Modelling crushing of granular materials as a poly-disperse mixture

Modélisation de la fracturation des matériaux granulaires comme un mélange poli-disperse

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ABSTRACT: This paper presents a new model to assess the evolution of the grain size distribution of granular materials during loading. This model is based on the theory of poly-disperse mixtures proposed by De Larrard, 2000. Using this model it is possible to evaluate the compacity of the mixture depending on the grain size distribution, the shape of the particles and the compaction energy. Markov processes are used to assess the evolution of the grain size distribution, Markovian transition probabilities for each grain size are evaluated experimentally using gyratory compaction. Finally the experimental results are compared with the results of the model showing a very good agreement.

RÉSUMÉ : Cet article présente un nouveau modèle qui permet de calculer l'évolution de la granulométrie des matériaux granulaires sous différents chargements. Le modèle est basé sur la théorie des mélanges poli-disperses proposée par De Larrard, 2000. Avec ce modèle, il est possible d'évaluer la compacité du mélange granulaire en fonction de la granulométrie du mélange, la forme des particules et l'énergie de compactage. Un procédé de Markov est ajouté au modèle pour obtenir l'évolution de la granulométrie, les probabilités de transition étant évaluées expérimentalement à l'aide d'une machine de compactage giratoire. Finalement, les résultats du modèle sont comparés avec les résultats expérimentaux et montrent une très bonne correspondance.

KEYWORDS: granular materials, crushing, abrasion, poly-disperse mixtures, compaction.

1 INTRODUCTION

Particle fracture plays a major role in the behaviour of granular materials that are used in engineering structures such as paved roads, railroads, highway embankments, and foundations. The most important engineering properties of granular materials in these structures depend on the amount of particle crushing that occurs due to static or dynamic loads (Lade et al. 1996). Particle breakage occurs as a result of these loads (Bolton 1999; Feda 2002; Hagerty et al. 1993; Hardin 1985; Lade et al. 1996). Grain crushing is influenced by grain angularity, grain size, uniformity of gradation, particle strength, porosity, stress level, and anisotropy (Bolton and McDowell 1997; Feda 2002; Hagerty et al. 1993; Hardin 1985; Lade et al. 1996; Lobo-Guerrero 2006; McDowell and Bolton 1998; Nakata et al. 1999; Nakata et al. 2001a; Yamamuro and Lade 1996).

Researches carried out in the past 10 years have proved the capability of discrete element models to analyse crushing of granular materials. These models works with individual particles and permits stress analysis in each one of those particles. However such models require restrictive assumptions with respect to the number and shape of each particle and a better approach to the actual state of the granular material requires high computational cost. The model presented in this paper use the theory of poly-disperse materials proposed by De Larrard (2000) and is a new possibility to assess the evolution of grain size distribution of granular materials without dealing with the difficulties of discrete element modelling.

2 DESCRIPTION OF THE MODEL

The evolution of the grain size distribution of granular materials under cyclic loading is the result of the abrasion and crushing of particles. The following aspects must evaluate to assess such evolution: (i) the stress level in each class of particle size within the granular material; (ii) the strength of the particles taking in consideration the number of loadings for cyclic loading; and (iii) the change of grain size distribution as a result of the crushing of particles.

In this model the first aspect is assessed with the aid of the poly-disperse theory proposed by De Larrard (2000), the second aspect uses the Weibull theory including a fatigue law for the cyclic loading, and the third one uses the theory of Markovian processes.

2.1 Description of a granular material as a polidisperse mixture

One of the most important variables having an effect on the crushing of a granular mixture is their unit weight. In fact huge experimental and theoretical evidence show that a large volume of voids in the granular material increases the stress within particles. As usual, the void volume could be characterized by the porosity n, the void ratio e or the compacity. The compacity of a granular mixture is defined as the solids volume of the grains Φ in a unit volume. As a result the compacity could be calculated using the porosity or the void ratio as follows:

