Influence of Ground Motion Variability on Seismic Displacement Uncertainty

Influence de la variabilité des mouvements de terrain sur l'incertitude des déplacements en régime sismique

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ABSTRACT: A series of probabilistic seismic displacement analyses were performed to understand how material property variability coupled with systematic changes in the type and complexity of ground motion variability affect the displacement prediction uncertainty of the Newmark rigid-block method.

RÉSUMÉ: Une série d'analyses probabilistes des déplacements en régime sismique ont été effectués pour comprendre comment les variations des propriétés des matériaux couplées avec le type et la complexité des variabilités des mouvements de terrain affectent les incertitudes dans les prédictions de déplacement avec la Méthode des blocs rigides de Newmark.

KEYWORDS: Newmark, displacement, seismic, probabilistic, ground motion, variability, uncertainty, Monte Carlo

1 INTRODUCTION

Seismic slope deformation methods are used to make predictions of earthquake-induced permanent displacements in natural slopes and man-made dams and embankments. The predictive capability of well-established methods such as rigid-block (Newmark 1965) and decoupled (Makdisi and Seed 1978) procedures, however, are often associated with a high-degree of uncertainty which is a consequence of both parametric and modeling sources of variability. Parametric variability describes a method's sensitivity to the range of input parameters (e.g., shear strength, groundwater and earthquake ground motions) and is a function of the number of input parameters as well as the amount of variability in each parameter. Modeling variability is related to how well the method captures the actual physical mechanism of seismic-induced deformation when all input parameters are fully known. Although the majority of deformation-based method available today have a common conceptual origin in the sliding-block model proposed by Newmark (1965), differences in their analytical formulation, procedural structure, underlying assumptions and mathematical or regression functional form can result in different predictive capabilities and sensitivities to parametric variability (Strenk and Wartman 2011).

In seismic slope deformation analyses, parametric variability comes from parameters characterizing the seismic demand (earthquake ground motions) and those characterizing the slope's seismic resistance (represented by the seismic yield coefficient, k_y which is a function of the slope geometry, shear strength and groundwater conditions). In a probabilistic framework, the interplay between these two sources of parametric variability can make evaluating their relative contributions to total displacement uncertainty a difficult task. The main focus of this study is to examine how ground motion variability influences the prediction uncertainty of the Newmark (1965) rigid-block method. To that end, a series of probabilistic seismic slope deformation analyses were performed on an idealized slope for a scenario earthquake event. Displacement uncertainty was quantified for several scenarios designed with increased levels of ground motion variability. Variability in the seismic resistance of the slope was also included. In this approach, realistic levels of variability in both seismic resistance and demand are systematically changed to evaluate their collective effect on displacement prediction uncertainty.

2 PROBABILISTIC DISPLACEMENT ANALYSIS

Performing a rigid-block analysis consists of the following steps: (1) a limit-equilibrium pseudostatic slope stability analysis to compute k_y; and (2) characterization of the earthquake-induced shaking at the site. The seismic yield coefficient represents the minimum acceleration required to initiate down-slope displacement of a slide mass. In the rigid-block method, earthquake shaking is characterized by acceleration time-histories that represent a rock outcropping condition which is consistent with the concept of slide mass rigidity assumed by Newmark (1965). Each of these analyses was implemented in a probabilistic framework using Monte-Carlo simulation. All simulations were performed for 1000 iterations using Latin-Hypercube sampling of the input distributions.

The idealized slope model has a height of 20 m with a slope face inclined at an angle of 18 degrees. The failure surface shown in Figure 1 is intended to represent a first-time, shallow translational landslide. Shear strength of the landslide material (unit weight, $\gamma = 20$ kN/m^3) was assumed to be controlled by the peak friction angle (ϕ'_{peak}) only.

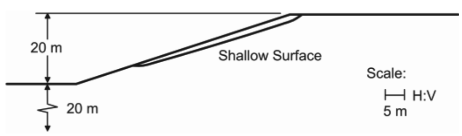

Figure 1. Cross-section of the idealized slope model with a shallow failure surface (maximum thickness of 2 m).

The scenario earthquake used for this analysis is the 1994 Northridge event (moment magnitude, $M_w = 6.7$), in California, USA. The slope was assumed to be located 28 km to the northwest of the epicenter. Based on the assumed site location, acceleration time histories were selected from recording stations that recorded the Northridge event. Four stations were selected: (1) Lake Hughes 12A (LHA); (2) Castaic-Old Ridge Road (ORR); (3) Vasquez Rock Park (VAS); and (4) Newhall-West

Pico Canyon Road (WPI). These time-histories were rotated to their fault-normal/parallel components and only the fault-normal components were used in the analysis.

