu_g$  are the particle's shear modulus and Poisson's ratio;  $a$  is the radius of the inter-particle contact.

When the tangential contact force is involved in the model, the tangential contact displacement is given by (Duffy and Mindlin, 1957):

$$\delta_r = \frac{3\mu f_n (2 - \nu_g)}{8G_g a} \left[ 1 - \left( 1 - \frac{f_r}{\mu f_n} \right)^{2/3} \right] \quad (6)$$

where  $\mu$  is the coefficient of inter-particle friction.

The tangential contact stiffness can be determined by partial derivation of tangential contact force to the tangential contact displacement:

$$k_r = \frac{\partial f_r}{\partial \delta_r} \quad (7)$$

The ratio for tangential contact stiffness to normal contact stiffness is a constant which only relates to the particle grain's Poisson ratio  $\nu_g$ :

$$\rho = \frac{k_r}{k_n} = 2 \left( \frac{1 - \nu_g}{2 - \nu_g} \right) \quad (8)$$

Contact stiffness ratio is a critical factor influencing the mechanical behaviour of granular materials. Contact stiffness ratio  $\rho$  is only theoretically related to the particle Poisson's ratio. To consider the particle rotation, an internal parameter  $\alpha_r$  is considered to reduce the interparticle contact stiffness ratio (Fleischmann et al., 2013b). The internal parameter is related to the contact density and particle diameter given by

$$\rho = 2 \left( \frac{1 - \nu_g}{2 - \nu_g} \right) (0.164\beta D^3 - 0.143) \quad (9)$$

where  $\beta$  is contact number per unit volume and  $D$  is the particle diameter.

### 2.2 Granular assembly constitutive relation

Kinematic hypothesis and static hypothesis are two types of homogenization techniques adopted in the micromechanical model (Hicher and Chang, 2005).

For the kinematic hypothesis, uniform strain field  $\boldsymbol{\varepsilon}$  is assumed for the granular assembly. The inter-particle contact displacement vector is determined by the production of strain tensor and branch vector. For granular assembly with uniform size spheres, branch vector is defined as the product of particle diameter and inter-particle contact normal vector:

$$\mathbf{l} = D\mathbf{n} \quad (10)$$

$$\boldsymbol{\delta} = \boldsymbol{\varepsilon}\mathbf{l} \quad (11)$$

Average stress tensor could be determined by volume average of tensor product of contact force vector and branch vector (Christoffersen et al., 1981):

$$\sigma_{ij} = \frac{1}{V} \sum_c f_i l_j \quad (12)$$

Stress strain constitutive relation of granular assembly is given by:

$$\boldsymbol{\sigma} = \mathbf{L}\boldsymbol{\varepsilon} \quad (13)$$

The stiffness tensor can be determined as

$$L_{ijkl} = \frac{1}{V} \sum_c K_{jk} l_i l_l \quad (14)$$

For the static hypothesis, the contact force is obtained based on the external stress (Liao et al., 1997):

$$f_i = \sigma_{ij} l_n \left[ \frac{1}{V} \sum_c l_j l_n \right]^{-1} \quad (15)$$

The strain tensor is obtained by the contact displacement:

$$\varepsilon_{ij} = \frac{1}{V} \sum_c \delta_i l_n \left[ \frac{1}{V} \sum_c l_j l_n \right]^{-1} \quad (16)$$

The compliance tensor can be determined. The coordination number is determined based on the void ratio  $e$  of the granular soils (Zhang, 2023):

$$Z = 24.2 \exp(-2.4e) \quad (17)$$

### 2.3 Fabric anisotropy

The natural granular soils are always inherent anisotropic. The fabric anisotropy can be represented by the spherical harmonics function truncated to second order (Kanatani, 1984):

$$E(\theta, \varphi) = \frac{1}{4\pi} \left[ 1 + \frac{A_{20}}{4} (3\cos 2\theta + 1) + 3\sin^2\theta (A_{22}\cos 2\varphi + B_{22}\sin 2\varphi) \right] \quad (18)$$

where  $\theta$  is the polar angle;  $\varphi$  is the azimuthal angle;  $A_{20}$ ,  $A_{22}$  and  $B_{22}$  are parameters quantifying the degree of fabric anisotropy.