$$n = 1 - \Phi \qquad e = \frac{1}{\Phi} - 1 \qquad \text{(1a, b)}$$

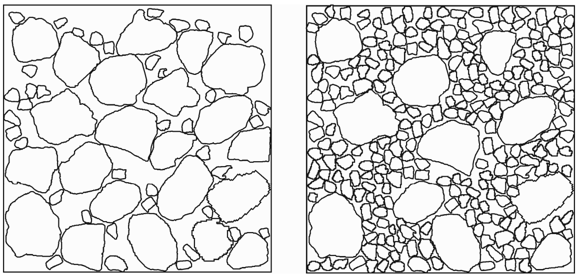
The model proposed by De Larrard (2000) permits obtaining the compacity of a granular mixture depending on the volumetric proportion (y_i) of particles of size d_i. This calculation is based on the knowledge of the residual compacity β_i of each grain size that represent the maximum compacity obtained experimentally for each class individually.

The virtual compacity γ was defined by De Larrard as the maximum compacity theoretically reachable for a particular granular mixture without any alteration of the original shape of the particles. Details about the derivation of the relationships to obtain the virtual compacity of a poly-disperse mixture are presented in De Larrard (2000), this paper present only a succinct description.

2.1.1 Compacity of granular mixtures

For binary mixtures with grain size d_1 and d_2 three cases could be considered depending upon the interaction between the two types of grains: no interaction, full interaction and partial interaction. Figures 1(a) and 1(b) illustrates the case of a binary mixture with no interaction between particles. For this case the virtual compacity γ could be obtained knowing the volumetric concentration, y_1 and y_2, and the residual compacity, β_1 and β_2, of each class size as follows:

The total volume is: $$y_1 + y_2 = 1 \qquad (2)$$



(a) (b)

Figure 1. Case of binary mixtures with no interaction.

The partial volume of each class Φ is the volume of the class in the unit volume, then:

$$y_1 = \frac{\Phi_1}{\Phi_1 + \Phi_2} \quad \text{and} \quad y_2 = \frac{\Phi_2}{\Phi_1 + \Phi_2} \qquad (3a, b)$$

The virtual compacity for a binary mixture becomes:

$$\gamma = \Phi_1 + \Phi_2 \qquad (4)$$

When the bigger grains are dominant, the virtual compacity is γ_1. In this case, the bigger grains fill up the volume with no interaction with the small grains. For this reason the partial volume Φ_1 is the same than the residual compacity β_1, so $\Phi_1 = \beta_1$. Then accordingly with equations 2 to 4, the virtual compacity γ_1 becomes:

$$\gamma_1 = \beta_1 / (1 - y_2) \qquad (5)$$

When the smaller grains are dominant the virtual compacity is γ_2. In this case the smaller grains fill the voids existing between the bigger grains with their maximum individual compacity β_2. In this case the virtual compacity γ_2 is:

$$\gamma_2 = \beta_2 / [1 - (1 - \beta_2) y_1] \qquad (6)$$

For the binary mixture only one virtual compacity is possible, this compacity is the minimum between γ_1 and γ_2. In fact if γ is higher than γ_1 grains 2 penetrate into grains 1 and vice versa. For this reason the only possible arrangement correspond to the minimum virtual compacity. This condition is called the impenetrability condition:

$$\gamma = \inf(\gamma_1, \gamma_2) \qquad (7)$$

Different interactions between particles can be considered: a binary mixture having total interaction occurs when the size of the particles is identical but the residual compacities are different: $d_1 = d_2$; $\beta_1 \neq \beta_2$. On the other hand, for the case of binary mixtures with $d_1 > d_2$ two physical effects could appear: de-compaction effect and boundary effect. Taking in to account these two effects, De Larrard (2000), calculate the virtual compacity γ_1 and γ_2 as follows:

$$\gamma_1 = \frac{\beta_1}{1 - \left(1 - a_{12} \beta_1 / \beta_2\right) y_2} \qquad (8)$$

$$\gamma_2 = \frac{\beta_2}{1 - \left[1 - \beta_2 + b_{21} \beta_2 \left(1 - 1/\beta_1\right)\right] y_1} \qquad (9)$$

Where a_{12} is the de-compaction coefficient and b_{12} is the boundary effect coefficient.

