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# Test study and constitutive modelling of the time-dependent stress-strain behavior of soils

## Caractérisation expérimentale et modélisation du comportement contrainte-déformation à dépendance du temps

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**ABSTRACT:** This paper reports firstly the experimental data from one-dimensional (1D) straining oedometer tests and 2D consolidated undrained compression (CU) tests on a bentonite (clay mineral montmorillonite) mixed with different percentages of silicon sand. The 1D tests include (i) multi-staged loading tests with unloading/reloading and with creep and swelling and (ii) step-changed constant rate of strain compression tests with unloading/reloading in 1D straining as well. From the test data, creep, swelling and strain rate effects are observed and discussed. This paper introduces a new 1D Elastic Visco-Plastic model considering both creep and swelling (called 1D EVPS) for soils in 1D straining based on the previous 1D Elastic Visco-Plastic (1D EVP) models. The 1D EVPS model is used to simulate a step-changed constant rate of strain compression test with unloading/reloading in 1D straining with comparison with measured data. In addition, the 1D EVPS model is used to simulate single-staged constant rate of strain CU tests and relaxation tests. The typical results from a selected 2D CU test on a bentonite mixed 70% silicon sand under effective cell pressure of 100 kPa in a triaxial state compressed with step-changed constant rate of strain and unloading-reloading are presented and discussed in the paper. The 2D CU test data show clearly strain rate effects and unloading-reloading loops. The main conclusions from the study are: (a) the stress-strain behaviour of a bentonite mixed with different percentages of silicon sand exhibits strong dependence on time and strain rate; (b) in 1D straining condition, there exists a creep region when stress-strain state closer to the normal consolidation line (NCL) and a swelling region when far away from the NCL; (c) the 1D EVPS model can re-produce well the time-dependent stress-strain behaviour of the soil including creep, swelling, strain effects, relaxation, and unloading-reloading loops in 1D straining condition; and (d) the data from 2D consolidated undrained triaxial compression tests on the bentonite-sand mixture also show the strong strain effects and unloading-reloading loops.

**RÉSUMÉ :** Cet article rend compte d'une part les données expérimentales provenant unidimensionnel (1D) rude épreuve essais oedométriques et 2D consolidés non drainés de compression (CU) des tests sur une bentonite (montmorillonite minérale) mélangé avec des pourcentages différents de sable de silicium. Les tests 1D comprennent: (i) multi-étagées essais de charge avec déchargement / rechargement et avec un taux constant de fluage et gonflement et (ii) étape changé d'essais de compression de déformation avec le déchargement / rechargement en 1D forcer ainsi. D'après les données d'essai, les effets du taux de fluage, gonflement et la tension sont observées et discutées. Cet article présente une nouvelle 1D élastique visco-plastique modèle considérant à la fois au fluage et gonflement (appelé EVP 1D) pour les sols en 1D égouttage sur la base de l'élastique 1D précédente visco-plastique (1D EVP) des modèles. Le modèle 1D EVP est utilisé pour simuler une étape changé taux a changé constante de l'essai de compression souche avec déchargement / rechargement en 1D forcer la comparaison avec les données mesurées. En outre, le modèle 1D EVP est utilisé pour simuler une seule mise en scène constante de vitesse de déformation essais CU et des tests de relaxation. Les résultats typiques d'un test sélectionné CU 2D sur une bentonite mélangée sable de silicium 70% sous la pression de cellule effective de 100 kPa dans un état comprimé triaxial avec sa belle-changé vitesse de déformation constante et le déchargement-rechargement sont présentés et discutés dans le document. Les données 2D essais CU montrent clairement la souche effets de taux et de déchargement-rechargement des boucles. Les principales conclusions de l'étude sont les suivants: (a) le comportement contrainte-déformation d'une bentonite mélangée avec différents pourcentages de sable de silicium présente une forte dépendance du temps et de la vitesse de déformation, (b) dans un état tendu 1D, il existe une région de fluage lorsque le stress de souche état plus proche de la ligne de consolidation normale (NCL) et une zone d'expansion quand loin de la NCL, (c) le modèle 1D EVP peut re-produire et dépendant du temps comportement contrainte-déformation du sol, y compris au fluage, de l'enflure, les effets de déformation, de relaxation et de déchargement-rechargement des boucles dans un état tendu 1D et (d) les données provenant 2D consolidés non drainés des essais de compression triaxiale sur le mélange de bentonite et de sable montrent également les effets de contrainte forts et de déchargement-rechargement des boucles.