When the parameters  $A_{22}$  and  $B_{22}$  are zero, the inherent fabric becomes transverse anisotropy as visualized in Figure 3. The fabric is isotropic as given in Figure 3(c) which is a sphere under the special case with all three fabric parameters equal to zero. When the parameter  $A_{20}$  increases from 0 to 2, the fabric starts to concentrate along the vertical direction indicating more inter-particle contacts at the vertical direction.

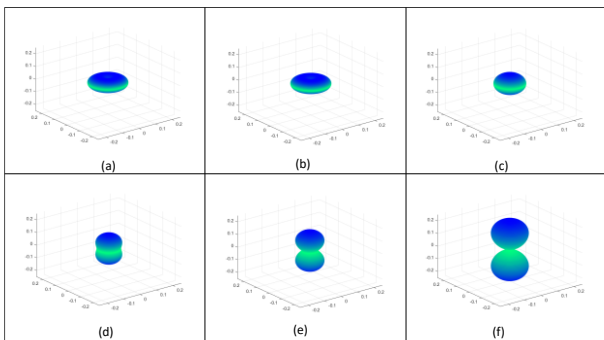


Figure 3. Fabric representation of different degrees of fabric anisotropy: (a)  $A_{20} = -1$ ; (b)  $A_{20} = -0.5$ ; (c)  $A_{20} = 0$ ; (d)  $A_{20} = 0.5$ ; (e)  $A_{20} = 1$ ; (f)  $A_{20} = 2$ .

## 3 AT-REST EARTH PRESSURE COEFFICIENT

Numerical one-dimensional compression test is conducted to determine the earth pressure coefficient at-rest based on the two types of micromechanical models programmed with MATLAB. The change of strain in horizontal direction is kept to zero replicating the at-rest condition and axial loading is

exerted on the sample along the vertical direction to model the real one-dimensional compression test in laboratory. The horizontal loading induced will be adjusted to keep the zero change in horizontal strain. For the kinematic hypothesis based micromechanical model, the vertical loading is exerted on the sample by increase of the axial strain as shown in Figure 1(b). For the static hypothesis based micromechanical model, the vertical loading is exerted on the sample by increasing the axial stress as shown in Figure 1(c). The numerical test is conducted to the vertical strain to  $10^{-4}$  level to maintain the elastic assumption in the model. Table 1 is the parameters adopted in the micromechanical model.

As shown in Figure 4, for both models, the at-rest earth pressure coefficient is significantly influenced by the inter-particle contact stiffness ratio  $\rho$  and fabric anisotropy parameter  $A_{20}$ . For the kinematic hypothesis-based micromechanical model, the maximum at-rest earth pressure coefficient  $K_0$  is predicted to be 0.5 which is far smaller than 2.0 given by the static hypothesis-based micromechanical model. The difference is consistent with the assumptions adopted by the two models. For kinematic hypothesis, a uniform strain field is assumed which exerts a virtual constraint to the granular assembly. The virtual constraint strengthens the granular assembly which causes the induced horizontal stress and at-rest earth pressure coefficient to be small equivalent to the Voigt hypothesis adopted in Fleischmann et al. (2013a). The static hypothesis-based model provides a more reasonable range of at-rest earth pressure coefficient  $K_0$  based on the analysis of the numerical test results. The detail analysis will be based on the static hypothesis-based model.

Table 1. Model parameters.

Parameter	Symbol	Value	Unit
Young's modulus	$E$	29	MN/m <sup>2</sup>
Poisson's ratio	$\nu$	0.2	-
Particle diameter	$D$	2	mm

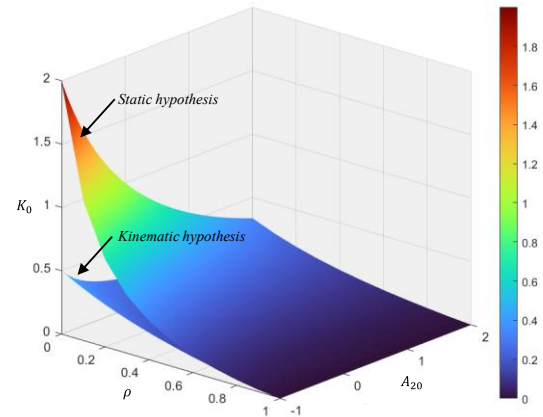


Figure 4. At-rest earth pressure coefficient  $K_0$  obtained by one-dimensional loading test.