In the case of a poly-disperse mixture with n_c granular classes and with $d_1 > d_i > d_{nc}$, the grains $d > d_i$ in the mixture undergo a de-compaction effect due to the grains which size $d < d_i$ and the mixture undergo a boundary effect due to the grains which size is $d < d_i$. The virtual compacity, considering the grain size *i* as the dominant grains is, (De Larrard, 2000):

$$\gamma_i = \frac{\beta_i}{1 - \sum_{j=1}^{i-1} \left[1 - \beta_i + b_{ij} \beta_i \left(1 - 1/\beta_j\right)\right] y_j - \sum_{j=i+1}^{n} \left[1 - a_{ij} \beta_i / \beta_j\right] y_j} \qquad (9)$$

Similar than the case of binary mixtures, the impenetrability restriction is applicable. This restriction becomes:

$$\gamma = \inf_{1 \le i \le n} \gamma_i \qquad (10)$$

Once calculating γ_i considering each class *i* as the dominant class (using equation 10), the actual dominant grain size is the one for which the minimum γ_i is obtained. From the geotechnical point of view, all the grains with $d < d_i$ are the matrix of the mixture, and the grains with $d > d_i$ are dispersed grains in the mixture. The virtual compacity γ is unreachable experimentally. For this reason it is necessary to obtain the actual compacity, Φ, which is more or less close to the virtual compacity depending upon the compaction method ($\Phi < \gamma$). For a real mixture the compacity Φ is the accumulation of the compacitys corresponding to each class:

$$\Phi = \sum_{i=1}^{n} \Phi_i \qquad (11)$$

In the mixture the dominant grain has the maximum compacity, taking in to account the presence of the other grains, this compacity is Φ_i^* .Therefore the compacities in the mixture are: $\Phi_0 \ldots \Phi_{i-1}, \Phi_i^*, \Phi_{i+1} \ldots\ldots \Phi_n$.

To obtain the relationship between the virtual compacity and the actual compacity, De Larrard, 2000, proposes a compaction coefficient *K*. This compaction coefficient is the addition of the compaction coefficient corresponding to each grain size:

$$K = \sum_{i=1}^{n} K_i \qquad (12)$$

The compaction coefficient for each grain size is obtained as follows:

$$K_i = \frac{\Phi_i / \Phi_i^*}{1 - \Phi_i / \Phi_i^*} = \frac{y_i / \beta_i}{1/\Phi - 1/\gamma_i} \qquad (13)$$

2.2 Probability of crushing of particles depending on its compacity

The relationship Φ_i/Φ_i^* is a powerful parameter to assess the stress level supported by the particles of size d_i within the granular material. In fact, Φ_i is the volume filled by the particles d_i, and Φ_i^* is the maximum volume that the particles d_i can fill without any alteration of the mixture. Therefore the relationship Φ_i/Φ_i^* describes the proportion of voids that is filled by particles *i* with respect to the available void space for these particles (Figure 2).

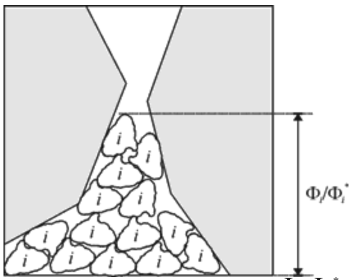


Figure 2. Schematic drawing of the relationship Φ_i/Φ_i^*.

Regarding stresses, a relationship Φ_i/Φ_i^* close to 1 indicates that particles *i* are filling well the void space reserved for those particles and therefore it is highly probably that the stress chains within the granular material flow along these particles. On the other hand, a relationship Φ_i/Φ_i^* close to zero indicates that particles *i* are in a loose state within the granular material and therefore doesn't support significant level of stress.

Regarding crushing, the relationship Φ_i/Φ_i^* take action with two opposite trends: (i) when Φ_i/Φ_i^* is close to 1 particles take high level of stresses that increase the probability of crushing but these particles are well confined by other particles that reduces the probability of crushing; on the other hand when Φ_i/Φ_i^* is close to zero, particles are with low level of stress but have a reduced number of contact points with other particles. These two opposite trends suggest that there is a particular value of Φ_i/Φ_i^* for which the probability of crushing is maximum, this can be set as the compacity relationship for maximum crushing $(\Phi_i/\Phi_i^*)_{mc}$.

