Rational expression of time-dependent behavior from normally consolidated soil to naturally deposited soil

Expression rationnelle du comportement dépendant du temps des sols normalement consolidés et déposés naturellement

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ABSTRACT: Fundamental concepts of the ordinary elasto-viscoplastic modelings, which are the non-statonary flow surface model and the over-stress type model, are explained in one-dimensional (1D) compression of normally consolidated soil for easy understading. Then, a new approach to describe various time-dependent behaviors of soils is presented extending the advanced elastoplasitic theory based on the subloading surface concept without employing usual elasto-viscoplastic theories. Throughout the simulations of various time-dependent behaviors of normally consolidated clay, the features of these three types of time-denpendent models are clarified. In addtoin, 1D anlyses using the proposed model are carried out not only for normally consolidated clay but also for over-consolidated clay and strauctured clay such as nuturally deposited clay. The analytical results simulate well the features of time-dependent behavior of various clays including the delayed settlement of sturctured clay.

RÉSUMÉ: Les concepts fondamentaux de la théorie classique de l'élasto-viscoplasticité que sont la surface potentiel et la notion de surcontrainte sont expliqués pour une meilleure compréhension à partir d'une compression 1D sur un sol normalement consolidé. Ensuite, une nouvelle approche permettant de décrire différents comportements fonction du temps des sols est présentée, étendant la théorie de l'élasto-plasticité basée le concept de la subsurface de charge sans s'appuyer sur les concepts usuels de l'élasto-viscoplasticité. A partir de simulations de différents comportements dépendant du temps d'argiles normalement consolidées, les éléments propres à ces trois types de modèles sont clarifiés. De plus, des analyses 1D à partir du nouveau modèle sont menées non seulement sur des argiles normalement consolidées mais aussi surconsolidées et sur des argiles structurées, telles que des argiles naturelles. Les résultats analytiques démontrent que les principales caractéristiques du comportement dépendant du temps de ces sols sont bien reproduites, y compris le tassement différé des argiles structurées.

KEYWORDS: constitutive modeling, time-dependent behavior, nornally consolidated clay, over consolidated clay, structured clay

1 INTRODUCTION

Previous time-dependent models for soils are mostly formulated based on one of the following two viscoplastic concepts: the non-stationary flow surface type and the over-stress type. In the non-stationary flow surface models (e.g., Sekiguchi 1977), the flow surface is function of time in addition to stress and plastic strain, and flow rule is assumed on this flow surface, in the same way as elastoplastic models. Another type of model is based on the over-stress viscoplastic theory by Perzna (1963) (e.g., Adachi and Oka 1982), in which the strain rate effects are described by assuming a Bingham like body and utilizing the difference of sizes of the static yield surface related to the current plastic strains and the dynamic yield surface related to the current real stresses.

In the present paper, for understanding the features of the above models, one-dimensional (1D) formulation of these models are firstly described for the case of a normally consolidated clay. A simple model to describe time-dependent behavior of normally consolidated soils, over consolidated soils and structured soils such as naturally deposited clays is presented, not using the usual viscoplastic theories but focusing on the experimental results that normally consolidation line (NCL) on e–lnσ plane shift depending on the strain rate and using the subloading surface concept by Hashiguchi (1980).

2 EXPLANATION OF ORDINARY VISCOPLASTIC MODELS IN ONE-DIMENSIONAL CONDITION

Formulation of 1D time-dependent models for normally consolidated soil based on the ordinary viscoplastic theories is shown here. Figure 1 show the normal consolidation line (NCL)

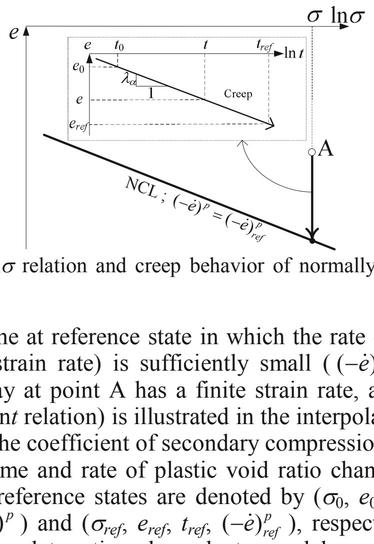

Figure 1. e-lnσ relation and creep behavior of normally consolidated clay

on e-lnσ plane at reference state in which the rate of void ratio change (or strain rate) is sufficiently small ($(-\dot{e})^p = (-\dot{e})^p_{ref}$). Here, the clay at point A has a finite strain rate, and its creep behavior (e-ln$t$ relation) is illustrated in the interpolated diagram, where λ_α is the coefficient of secondary compression. The stress, void ratio, time and rate of plastic void ratio change at initial, current and reference states are denoted by (σ_0, e_0, t_0, $(-\dot{e})^p_0$), (σ, e, t, $(-\dot{e})^p$) and (σ_{ref}, e_{ref}, t_{ref}, $(-\dot{e})^p_{ref}$), respectively. Now, let us formulate time-dependent models of normally consolidated soil based on the ordinary viscoplasticities under the condition of Figure 1.