3 GROUND MOTION VARIABILITY

Shaking experienced at the hypothetical site was characterized using published ground motion prediction equations (GMPE) using a pseudo-probabilistic approach to model variability in the ground motions. In this approach, the Abrahamson and Silva (2008) GMPE was used to obtain a median acceleration response spectrum and the intra-event component of standard deviation (σ) (i.e., single-event variability) for the median prediction. Variability was modeled by varying the number of standard deviations (n_σ) around the median spectrum ($n_\sigma = 0$) as a normally-distributed random variable ranging from $n_\sigma = -3$ to +3. The use of a normal distribution achieves symmetry in the range of n_σ generated while preserving the underlying log-normal distribution of the spectral ordinates. Different target response spectra were obtained by multiplying n_σ by the intra-event standard deviation and adding or subtracting this from the medium spectrum. The median, $\pm$ 1σ and $\pm$ 3σ spectra are shown in Figure 2; the *PGA* ranges from 0.07 g to 1.15 g with a median value of 0.28 g. A rock outcropping condition was simulated by assuming a shear wave velocity over 30 m (v_{s30}) of 1100 m/s and depth of rock of 0 m (ground surface).

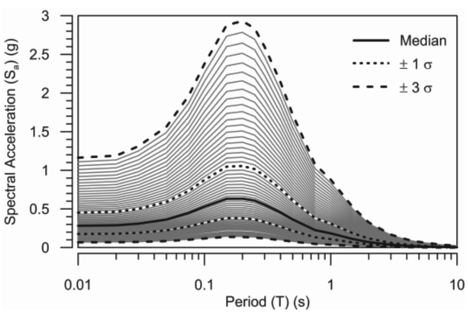

Figure 2. Illustration of the range of acceleration response spectra for the hypothetical site

To understand how ground motion variability affects the displacement uncertainty, the pseudo-probabilistic approach described above was implemented in the seismic displacement analysis in a systematic way by performing multiple probabilistic analyses for four different scenarios. These scenarios were designed to progressively increase in their inherent complexity by introducing different types of ground motion variability to the seismic displacement analysis. The four scenarios evaluated are described below.

No variability – This is a baseline condition with no variability where the ground motion is held at a constant intensity. Each parent motion (LHA, ORR, VAS, WPI) was spectrally-matched to the median target response spectra generating four separate motions each with a *PGA* of 0.28 g. All spectral-matching was performed using a wavelet-based algorithm developed by Mukherjee and Gupta (2002).

Variable intensity – For this scenario, variability in the ground motion intensity is introduced. Using the median spectrally-matched motions from the previous scenario, in this approach, variation in intensity is modeled by simple uniform-scaling to the target $\pm$ 3σ *PGA* values generated from the pseudo-probabilistic approach. For each parent motion, a suite of 1000 *PGA*-scaled acceleration time-histories were generated.

Variable intensity and frequency – For this case, additional variability in form of richer ground motion frequency content is combined with the variation of intensity. This form of variability was introduced by considering the variation of spectral acceleration (S_a) with period (T) provided by the full target response spectra. Variability in the frequency content is modeled in an approximate manner through variation of the spectral shape and bandwidth of the response spectra between the $\pm$ 3σ bounds (Figure 2). This additional variability was achieved by spectrally-matching a single parent ground motion to randomly-selected response spectrum between $\pm$ 3σ bounds generated from the pseudo-probabilistic approach. For each parent motion, 1000 spectra-compatible acceleration time-histories were generated.

Variable, intensity, frequency and waveform – In this approach, additional variability is introduced by using all four parent ground motions in the spectral-matching process. Since the wavelet-based spectrally-matching algorithm generally maintains the non-stationary characteristics of the parent motions, this scenario provides variability in the core waveforms that are propagated through the displacement analysis. To accomplish this, randomly selected parent motions are paired with randomly selected response spectra (between the $\pm$ 3σ bounds generated from the pseudo-probabilistic approach) and spectrally-matched. A total of 1000 spectra-compatible acceleration time-histories were generated that incorporate all four parent ground motions.