**KEYWORDS:** Stress-strain, time-dependent, creep, swelling, relaxation, strain-rate, visco-plastic.

### 1. INTRODUCTION

The stress-strain behavior of all geomaterials is time- (or rate)-dependent. The time-dependence of the stress-strain may be neglected for some geomaterials such as hard rock and sand, but cannot be ignored for some geomaterials such as soft rock and clayey soils. The phenomena of the time-dependence include creep, relaxation, the increase of the pre-consolidation pressure with strain rates, swelling, *etc.* Among these, creep is the compression with time under a constant stress; while swelling is expansion with time under a constant stress, opposite to the

creep. It is found that some clayey soils exhibit both creep and swelling such as soils containing clay mineral montmorillonite. In this paper, the authors will report first the experimental data from one-dimensional (1D) straining oedometer tests and consolidated 2D undrained compression tests on a bentonite mixed with different silicon sand contents. The 1D tests include (i) multi-staged loading tests with unloading/reloading and with creep and swelling and (ii) step-changed constant rate of strain compression tests with unloading/reloading in 1D straining as well. From the test data, creep, swelling and strain rate effects are observed and discussed. The 2D tests include (i) multi-

staged isotropic loading tests with unloading/reloading and with creep and swelling, and (ii) step-changed constant rate of strain consolidated undrained compression tests (CU tests) in triaxial state. From these test data, creep, swelling, relaxation, and strain rate effects are investigated and discussed.

Based on the 1D Elastic Visco-Plastic (1D EVP) models developed by Yin and Graham (1989, 1994, 1999), the authors have proposed a new 1D Elastic Visco-Plastic model considering both creep and swelling (called 1D EVPS) (Yin and Tong 2011). The data from the multi-staged oedometer tests are used to calibrate the new 1D EVPS model. After this, 1D EVPS model is used to simulate the step-changed constant rate of strain compression tests in 1D straining and other types of tests and make comparisons.

## 2. OEDOMETER TESTS ON BENTONITE MIXED WITH DIFFERENT SAND CONTENTS AND RESULTS

The authors have done a series of oedometer tests on a mixture of bentone and silicon sand with silicon sand contents of 50%, 60%, 70%, 80% and 90%. Table 1 gives a simmay of basis properties of the mixtue with 60%, 70%, and 80% (Tong and Yin 2011).

Table 1 Basic properties for different bentonite-sand mixed samples

Sand proportion	50%	60%	70%
Water Content (%)	211.5	159.0	124.5
Liquid Limit $W_L$ (%)	146.0	106.0	82.6
Plastic Limit $W_P$ (%)	29.9	28.7	27.4
Plastic Index $I_P$ (%)	121.1	77.3	55.2

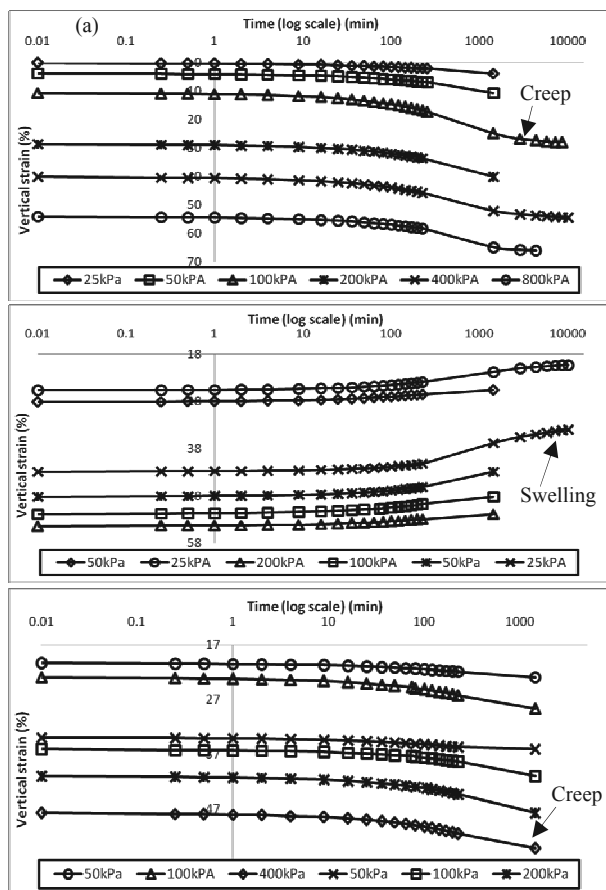


Figure 1. Vertical strain against time in log scale for 50% sand mixed with 50% bentonite (a) loading, (b) unloading, and (c) reloading