2.1 *Non-stationary flow surface model (1D Sekiguchi model)*

It is experimentally known that for normaly consolidated clays there is a unique relation between stress, void ratio (or strain) and strain rate in one-dimensional compresson (e.g., Bjerrum 1967), and the well-known linear relation in e-ln σ plane (NCL) shift due to the stain rate. Now, points I and P in Figure 2

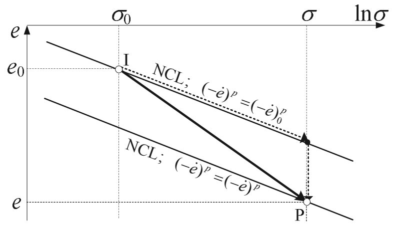

Figure 2. Explanation of non-stationary flow surface model

indicate the initial (σ=σ_0, e=e_0) and the current (σ=σ, e=e) states, respectively. The plastic change in void ratio from the initial state to the current state can be expressed by

$$(-\Delta e)^p = e_0 - e - (-\Delta e)^e = F + \lambda_\alpha \ln\frac{t}{t_0}$$
$$= F + \lambda_\alpha \ln\frac{(-\dot{e})_0^p}{(-\dot{e})^p} \quad \left(\text{where,}\ \ F = (\lambda-\kappa)\ln\frac{\sigma}{\sigma_0}\right) \qquad (1)$$

Here, λ and κ denote compression and swelling indices. By solving this ordinary differential equation with variables of time and plastic void ratio change (plastic strain), the plastic change in void ratio is given in the following form as a function of stress term F and time t (Sekiguchi 1977):

$$(-\Delta e)^p = \lambda_\alpha \ln\left\{\frac{(-\dot{e})_0^p\cdot t}{\lambda_\alpha}\exp\left(\frac{F}{\lambda_\alpha}\right)+1\right\}$$
$$= \lambda_\alpha \ln S \quad \left(\text{where,}\ \ S = \frac{(-\dot{e})_0^p\cdot t}{\lambda_\alpha}\exp\left(\frac{F}{\lambda_\alpha}\right)+1\right) \qquad (2)$$

Then, the increment of plastic void ratio is calculated as

$$d(-e)^p = \frac{\partial(-\Delta e)^p}{\partial\sigma}d\sigma + \frac{\partial(-\Delta e)^p}{\partial t}dt$$
$$= (\lambda-\kappa)\left(1-\frac{1}{S}\right)\frac{1}{\sigma}d\sigma + \lambda_\alpha\left(1-\frac{1}{S}\right)\frac{1}{t}dt \qquad (3)$$

The elastic increment of plastic void ratio change is given by

$$d(-e)^e = \kappa\frac{1}{\sigma}d\sigma \qquad (4)$$

Total increment is given by a summation of the plastic component of Equation (3) and the elastic component of Equation (4). As is seen from Equation (3), the non-stationary flow surface model contains time variable t, which is not objective. The multi-dimensional model in which the stress term F is replaced by the yield function of the Cam clay using (p, q) corresponds to so-called Sekiguchi model (1977).

2.2 *Over-stress type model*

As is seen from Figure 3, viscoplastic strain rate of the Bingham body is given by

$$\dot{\varepsilon}^p = \frac{1}{\mu}\langle\sigma-\sigma_c\rangle = \frac{\sigma_c}{\mu}\left\langle\frac{\sigma}{\sigma_c}-1\right\rangle \qquad (5)$$

Here, the symbol < > denotes the Macaulay bracket, i.e. $<A>=A$ if $A>0$; otherwise $<A>=0$, and σ_c and μ are the yield stress and the coefficient of viscosity. The over-stress type of viscoplasticity gives the viscoplastic strain rate as follows generalizing the Bingham viscosity:

$$\dot{\varepsilon}^p = \gamma\langle\Phi(\xi)\rangle$$
$$\left(\text{where}\ \ \langle\Phi(\xi)\rangle = \Phi(\xi)\ \text{if}\ \xi>0,\ \ \langle\Phi(\xi)\rangle = 0\ \text{if}\ \xi\le 0\right) \qquad (6)$$