## 4 MICROSCALE PARAMETERS

At-rest earth pressure coefficient is influenced by many parameters both microscopically and macroscopically including the void ratio and fabric anisotropy. The effect of these factors and the underlying mechanism will be studied based on the micromechanical model results.

#### 4.1 Effect of void ratio

Void ratio is a typical density parameter for soil and mostly considered for design in the geotechnical engineering projects. According to the test results given in Figure 5, it can be shown that the at-rest earth pressure coefficient  $K_0$  increase with the increase of void ratio. When the soil is very dense with void ratio around 0.6, the coefficient of earth pressure at-rest is 0.22 which is a quite small value. For a very loose case with void ratio 0.95, the coefficient of earth pressure at-rest is 0.8 which indicates high in-situ horizontal earth pressure.

For relative narrow range of void ratio such as from 0.5 to 0.75 or from 0.75 to 1 shown in Figure, the relation between void ratio and earth pressure coefficient at-rest  $K_0$  is approximately linear which is consistent with results of previous research studies (Chu and Gan, 2004; Okochi and Tatsuoka, 1984). The results indicate a smaller at-rest earth pressure coefficient for dense material and larger at-rest earth pressure coefficient for loose material. Based on the micromechanical method-based analysis, the main reason for causing this is due to the change of coordination number or scalar fabric with the density of the material. For dense material, the value of coordination number is large which means more inter-particle contacts joining in the force chain. The vertical loading can be more efficiently transferred through the force chain within the soil. Induced horizontal stress required for stability of the force chain would be relatively small which gives a small coefficient of earth pressure at-rest. For loose material, the value of coordination number is small which means less inter-particle contacts joining in the force chain. The vertical loading can only be transferred through the limited force chain within the soil. Induced horizontal stress would be relatively large to maintain the stability of the force chain which gives a large coefficient of earth pressure at-rest.

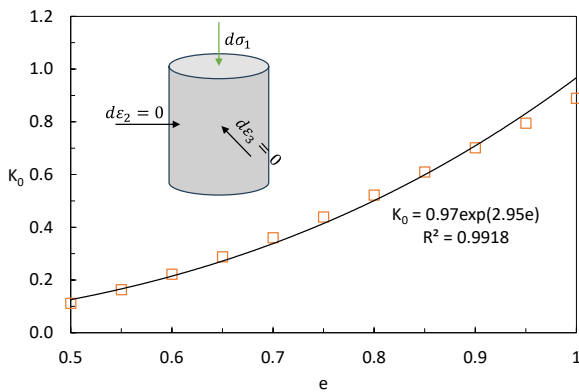


Figure 5. Effect of void ratio on the  $K_0$ .

The effect of void ratio at the macroscale is closely related to the microscale parameter coordination number. Through the control of coordination number or scalar fabric within the granular system, the void ratio directly controls the earth pressure coefficient at-rest. Similar results was also obtained by DEM simulation (Gu et al., 2015). Figure 6 is the change of coefficient of earth pressure at rest with coordination number. The earth pressure coefficient at-rest decreases with the increase of coordination number. The change of coordination number affects the at-rest earth pressure coefficient  $K_0$  at a similar trend with the void ratio. In geotechnical engineering practice, compaction of fill material to certain density is a general requirement for the stability of geotechnical structures. The larger the density of the granular materials, the lower the void ratio of the materials. The effect of void ratio on the coefficient of earth pressure at-rest obtained from theoretically

model and laboratory test results have good agreement with the practice.

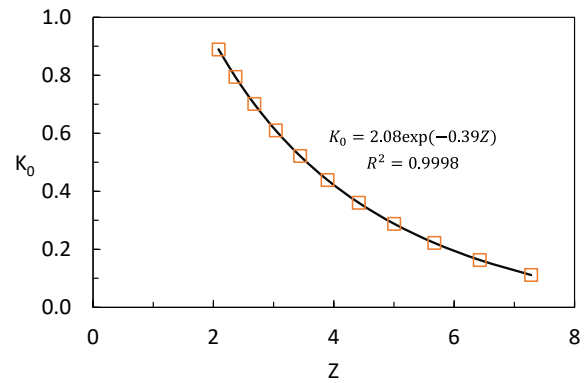


Figure 6. Effect of coordination number on the  $K_0$ .