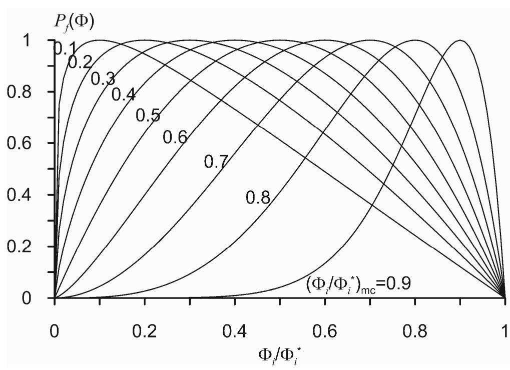


Figure 3. Probability of crushing of particles depending on the relationship Φ_i/Φ_i^*.

A function that can describe the probability of crushing of particles depending the on the relationship Φ_i/Φ_i^* is presented in equation 14 (Ocampo 2009), Figure 3:

$$p_{f_i}(\Phi) = 4\left\{\left(\Phi_i/\Phi_i^*\right)^{\alpha}\left[1-\left(\Phi_i/\Phi_i^*\right)^{\alpha}\right]\right\} \quad (14)$$

Parameter α is directly related with the compacity relationship for maximum crushing $(\Phi_i/\Phi_i^*)_{mc}$ as follows:

$$\alpha = -\ln(2)/\ln(\Phi_i/\Phi_i^*)_{mc} \quad (15)$$

2.3 *Probability of crushing of particles depending on its strength*

The effect of the particles size on the strength of particles can be captured using the Weibull relationship. Furthermore as the model presented in this paper focuses on cyclic loading, the strength of the particles depends on the number of loading cycles through a Wohler law. Equation 16 represent the crushing probability of any particle that is the result of including the Wohler law into the Weibull relationship:

$$p_{f_i}(d_i) = 1-\exp\left[-\left(\sigma/(\sigma_1 N^{b_f} d_i^{-b_c})\right)^m\right] \quad (16)$$

Where m is the Weibull Law parameter, σ is the applied stress, σ_1 is the strength of a particle having unit diameter and for one loading cycle, N is the number of cycles, b_f is the slope of the Wohler fatigue law, and b_c is the fracture parameter proposed by Lee (1992).

2.4 *Combined probability of crushing*

The crushing probabilities depending on the compacity and on the strength correspond to independent process; as a result the combined probability of crushing can be assessed as the product of both probabilities:

$$p_f = p_{f_i}(\Phi)\cdot p_f(d_i) \quad (17)$$

2.5 *Grain size distribution of crushed particles*

When original particles break, they produce a set of smaller sub particles modifying the grain size distribution of the mixture. Afterwards, for another loading cycle, these sub particles can break again. This process is repetitive for different loading cycles and can be described by a Markov process. In such type of process an initial particle having d_i size (state 1) breaks and produce a set of particles having sizes $d_j<d_i$ (states 2 to 11), this process is described in Figure 4.

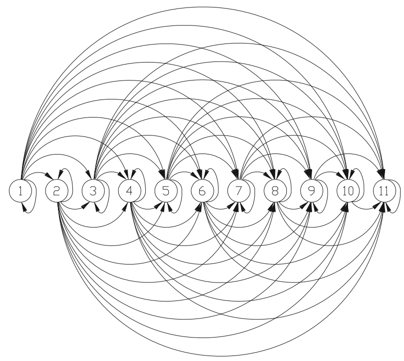


Figure 4. Diagram of state transition in a Markovian process.

In a Markovian process the transition between states is described by the Transition probability Matrix π as follows:

$$\pi = \begin{bmatrix} p_{1,1} & . & p_{1,n} \\ 0 & p_{i,i} & p_{i,n} \\ 0 & 0 & p_{n,n} \end{bmatrix} \quad (18)$$

In this matrix components p_{ii} are the probability of no failure of particle i that is obtained from equation 17 as $p_{ii}=1-p_{fi}$. Elements p_{ij} for $j<i$ represent the transition probability of a particle having an initial size i to a size j. Transition probabilities of particles during crushing was studied experimentally by Ocampo 2009 using particles with different colour for each initial size. Figure 5 represent the fitting of the experimental results using a beta function.