The straight line with slope of λ in Figure 4 indicates NCL at reference state in which the strain rate is sufficiently small. The condition on this line is called "static". Now, it is assumed that clay at point P in Figure 4 satisfies the creep behavior in Figure 1. From Figure 4, the difference between the current void ratio and that on NCL (-ρ) at the same stress is expressed as

$$-\rho = e - e_{ref} = (\lambda-\kappa)\ln\frac{\sigma}{\sigma_s} \qquad (7)$$

Also, the following equation is obtained from the interpolated diagram in Figure 1:

$$-\rho = \lambda_\alpha \ln\frac{t_{ref}}{t} = \lambda_\alpha \ln\frac{(-\dot{e})^p}{(-\dot{e})_{ref}^p} \qquad (8)$$

From Eqatoins (7) and (8), the rate of plastic void ratio change is expressed as

$$(-\dot{e})^p = (-\dot{e})_{ref}^p \exp\left(\frac{-\rho}{\lambda_\alpha}\right) = (-\dot{e})_{ref}^p \exp\left(\frac{\lambda-\kappa}{\lambda_\alpha}\ln\frac{\sigma}{\sigma_s}\right) = (-\dot{e})_{ref}^p\left(\frac{\sigma}{\sigma_s}\right)^{\frac{\lambda-\kappa}{\lambda_\alpha}} \qquad (9)$$

Here, the current stress σ is called "dyanamic stress", and the stress which is determined from the current void ratio σ_s is called "static stress". These correspond to "dynamic yield function" and "static yield function" in multi-dimensional over-stress type vicsoplastic model. If it is assumed that Equation (9) holds not only under creep condition but also under other conditions, the increment of plastic void ratio change is given by

$$d(-e)^p = (-\dot{e})_{ref}^p\left\langle(\xi+1)^{\frac{\lambda-\kappa}{\lambda_\alpha}}\right\rangle dt \quad \left(\text{where}\ \ \xi = \frac{\sigma}{\sigma_s}-1\right) \qquad (10)$$

The elastic component is given by Equation (4). it is also shown by Mimura and Sekiguchi (1985) that Equation (10) corresponds to Equation (3) in which the term of stress increment is eliminated. Since the plastic strain increment is related with time alone, there is no loading condition in this type of models.

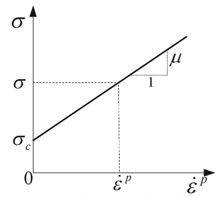

Figure 3. Bingham body

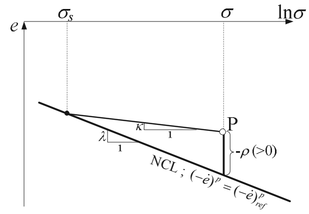

Figure 4. Explanation of over-stress type model

3 NEW TYPE OF TIME-DEPENDENT MODEL

As mentioned above, NCL shifts depending on the strain rate. The two straight lines in Figure 5 indicate NCLs corresponding to the initial state (point I; σ=σ_0, e=e_0, $(-\dot{e})^p=(-\dot{e})_0^p$) and the current state (point P; σ=σ, e=e, $(-\dot{e})^p=(-\dot{e})^p$). The difference of these two lines are expressed as (ψ-ψ_0). To model not only normally consolidated clay but also over consolidated clay, the state variable ρ (=e_N-e) which is the difference between the current void ratio and that on NCL is introduced. Furthermore, to describe the behavior of structured clay such as naturally deposited clay, another state variable ω, which represent an imaginary increase of density due to bonding effect, is employed. In the case of ρ=0 and ω=0, Figure 5 results in Figure 2. From Figure 5, the plastic change of void ratio is expressed as

$$(-\Delta e)^p = (-\Delta e) - (-\Delta e)^e$$
$$= \{(e_{N0}-e_N) - (\rho_0-\rho) - (\psi_0-\psi)\} - (-\Delta e)^e \qquad (11)$$
$$= \lambda\ln\frac{\sigma}{\sigma_0} - (\rho_0-\rho) - (\psi_0-\psi) - \kappa\ln\frac{\sigma}{\sigma_0}$$