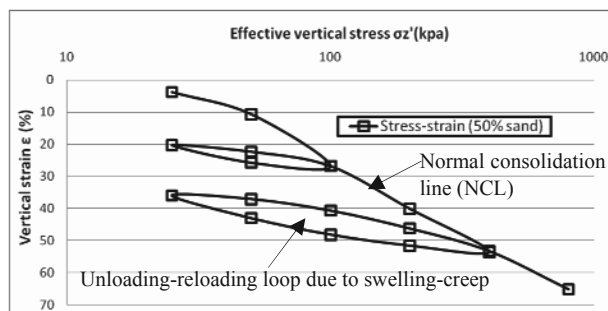


Figure 2. Vertical strain against vertical effective stress in log scale from tests on bentonite-sand mixture (50% sand content)

The curves of log(time) and vertical strains for a mixture of 50% bentone and 50% sand are shown in Figure 1. The oedometer test was carried out in stages and with unloading-reloading.

It is clearly seen from Figure 1(a) and (b) that the mixture exhibits both time-dependent creep and swelling. It is noted that “swelling” here is neither the rebound due to unloading, or the expansion of an unsaturated clay due to absorbing water. Instead, the “swelling” here is defined expansion of a saturated clay due to swelling potential of the clay under a constant vertical effective stress. The “swelling” here is opposite to teh creep.

Figure 2 shows the curve of vertical strain against vertical effective stress in log scale from tests on bentonite-sand mixture (50% sand content) with duration of 24 hours for each loading. It is seen from the figure that there are two unloading-reloading loops. The authors believe that the swelling and creep contribute to the loops. This is because that when the specimen is unloaded to very stress-strain state far from the normal consolidation line (NCL), the swelling potential of the mixture has caused the clay to expand and caused the time-depedeint reduction of strain. When the specimen is re-loaded to the stress-strain state closer to the NCL, the mixtuer will have creep compression. The authors also consider that the partical movements of the mixture may also contribute to the formation of the loops.

It is noted that the curves of log(time) and vertical strains and the curves of vertical strain against vertical effective stress in log scale of other mixtures are similar to those in Figures 1 and 2. From all test data, the authors have detrimed values of compression index  $C_c$ , rebound index  $C_r$ , creep coefficient  $C_{\square}$  and swelling coefficient  $C_s$ . The definitions of those parameters are:

$$C_c = \frac{-\Delta e}{\Delta \log \sigma'_z}; \quad C_r = \frac{\Delta e}{\Delta \log \sigma'_z} \tag{1}$$

$$C_{\square} = \frac{-\Delta e}{\Delta \log t}; \quad C_s = \frac{\Delta e}{\Delta \log t}$$

Values of  $C_c$ ,  $C_r$ ,  $C_{\square}$  and  $C_s$  are presented in Table 1. In Table 1, the ratio of  $C_{\square}/C_c$  and  $C_s/C_r$  are also given. The following ovservations can be obtained from Table 2:

- Both the compression index  $C_c$  and rebound index  $C_r$  decrease with the increase of the sand content. This means that the more the sand content, the less compression of the mixture.
- In generally speaking, the creep coefficient  $C_{\square}$  decreases with the increase of the sand content. This means that the more the sand content, the less creep of the mixture. In addition, the coefficient  $C_{\square}$  also decreases with the increase of the vertical effective stress.
- In generally speaking, the swelling coefficient  $C_s$  decreases with the increase of the sand content. This means that the more the sand content, the less swelling of the mixture. In addition, the coefficient  $C_s$  increases with the decrease of the vertical effective stress.

- The ratios of  $C_a/C_c$  and  $C_s/C_c$  also decrease with the sand content.