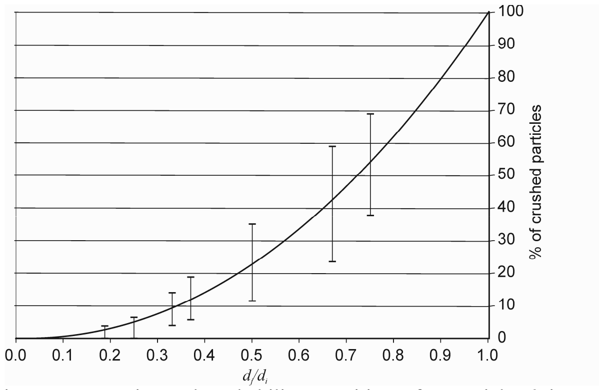


Figure 5. Experimental probability transition of a particle d_i in to a particle d_j.

Elements p_{ij} of the transition probability matrix can be calculated as follows:

$$p_{ij} = p_f \cdot Beta(d_i, d_j) \quad (19)$$

Finally the evolution of the grain size distribution of a granular mixture results from the product of the transition probability matrix (transposed) and the grain size distribution before the loading cycle:

$$[y_i]_N = [\pi]^T [y_i]_{N-1} \quad (20)$$

3 RESULTS

To verify the predictions of the model three different granular materials were tested in a gyratory compactor. This apparatus reproduce the stress rotation during field compaction of granular layers. Compaction was performed to different number of loading cycles and then the grain size distribution was analyzed.

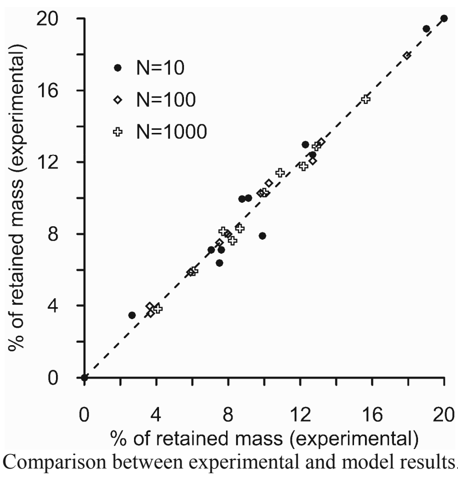


Figure 6. Comparison between experimental and model results.

Figure 6 present a comparison of the experimental and numerical tests for one of the tested materials. This figure shows the mass of particles retained in each sieve size. Each point in the figures corresponds to a particle size at the end of a specific number of loading cycles. These figures show the very good agreement between the model and the experimental data.

Figure 7 represent an extrapolation of the results of the model up to a million of loading cycles showing the capacity of this analytical model to calculate the evolution of grain size distribution when a high number of cycles are applied to the material.

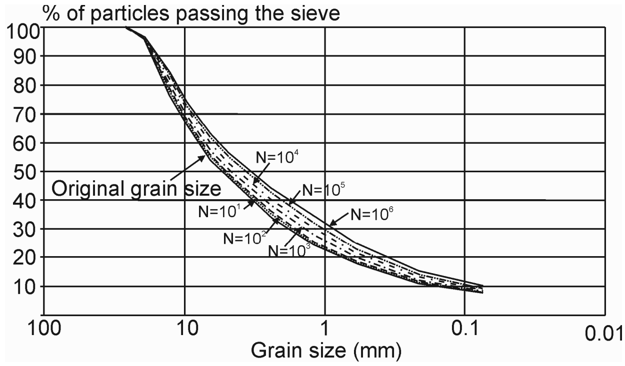


Figure 7. Evolution of the grain size distribution for different number of loading cycles.

4 CONCLUSION

This paper presents a new model to assess the process of changes of the granular material properties as fracturing and abrasion occur as a result of cyclic loading. The model has shown that it can accurately predict the deterioration process of unbound granular materials subject to cyclic loading. This analytical model is based on the theory of poly-disperse mictures and therefore calculations up to high number of loading cycles can be performed without the difficulties of discrete element modelling. The results show very good agrement with the experimental tests illustrating the possibilities of this new model.

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