Denoting $F = (\lambda-\kappa)\ln(\sigma/\sigma_0)$ in the same way as Equation (1) and H=(-Δe)^p, the following equaton (1D yield function) is obtaied:

$$f = F - \{H + (\rho_0-\rho) + (\psi_0-\psi)\} = 0 \qquad (12)$$

From the consitency condition (df = 0),

$$df = dF - \{dH - d\rho - d\psi\} = 0 \qquad (13)$$

Now, it can be considered that the evolution rule of ρ with the development of plastic deformation for structured soil is determined not only by the state variable ρ related to real

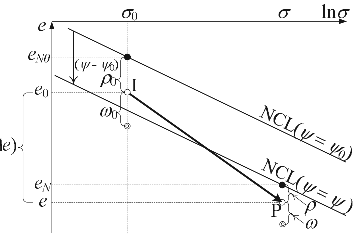

Figure 5. Explanation of the proposed model

density but also by the state variable ω related to the imaginary increase of density (bonding effect). Larger values of the state variables ρ and ω can be assumed to have more effects for the degradation of ρ and ω. Then, the evolution rules of ρ and ω can be given in the following form, using increasing functions $G(\rho)$ and $Q(\omega)$ which satisfy $G(0)=0$ and $Q(0)=0$, respectively:

$$d\rho = -\left\{G(\rho) + Q(\omega)\right\} \cdot d(-e)^p \tag{14}$$

Simply, the evolution rule of ω is also given using $Q(\omega)$ by

$$d\omega = -Q(\omega) \cdot d(-e)^p \tag{15}$$

The distance (ψ-ψ_0) of the two NCLs in Figure 5 is expressed as a function of the elapsed time t or the rate of plastic change in void ratio $(-e^{\cdot})^p$, referring to the interpolated diagram in Figure 1.

$$\psi - \psi_0 = \lambda_\alpha \ln\frac{t}{t_0} = \lambda_\alpha \ln\frac{(-\dot{e})_0^p}{(-\dot{e})^p} = \left\{-\lambda_\alpha \ln(-\dot{e})^p\right\} - \left\{-\lambda_\alpha \ln(-\dot{e})_0^p\right\} \tag{16}$$

Its increment is given by

$$d\psi = \frac{\partial \psi}{\partial t}dt = \lambda_\alpha \frac{1}{t}dt = (-\dot{e})^p\, dt \tag{17}$$

Now, it is assumed that Equations (16) and (17) hold not only for normally consolidated soils but also for over consolidated soils and naturally deposited soils. Substituting Equations (14) and (17) into Equation (13), the increment of the plastic void ratio can be obtained as

$$d(-e)^p = \frac{(\lambda-\kappa)\frac{1}{\sigma}d\sigma + (-\dot{e})^p \cdot dt}{1+G(\rho)+Q(\omega)} \cong \frac{(\lambda-\kappa)\frac{1}{\sigma}d\sigma + (-\dot{e})^{p*} \cdot dt}{1+G(\rho)+Q(\omega)} \tag{18}$$

Here, $(-\dot{e})^{p*}$ denotes the rate of the plastic void ratio change at the step immediately before the current calculation step. Finally, the total increment of void ratio is given by the following equation:

$$\begin{aligned} d(-e) &= d(-e)^p + d(-e)^e \\ &= \left(\frac{\lambda-\kappa}{1+G(\rho)+Q(\omega)} + \kappa\right)\frac{d\sigma}{\sigma} + \frac{(-\dot{e})^{p*}}{1+G(\rho)+Q(\omega)}dt \end{aligned} \tag{19}$$

In order to simplify the numerical calculations, the known rate $(-\dot{e})^{p*}$ in the previous calculation step can be used, instead of the current rate, as described. The error caused by using the previous known rate is automatically corrected by the subloading surface concept and is negligible in the calculations, because an incremental method with small steps is used in non-linear analysis. Time variable t which is not objective is not included, and the loading condition is given as $d(-e)^p > 0$. The details of the present modeling are described in Nakai et al. (2011a) and Nakai (2012).

4 SIMULATIONS OF TIME-DEPENDENT BEHAVIOR

4.1 *Analysis of normally consolidated clay by three models*

Table 1 shows material parameters for Fujinomori clay, which are common to the three models. The initial rate of plastic void ratio change is assumed as $(-\dot{e})_0^p = (-\dot{e})_{ref}^p$ =1.0x10^{-7}/min. Linear increasing functions, $G(\rho)=100\rho$ and $Q(\omega)=40\omega$, are employed for the case to consider density and bonding effects.