#### 4.2 Effect of inherent fabric anisotropy

Soil inherently exhibits fabric anisotropy as a consequence of gravity-driven deposition, such that granular assemblies display direction-dependent mechanical responses. This inherent anisotropy arises from preferred particle orientations and the distribution of interparticle contact normals established during deposition (Arthur and Menzies, 1972). Subsequent processes—such as consolidation, cementation, aging, and erosion—can further modify the fabric and thereby induce additional anisotropy, which may be considered part of the overall inherent anisotropy. The resulting fabric frequently manifests as cross anisotropy (transverse isotropy), a condition commonly observed in natural granular materials including sands, clays, and rocks, as well as in many engineered composites. In laboratory investigations, variations in sample preparation produce distinct fabrics (Yang et al., 2008) and correspondingly different coefficients of earth pressure at rest (Okochi and Tatsuoka, 1984; Wanatowski and Chu, 2007), as illustrated in Figure 7 and Figure 8. Within micromechanical modelling frameworks, these fabric differences are quantified by a fabric anisotropy parameter,  $A_{20}$ ; the representation of fabric using  $A_{20}$  is presented in Figure 3.

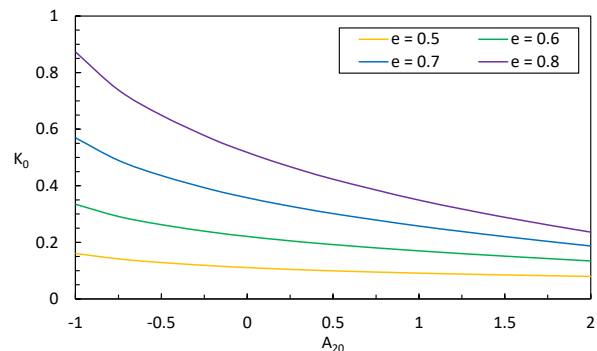


Figure 7. Effect of fabric anisotropy on the  $K_0$  under different void ratio.

Micromechanical model results demonstrate that the coefficient of earth pressure at rest  $K_0$  is strongly governed by material fabric anisotropy. For a fixed void ratio,  $K_0$  decreases monotonically with increasing fabric anisotropy parameter  $A_{20}$  as shown in Figure 7; this decline is most pronounced for loose specimens exhibiting low coordination and initially small  $A_{20}$  values. The model captures the influence of sample preparation on fabric anisotropy: specimens prepared by moist tamping exhibit higher fabric anisotropy and correspondingly

lower  $K_0$  than those prepared by air pluviation or water sedimentation shown in Figure 8 (Okochi and Tatsuoka, 1984; Wanatowski and Chu, 2007). Conversely, at a given level of fabric anisotropy,  $K_0$  increases with void ratio, and this sensitivity to void ratio is amplified for loose materials characterized by small  $A_{20}$ .

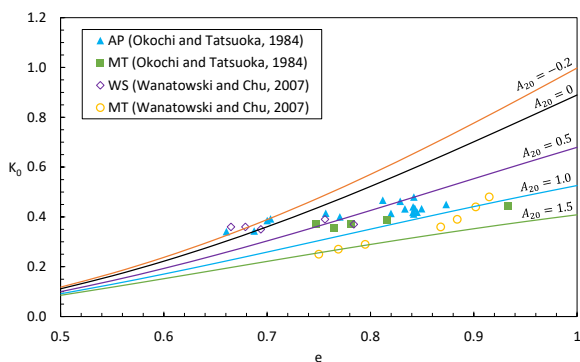


Figure 8. Effect of void ratio under different degrees of inherent fabric anisotropy.

The observed behavior is attributable to a redistribution of interparticle contacts within the granular assembly. As the fabric anisotropy parameter  $A_{20}$  increases, contacts progressively align with the vertical direction; in the limiting case  $A_{20}$  equals to two, transverse contacts vanish and all contacts are oriented either vertically or obliquely. This increase in vertically oriented contacts enhances the assembly's capacity to transmit vertical loads, thereby reducing the transverse stress required to preserve structural stability. Consequently, the external force necessary to maintain zero horizontal strain is diminished, resulting in a lower earth-pressure coefficient at-rest.