Table 2 Creep and swelling indexes for different bentonite samples

Sand proportion	50%	60%	70%	80%	90%
Compression index $C_c$	0.315	0.296	0.260	0.243	0.104
Rebounding index $C_r$	0.130	0.113	0.092	0.054	0.021
Creep coefficient $C_{a1}$ ( $\sigma_z=100\text{kPa}$ )	0.0304	0.0263	0.0227	0.0062	0.0025
Creep coefficient $C_{a2}$ ( $\sigma_v=400\text{kPa}$ )	0.0191	0.0141	0.0120	0.0051	0.0023
$C_a/C_c$ ( $\sigma_z=100\text{kPa}$ )	0.0965	0.0889	0.0873	0.0255	0.0240
$C_a/C_c$ ( $\sigma_z=400\text{kPa}$ )	0.0606	0.0476	0.0462	0.0210	0.0221
Swelling coefficient $C_{s1}$ ( $\sigma_z=10\text{kPa}$ )	0.0284	0.0251	0.0160	0.0085	0.0033
Swelling coefficient $C_{s2}$ ( $\sigma_z=50\text{kPa}$ )	0.0173	0.0131	0.0124	0.0062	0.0024
$C_{s1}/C_r$ ( $\sigma_z=100\text{kPa}$ )	0.2185	0.2221	0.1739	0.1574	0.1571
$C_{s2}/C_r$ ( $\sigma_z=400\text{kPa}$ )	0.1331	0.1159	0.1048	0.1148	0.1143
$C_{s1}/C_c$ ( $\sigma_z=100\text{kPa}$ )	0.0902	0.0848	0.0615	0.0350	0.0317
$C_{s2}/C_c$ ( $\sigma_z=400\text{kPa}$ )	0.0549	0.0443	0.0477	0.0255	0.0230

### 3. A NEW 1D ELASTIC VISCO-PLASTIC MODEL FOR SOILS EXHIBITING BOTH CREEP AND SWELLING

Yin and Tong (2012) has developed a new Elastic Visco-Plastic model considering both creep and swelling of the clays in one-dimensional straining, called 1D EVPS model. This new model is based on the previous 1D EVP models proposed by the first author and his co-workers (Yin 1990; Yin and Graham 1989, 1994). Important concepts of “elastic” time line, “reference” time lines in both creep and swelling regions, and “equivalent” time lines are shown in Fig.3 in the plot of the vertical effective stress in ln-scale and vertical strain. The “elastic” line has a slope  $\kappa/V$  and the two “reference” time lines in both creep and swelling regions have the same slope  $\lambda/V$  where  $V=1+e_o$  in which  $e_o$  is the initial void ratio. Other parameters are also shown in the figure such  $(\sigma_{z0}^{rs}, \varepsilon_{z0}^{rs})$  and  $(\sigma_{z0}^{rc}, \varepsilon_{z0}^{rc})$ . Yin and Tong (2012) used the natural logarithmic functions to fit the “elastic” time line, “reference” time lines, and creep compression using equivalent time concept. The 1D EVPS model can be derived and expressed as:

$$\dot{\varepsilon}_z = \frac{\kappa}{V} \frac{\dot{\sigma}_z}{\sigma_z} + \frac{\psi^c}{V} \frac{1}{t_o^c} \exp[-(\varepsilon_z - \varepsilon_{z0}^{rc}) \frac{V}{\psi^c} \left( \frac{\sigma_z}{\sigma_{z0}^{rc}} \right)^{\frac{\lambda}{\psi^c}}] - \frac{\psi^s}{V} \frac{1}{t_o^s} \exp[(\varepsilon_z - \varepsilon_{z0}^{rs}) \frac{V}{\psi^s} \left( \frac{\sigma_z}{\sigma_{z0}^{rs}} \right)^{\frac{\lambda}{\psi^s}}] \quad (2)$$

where the parameters  $(\psi^c, t_o^c)$  are for creep compression and  $(\psi^s, t_o^s)$  are for swelling of the clays. Eq.(2) is a general constitutive model for the time-dependent stress-strain behaviour of soils exhibiting both creep and swelling in 1D straining. This model is valid for all loading conditions such as constant rate of strain (CRSN) loading, relaxation, unloading, reloading etc.

Yin and Tong (2012) has used data of a multi-staged oedometer test on a bentonite mixed with 70% silicon silica sand mixture (SMB) to determine values of all parameters in Eq.(2). The basic properties can be found in Table 1. The initial void ratio  $e_o$  is 5.5 (after pre-consolidation but before oedometer testing). All values of these parameters are presented in Table 3.