Figure 6 shows the results of one-dimensional compression behavior of normally consolidated (NC) clay for different strain rates, arranged in terms of the e-logσ relation. In the figure, the solid straight lines (no creep) show the relation without time effect, and the thick lines show the results in which strain rate decreases and increases again along the way. In every model, the lines of constant strain rate are parallel to each other, which is a good agreement with published experimental results (e,g., Bjerrum, 1967). It is also seen that when the strain rate decreases at a certain point, the curve follows exactly the same path which is supposed to follow for the new strain rate. However, when the strain rate increases again to the previous rate, the result of the non-stationary flow surface model does not return to the target curve corresponding to the respective strain rate which is called the phenomenon of 'isotache'. The result of the over-stress model shows sudden change of stress,

Table 1. Material parameters for clay

λ	0.104
κ	0.010
N (e_N at σ= 98kPa)	0.83
λ_α	0.003
$(-\dot{e})_{ref}^p$	1 x 10^{-7}/min.

N.B. N is the void ratio of NCL at $(-\dot{e})^p = (-\dot{e})_{ref}^p$

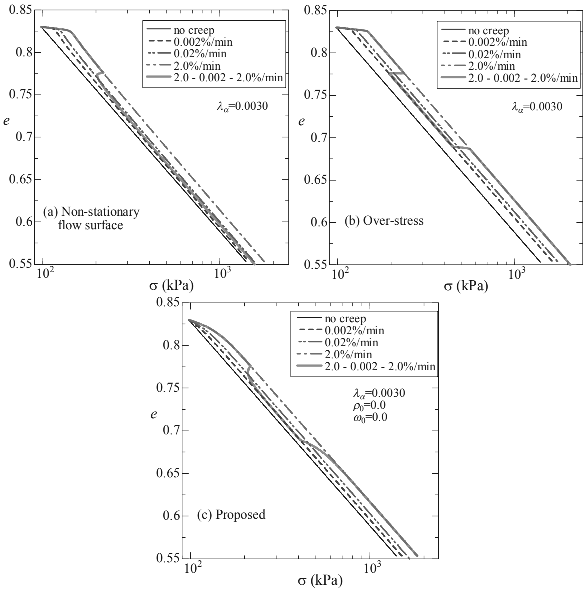

Figure 6. Simulations of strain rate effect of NC clay

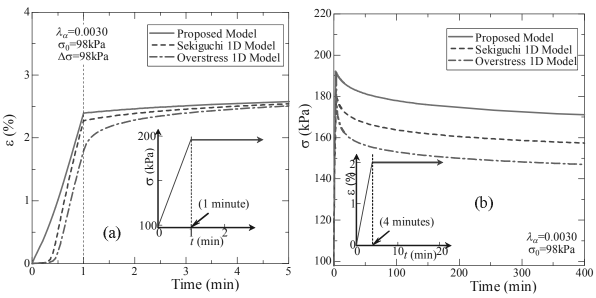

Figure 7. Simulations of creep and stress relaxation of NC clay

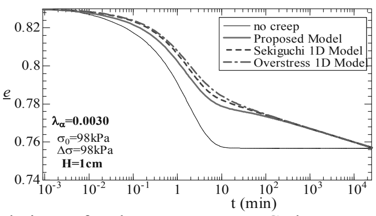

Figure 8. Simulations of oedometer test on NC clay

because the plastic strain increment does not related to the stress increment as described before. The result of the proposed model describes properly the strain rate effects known experimentally. Figure 7 shows the results of (a) creep behavior after loading with constant stress rate, and (b) stress relaxation behavior after loading with constant strain rate (loading processes are indicated in the interpolated diagram in each figure). Figure 8 shows the computed *e*-logt relation of conventional oedometer test by 1D soil-water coupled finite element analysis. Here, σ_0 is the initial stress, and $\Delta\sigma$ is the instantaneous stress increment. It can be seen that the three models describe the well-known consolidation behavior of normally consolidated clay including the secondary compression.

4.2 *Analysis of over consolidated clay and naturally deposited clay by the proposed model*

The non-satoionary flow surface model and the over-sress type model described in the above section are applicable to normally consolidated soil alone. On the ther hand, as can be seen from the derivaton process, the proposed model is valid for over consolidated soil and structured soil as well. In this subsection, some applications of the poposed model to over consolidated clay and structured clay are shown.