4 SEISMIC DISPLACEMENT NON-LINEARITY

While the intended purpose of this study is to examine ground motion variability, Strenk and Wartman (2011) demonstrated that predictive uncertainty of common seismic displacement methods is also a function of a slope's relative degree of stability. In the context of Newmark's sliding block analogy, the stability of a slope with respect to sliding is defined as the ratio of a slope's ability to resist sliding (k_y) over the peak value of the earthquake-induced acceleration (*PGA*) and is referred to as the acceleration ratio (k_y/PGA). For a variety of simplified, equation-based methods and even more complex decoupled methods, a highly-nonlinear relationship exists between the acceleration ratio and displacement (δ); this is illustrated in Figure 3.

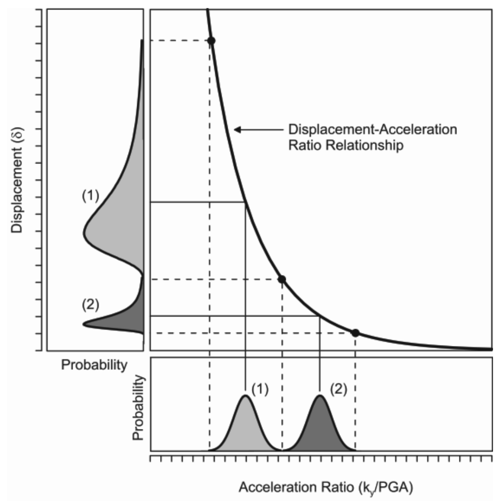

Figure 3. Illustration of the relationship linking the position of the acceleration ratio distribution to displacement prediction uncertainty.

Non-linearity is a consequence of Newmark's modeling decision to approximate the physical mechanism of earthquake-induced displacement after the inherently non-linear dynamic behavior of a sliding-block system. This inherent non-linearity manifests itself through the calculation procedure of "double-integrating" the acceleration time-history in excess of k_y (Newmark 1965) to compute the permanent displacement. In a probabilistic framework, non-linearity was observed to have a strong influence on how parametric variability (in seismic resistance and demand) is propagated through the analysis and can lead to varying degrees of displacement uncertainty (Strenk and Wartman 2011). This relationship is characterized by a threshold behavior whereby nominal changes in acceleration ratio are greatly amplified into large changes in predicted displacement and hence uncertainty (Figure 3).

In order to capture this, multiple simulations were performed with the purpose of generating three different distributions of k_y targeted to have low, moderate and high median values. To do this, probability distributions of ϕ'_{peak} were systematically varied through the pseudostatic analysis to control the slope's seismic resistance; these distributions are referred to as "low", "moderate" and "high" strength and are illustrated in Figure 4a.

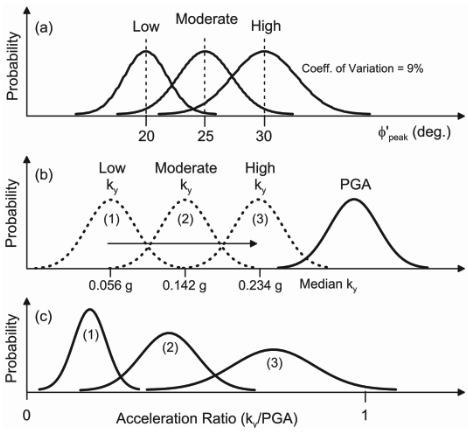

Figure 4. Illustration of the relationship between the k_y and *PGA* distributions and the resulting k_y/PGA distributions. Corresponding distributions of ϕ'_{peak} are shown for the three strength conditions.

By holding the distribution of *PGA* constant and varying the k_y distributions (Figure 4b), sampling of acceleration ratios was targeted to concentrate along different portions of the k_y/PGA scale (Figure 4c). For each ground motion variability scenario, three separate simulations for each strength condition were performed. This approach allowed displacement uncertainty to be quantified along different regions of displacement-acceleration ratio relationship and provided insight into how both method non-linearity and ground motion variability affect the level of prediction uncertainty.

5 DISPLACEMENT ANALYSIS RESULTS

Statistical measures of central tendency and spread were calculated for the displacement data obtained from the Monte-Carlo simulations of the different scenarios of ground motion variability. Overall, the displacement data was observed to be positively skewed as a result of several sources including inherent distributional assumptions (e.g. log-normal GMPE), truncation at prescribed limits (e.g. k_y below zero cannot exist, no k_y/PGA greater than 1) and method non-linearities (Figure 3). Non-parametric statistics based on percentiles that are less sensitive to data asymmetry and were used to describe the data.