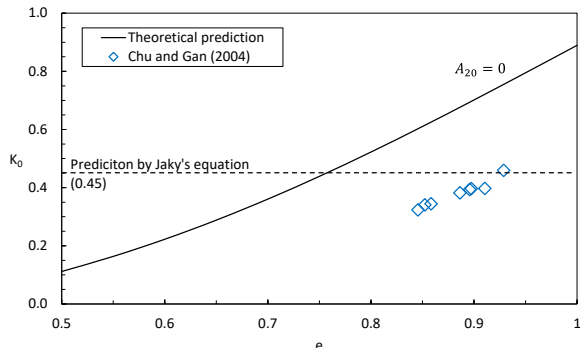


Figure 9. Prediction of  $K_0$  under isotropic condition compared with laboratory test data.

Results from the micromechanical model indicate a coupled influence of fabric anisotropy and void ratio as shown in Figure 4. At low void ratio, where the coordination number is high, the influence of fabric anisotropy on the at-rest earth pressure coefficient  $K_0$  is negligible; conversely, as the void ratio approaches unity the influence of fabric anisotropy becomes pronounced. Void ratio therefore modulates  $K_0$  indirectly through the coordination number, a scalar measure of fabric that reflects the internal contact topology of the granular assembly. In dense assemblies with large coordination numbers the abundance of interparticle contacts renders the directional distribution of contacts relatively unimportant, whereas in loose assemblies with low coordination numbers the limited number of contacts makes  $K_0$  highly sensitive to contact orientation. Overall, the at-rest earth pressure coefficient is strongly governed by the fabric of granular material.

Accounting for fabric anisotropy also explains discrepancies between model predictions and laboratory measurements. Under an isotropic fabric assumption with  $A_{20}$  equal to zero, predictions from Jaky's equation and the micromechanical model exceed the laboratory results reported by Chu and Gan (2004) for loose granular material (Figure 9). When sample anisotropy is incorporated and the fabric anisotropy parameter is calibrated to  $A_{20}$  equal to 1.3 (Figure 10), the model predictions align with the experimental observations.

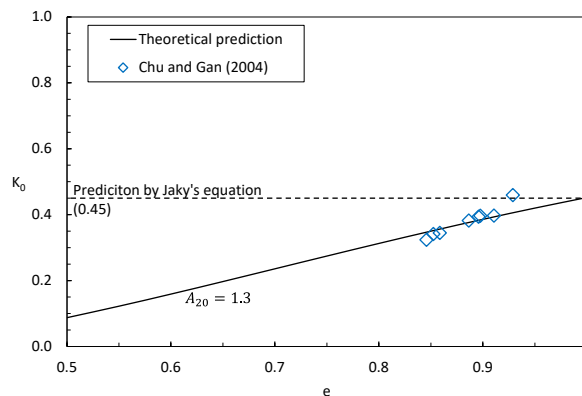


Figure 10. Prediction of  $K_0$  under anisotropic condition ( $A_{20} = 1.3$ ) compared with laboratory test data.

## 5 CONCLUSIONS

The theoretical study provides new insights on how the microscopic particle level parameters affect the at rest earth pressure coefficient of granular materials. There are four key findings through this study.

- Based on the micromechanical model analysis, the results directly show the key particle level parameters influencing the critical engineering parameter earth pressure coefficient at-rest. The predicted results have good agreement with the laboratory testing results.
- The coefficient of earth pressure at-rest increases with the increase of void ratio of the granular materials. The change in coefficient of earth pressure at-rest with void ratio is large and can't be neglected. For a small range of void ratio, the coefficient of earth pressure at-rest is roughly linearly increase with the void ratio.
- The coefficient of earth pressure at-rest is significantly influenced by the degree of inherent fabric anisotropy of the granular materials. With the increase of the degree of fabric anisotropy  $A_{20}$ , the coefficient of earth pressure would decrease significantly. The mechanism leading to this phenomenon is due to the redistribution of interparticle contacts. With the increase of the degree of fabric anisotropy  $A_{20}$ , more interparticle contacts would tend to along the vertical direction. This will strengthen the material along the vertical direction causing less force transferred to horizontal direction meaning that a smaller coefficient of earth pressure at-rest.
- Jaky's formula is widely used in engineering practice for the prediction of earth pressure coefficient at-rest. The study shows that prediction of Jaky's formula could only provide a rough estimate. Only one parameter critical state friction angle considered in Jaky's formula is not enough for complex natural granular soil.

## 6 ACKNOWLEDGEMENTS

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