This 1D EVP model has been used to simulate the stress-strain behavior of the same clay under step-changed constant-

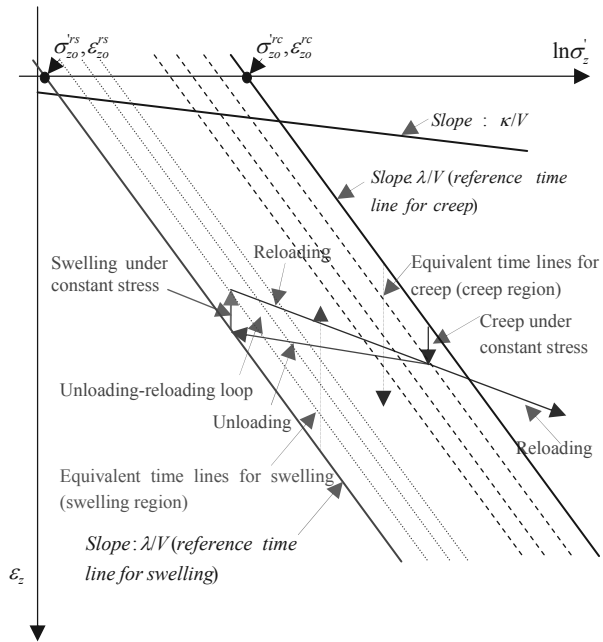


Figure 3. Conceptual illustration of creep, swelling, unloading-reloading loop, “elastic” line, “reference” time line, and “equivalent” time lines (extended from Yin 1990; Yin and Graham 1989, 1994, 1999)

rate of strain (CSR) loading with comparison with measured data as shown in Figure 3. It is seen from this figure that the 1D EBVPS model can simulate the strain effects, unloading/reloading loops, and the non-linear stress-strain behavior.

Table 3. Values of parameters in 1D EVPS model

Parameters	Bentonite with 70% sand
Elastic parameter $\kappa/V$	0.0542
Plastic parameter $\lambda/V$	0.1367
Creep parameter $\psi^c$	0.01956
Creep parameter $t_o^c$	24 hours
Swelling parameter $\psi^s$	0.01554
Swelling parameter $t_o^s$	24 hours
Interception stress $\sigma_{z0}^{rc}$	50 kPa
Interception strain $\varepsilon_{z0}^{rc}$	0
Interception stress $\sigma_{z0}^{rs}$	16 kPa
Interception strain $\varepsilon_{z0}^{rs}$	0

This 1D EVP model has also been used to simulate the stress-strain behavior of the same clay under singled staged constant-rate of strain (CSR) loading with CSR of  $\dot{\varepsilon}_z = 10^{-4}/\text{sec}, 10^{-5}/\text{sec}, 10^{-6}/\text{sec}, 10^{-7}/\text{sec}$ . It is seen from the figure that the higher the strain rate, the larger the effective stress. The pre-consolidation pressure increases with the strain rate.

In addition, the 1D EVP model has also been used to simulate the stress decreasing with time (stress relaxation) in the creep region and the stress increasing with time (also called stress relaxation) in the swelling region when the vertical strain is kept a constant (Yin and Tong 2012). When the initial stress-strain state point is in the swelling region (far away from the normal consolidation line, the clay has a swelling potential so that the stress will increase with time when the specimen is confined in vertical thickness (strain is constant).

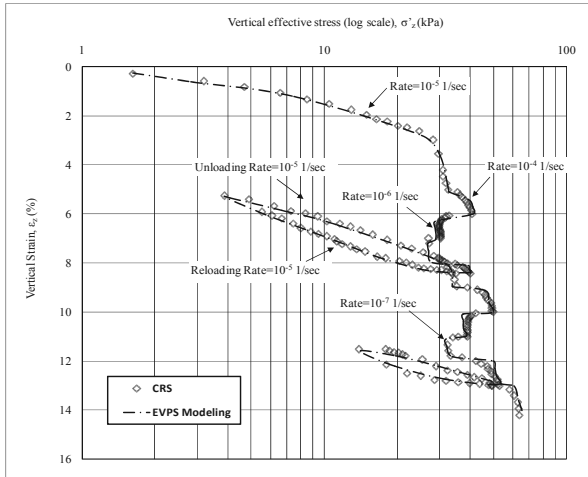


Figure 4. Comparison of modelling results with measured data from a CRS test with step-changed strain rates and with unloading and reloading

4. CONSOLIDATED UNDRAINED TRIAXIAL COMPRESSION TESTS ON BENTONITE MIXED WITH SAND AND RESULTS

The authors have done consolidated undrained triaxial tests on the same bentonite mixed with different ratios of silicon sand with effective confining pressure of 50 kPa, 100 kPa and 200 kPa. Curves of (a) deviator stress versus axial strain, (b) the excess porewater pressure versus axial strain, and (c) the deviator stress versus effective (or total) mean stress of a CU test on bentonite mixed with 70% of sand and effective cell pressure of 100 kPa are shown in Figure 6. It is seen from Figure 6 that the effects of strain rates are very significant. The unloading-reloading loops are also evident.