For this study, uncertainty in the displacement predictions was assessed using the interquartile range (*IQR*). This parameter represents the range of the middle 50% of the displacement data and is calculated as:

$$IQR = P_{75} - P_{25}$$

where, P_{75} and P_{25} are the 75th and 25th percentiles of the data. Interquartile range data is summarized in Table 1 for the different ground motion variability scenarios and strength conditions.

Table 1. Summary of displacement interquartile range for the different types of ground motion variability and strength conditions

Ground Motion Variability	*Displacement IQR (cm)*			
No Variability	LHA	ORR	VAS	WPI
Low Strength	8.2	7.5	9.1	9.4
Moderate Strength	0.4	0.5	0.9	0.8
High Strength	0.1	0.1	0.2	0.2
Varied intensity				
Low Strength	14.6	12.9	15.9	16.4
Moderate Strength	2.3	2.0	3.4	3.0
High Strength	0.7	0.7	1.3	1.0
Varied intensity-frequency				
Low Strength	15.3	13.9	17.1	17.5
Moderate Strength	2.6	2.3	3.9	3.5
High Strength	0.8	0.9	1.5	1.3
Varied intensity-frequency-waveform (all 4 motions)				
Low Strength	16.7			
Moderate Strength	3.0			
High Strength	1.0			

6 DISCUSSION

The seismic displacement results presented in Table 1 demonstrate that the combined variability from both ground motions and the soil properties have a marked influence on displacement prediction uncertainty. For the scenarios evaluated, uncertainty is also observed to increase as variability in the intensity, frequency content and waveform are incrementally introduced into the probabilistic seismic displacement analysis.

6.1 *No Variability*

For this scenario, displacement uncertainty is due *entirely* to the variability in the seismic resistance, specifically the k_y distributions. With no variability in the ground motions, the displacement-acceleration ratio relationship (δ-k_y/PGA) plots as a smooth, monotonically-increasing line; the δ-k_y/PGA relationship for the LHA parent motion is shown in Figure 5. At low acceleration ratios, this relationship is highly non-linear which causes parametric variability of k_y to be magnified into large displacement uncertainty (*IQR* of 8 to 9 cm at the low strength) that progressively reduces as the relative stability of the slope increases (moderate to high strength conditions). This is consistent with the median k_y/PGA that increases from about 0.20, 0.51 and 0.83 for the low, moderate and high strength k_y distributions. For this scenario, the δ-k_y/PGA relationship for the LHA motion is similar to that for the other parent motions which is due to the fact that all four motions were spectrally matched to the same median response spectrum and thus predict a similar response.

6.2 *Varied Intensity*

For this scenario, variability in k_y and intensity of the ground motions contribute to the overall displacement uncertainty. As these are scaled motions, the frequency content is constant in the displacement analysis of each parent motion. The δ-k_y/PGA relationship for the LHA motion is compared to the case of no ground motion variability in Figure 5. Significant scatter is observed in the δ-k_y/PGA relationship and the displacement uncertainty for each parent motion is higher than that for the case of no ground motion variability. By including variability in the shaking intensity, the displacement *IQR* is observed to increase by a factor of nearly 2 for the low strength condition. The *IQR* increases by factors of 4 and 5 for the moderate to high strength conditions.

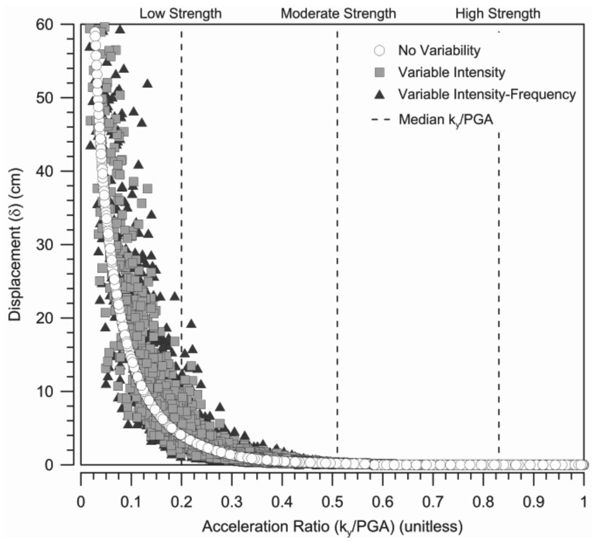

Figure 5. Comparison of the δ-k_y/*PGA* relationships for the LHA parent motion for the different ground motion variability scenarios

From this it may be inferred that the displacement uncertainty for marginally stable slopes is more sensitive to variability in the site conditions (i.e., input parameters that feed into k_y) while slopes with greater relative stability are more sensitive to ground motion variability. This suggests that for weaker slopes, greater resources and effort should be focused on constraining the variability of the subsurface conditions, whereas for more stable slopes the focus should be on characterizing the earthquake ground motions.