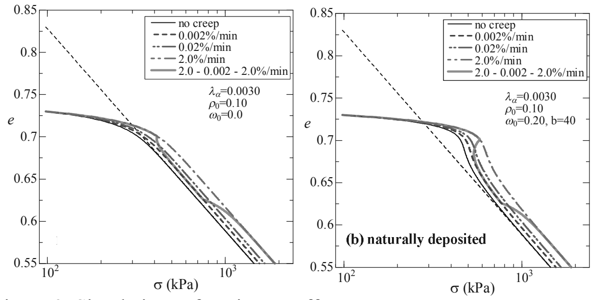

Figure 9. Simulations of strain rate effect

Figure 9 shows the results of one-dimensional compression behavior of these clays in the same way as Figure 6. It can be seen that the present model describe well the typical feature of *e*-lnσ relation for these clays and the time effect such that the quasi-yield stress becomes large with increasing strain rate and the phenomenon of isotache. Figure 10 shows the computed results of oedometer tests on non-structured (ω_0=0.0) and structured (ω_0=0.20) clays with two kinds of initial void ratios (ρ_0=0.0 and 0.10), arranged with the same manner as Figure 8. Here, thin curves and thick curves indicate ones without bonding (ω_0=0.0) and ones with bonding (ω_0=0.20). Diagrams (a), (b) and (c) shows the results of the ratio of stress increment to initial stress: $\Delta\sigma/\sigma_0$=1, 4 and 8, respectively. It is seen from diagrams (a) and (b) that although the behavior of the normally consolidated structured clay (ρ_0=0.0) is different from that of the normally consolidated non-structured clay under small stress increment ($\Delta\sigma/\sigma_0$=1), there is not much difference between them under large stress increment ($\Delta\sigma/\sigma_0$=4). On the other hand, the behavior of over consolidated clays (ρ_0=0.10) is highly influenced by the effect of structure (bonding) not under small stress increment but under large stress increment. It is also seen that from diagram (c) that when the stress increment is extremely large ($\Delta\sigma/\sigma_0$=8), there is not much difference between non-structured clay and structured clay regardless of the initial void ratio. Figure 11 shows the stress strain relation of these structured clays under large and small strain rate. The delayed settlements of structured clays may occur when the quasi-yield stress, which becomes small with decreasing strain rate, moves through the stress of the corresponding element (indicated by vertical dotted lines).

5 CONCLUSIONS

The feature of the ordinary viscoplastic models for clay are explained in 1D condition for easy understanding. A new approach to model without using ordinary viscoplasticiy is also shown. The new model is applicable not only to normally consolidated soil but also to over consolidated soil and naturally deposited soil. The applicabity of these models are discussed through the simulations of various time-dependent behavior of clays. Using the t_{ij} concept (Nakai and Mihara 1984), the present 1D model can be extended to 3D one (see Nakai et al. 2011b, Nakai 2012).

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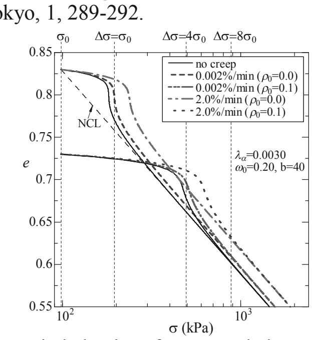

Figure 11. Stress-strain behavior of structured clays

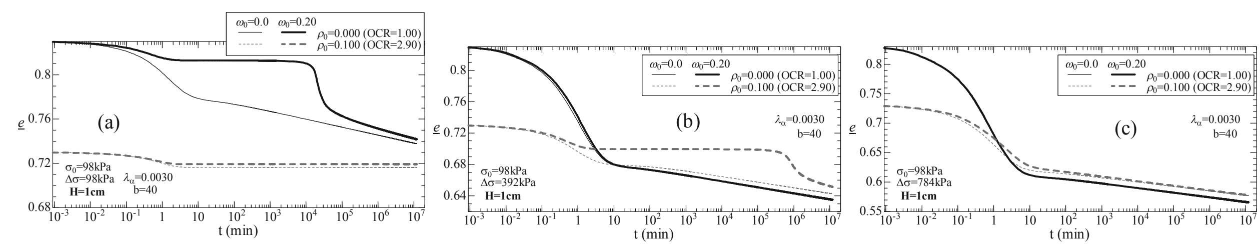

Figure 10. Simulations of oedometer tests on non-structured and structured clays with different initial void ratios