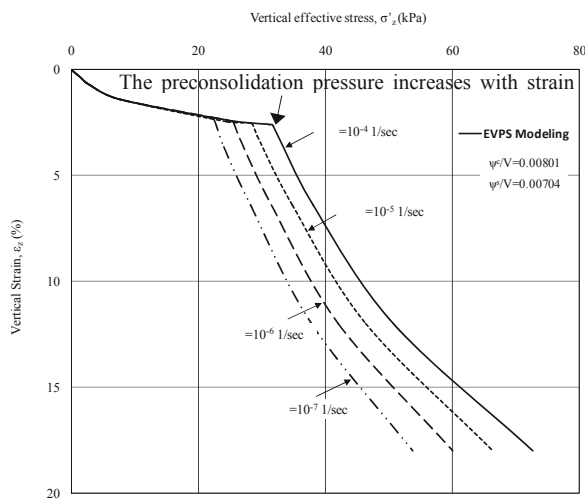


Figure 5. Simulation for strain rate dependent stress-strain behavior in CRS tests using the EVPS model

5. CONCLUSIONS

From the above presentation and study, the following conclusions can be drawn:

- (a) The stress-strain behavior of a bentonite mixed with different percentages of silicon sand exhibits strong dependence on time and strain rate.
- (b) Creep, swelling, relaxation and strain effects are clearly observed and are significant.
- (c) In 1D straining condition, there exists a creep region with stress-strain state closer to the normal consolidation line (NCL) and a swelling region far away from the NCL.

- (d) The 1D EVPS model can re-produce well the time-dependent stress-strain behavior of the soil including creep, swelling, strain effects, relaxation, and unloading-reloading loops in 1D straining condition.
- (e) The data from 2D consolidated undrained triaxial compression tests on the bentonite-sand mixture also show the strain effects and unloading-reloading loops.

6. ACKNOWLEDGEMENTS

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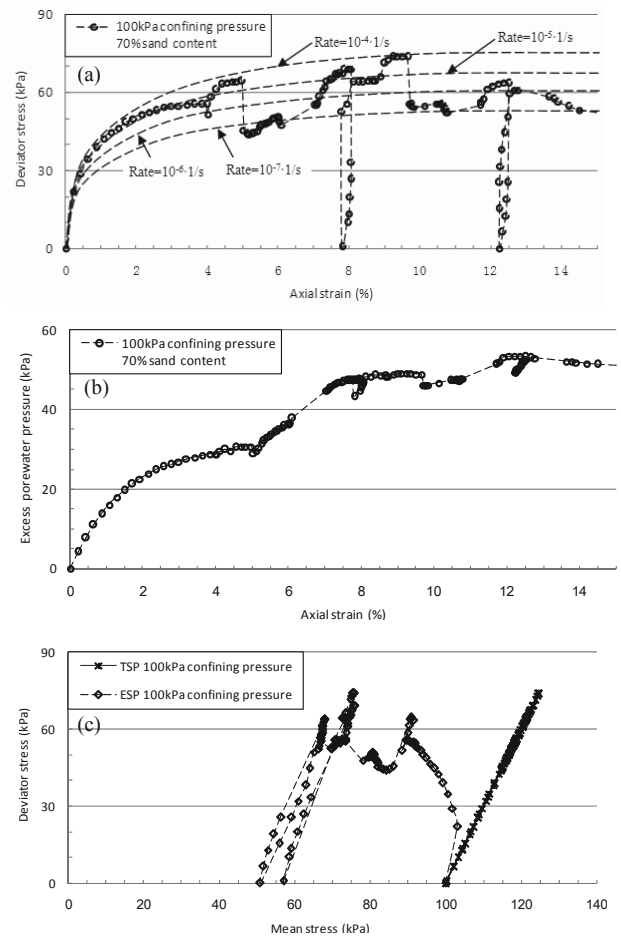


Figure 6. (a) Deviator stress versus axial strain, (b) the excess porewater pressure versus axial strain, and (c) the deviator stress versus effective (or total) mean stress – CU test on bentonite mixed with 70% of sand and effective cell pressure of 100 kPa