6.3 *Varied Intensity and Frequency*

When additional variability from frequency content is introduced into the analysis, marginal increases in the displacement uncertainty are observed. This is indicated by the slight increase in scatter in the δ-k_y/*PGA* data shown in Figure 5. For the four parent motions, displacement uncertainty increases by a factor of about 1.2, this is substantially less change than that observed when variability in intensity was introduced into the analysis. This is consistent with research that has demonstrated the efficiency of intensity ground motion parameters (e.g., *PGA*, peak ground velocity, *PGV* and Arias Intensity, I_a) over frequency content parameters for correlating with displacement predictions (Saygili and Rathje 2008).

While the range of response spectra simulated in the analysis (Figure 2) do show variations in their spectra shape, the peak spectral acceleration between the ± 3σ bounds remains constant at a predominant period (T_p) of about 0.2 s. This characteristic as well as the frequency-smoothing affect that spectral-matching has on the output motions could explain the marginal increase in displacement uncertainty. It is likely that displacement uncertainty would increase had the T_p of the simulated response spectra been allowed to take on a range of values. If, for example, the sub-model for site-conditions (specifically v_{s30}) in the Abrahamson and Silva (2008) GMPE had been modeled as a random variable this would have generated more diverse spectral shapes and hence additional variability in the ground motions. If this technique is implemented in a probabilistic analysis it should be compatible with the underlying assumptions of the method being used. For example, the Newmark (1965) rigid-block model assumes that the slope and slide mass are a non-deformable, rigid body. As such it would be inappropriate to use motions spectrally-matched to a response spectrum for site conditions representative of soft soil as this implies that the slope and slide mass can respond dynamically to the motions. In this case, decoupled-type analyses such as Makdisi and Seed (1978) would be appropriate.

6.4 *Variable Intensity, Frequency and Waveform*

When the intensity, frequency and waveform are varied, the combined affect yields displacement uncertainty that is marginally different from the previous case. Similar to frequency, introducing more diverse waveforms has little effect on the prediction uncertainty. However, it is interesting to note that *IQR* for this scenario is sometimes greater or less than the *IQR* for the individual parent motions. This is related to the wavelet-based spectral-matching process where the parent motion is decomposed, scaled up/down using wavelets and re-assembled such that modified time-history is compatible with the spectrum and non-stationary characteristics of the parent motion are preserved. Thus, even though the response spectra varies between ± 3σ bounds using only one parent motion yields a suite of spectra-compatible motions that have very similar non-stationary characteristics. This explains the systematic differences between the four parent motions for three previous ground motion variability scenarios. For example, the non-stationary characteristics of the VAS and WPI motions are such that when input into the "double-integration" computation of the rigid-block method, regardless of being scaled or spectrally matched, systematically yield higher median displacements and more prediction uncertainty than motions LHA and ORR. When all four motions are propagated through the displacement analysis, the effects of individual motions are tempered and a more average displacement response is computed. This average response, however, is not necessarily more precise than using a single parent motion.

7 CONCLUSIONS

Systematically increasing ground motion variability (and complexity) in the probabilistic displacement analysis showed a trend of increased displacement uncertainty. For the idealized slope, this trend was consistent across a range of relative stabilities (low, moderate and high). Ground motion variability had its most pronouced affect on displacement uncertainty at low acceleration ratios as a consequence of the non-linear δ-k_y/*PGA* relationship. Of the four scenarios considered, variability in ground motion intensity alone has the greatest impact on the displacement prediction uncertainty. When additional variation in frequency content and waveform is introduced, only minimal changes in uncertainty are observed. These results suggest that displacement uncertainty may be adequately characterized through simple scaling of ground motion intensity and that the effort required to create variability in both intensity and frequency through spectral-matching may be of limited utility. Also, the use of mutliple parent motions in a displacement analysis can overcome potential biases introduced by individual motions, but may not always lead to more constrained prediction uncertainty.

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