# Probabilistic Assessment of the Bearing Capacity of Shallow Strip Footings on Stiff-Over-Soft Clay

## Evaluation probabiliste de la capacité portante de semelles filantes peu profondes sur couche d'argile rigide recouvrant une couche d'argile molle

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ABSTRACT: This paper focuses on the probabilistic assessment of the resistance factor for bearing capacity for a strip footing on a stiff-over-soft clay profile. The analysis is performed by applying the Random Finite Element Method, which combines finite element simulation, spatial variability analysis and Monte Carlo simulation. Finite-element analyses are performed in the program ABAQUS on meshes in which undrained strength values are assigned on the basis of quantitative estimates of the vertical and horizontal spatial variability and the probabilistically modelled scatter of undrained strength itself. The stochastic implementation of the numerical analyses results in samples of bearing capacity factors which, when normalized by a deterministic bearing capacity factor, provide a set of tabulated factors calibrated to user-defined target reliability levels. The results have application for the prediction of foundation punch-through, where the footing pushes the upper strong layer of soil into the softer clay beneath.

RÉSUMÉ : Cet article porte sur l'évaluation probabiliste du facteur de résistance de la capacité portante d'une semelle filante posée sur une couche d'argile rigide recouvrant une couche d'argile molle. L'analyse applique la méthode des éléments finis aléatoires, qui associe simulation par éléments finis, analyse de variabilité spatiale et simulation par la méthode de Monte Carlo. Les analyses par éléments finis sont réalisées avec le programme ABAQUS, en utilisant des maillages pour lesquels la valeur de résistance au cisaillement non drainée est déterminée sur la base de l'estimation quantitative des variabilités verticale et horizontale, ainsi que sur la dispersion de la résistance au cisaillement non drainée modélisée de façon probabiliste. L'implémentation stochastique des analyses numériques conduit à des facteurs de capacité portante qui, lorsqu'on les normalise par le facteur de capacité portante déterministe, fournit un ensemble de facteurs étalonnés pour des niveaux de fiabilité prédéfinis. Les résultats obtenus trouvent une application pour la prédiction du poinçonnement des fondations, dans le cas où la fondation pousse la couche supérieure de sol dur dans la couche de sol mou située en dessous.

KEYWORDS: Bearing capacity, shallow foundation, spatial variability, probability, random finite element analysis, strip footing

## 1 INTRODUCTION

The bearing capacity of a shallow foundation on two layered clay soil is a classical problem in soil mechanics and one of importance to many applications, including the punch-through of offshore foundations. The problem being analyzed in this paper is defined in Figure 1: what is the vertical load carrying capacity of a strip footing of width ($B$) on a top layer of soil of undrained shear strength ($s_{ut}$) overlying a weaker bottom layer ($s_{ub}$). Using finite element analysis in conjunction with limit theorems, Merifield et al. (1999) published extensive bearing capacity factors ($N_c$*) defined to predict the vertical capacity as a function of the strip footing width and the undrained shear strength of the top clay layer. Conditions of varying top layer thickness and shear strength ratio were analyzed. However, these solutions were only provided for deterministic properties of soil with no spatial variability accounted for.

This paper makes use of the Random Finite Element Method (RFEM) (see Fenton and Griffith, 2008) to investigate the effect of the spatial variability in undrained shear strength on the bearing capacity of a shallow strip footing on two-layered stiff-over-soft clay. In the RFEM the characterization of the spatial variability enables the generation of random fields with spatially varying values, all of which are mapped onto a finite element mesh. The generation of multiple random fields associated with the soil domain allows the repeated implementation of finite element analysis, yielding multiple samples of outputs. These can subsequently be analysed statistically.

Although RFEM has been used to estimate the statistical distribution of the vertical undrained bearing capacity of a strip foundation on a single layer (Paice et al. 1996, Nobahar and Popescu 2000, Griffiths and Fenton 2001, Griffiths et al. 2002, Fenton and Griffiths 2003, Popescu et al. 2005, Kasama and Whittle 2011, Cassidy et al. 2012) it has yet to be applied to the two-layered condition. The aim of this paper is to (i) provide a methodology for doing so, (ii) discuss preliminary trends due to changing variation in undrained shear strength distributions, and (iii) estimate quantitatively the degree of unconservatism in using deterministic bearing capacity factors.

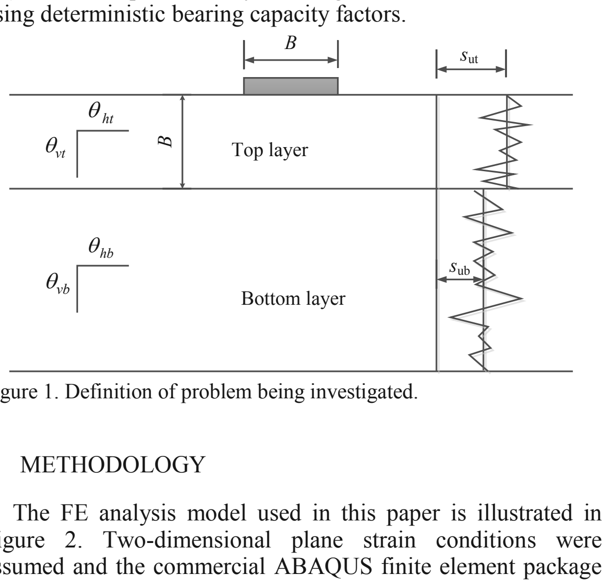



Figure 1. Definition of problem being investigated.

## 2 METHODOLOGY

The FE analysis model used in this paper is illustrated in Figure 2. Two-dimensional plane strain conditions were assumed and the commercial ABAQUS finite element package utilised (version 6.10, Dassault Systèmes 2010). A shallow foundation with width $B$ was founded on the surface of the two-layered soil, which was modelled by a linear-elastic perfectly-plastic Tresca constitutive law with an undrained shear strength ($s_u$). The elastic response was defined by the Young's modulus ($E = 500s_u$) and the Poisson's ratio set as 0.49. Corresponding to one of the analysis cases of Merifield et al. (1999), the soil contained a top layer of $1B$ thickness. For efficiency the infinite bottom layer of Merifield et al (1999). was shortened to $3.8B$; a depth deep enough, however, to ensure no boundary effects. The analysis width was $6B$. The lateral soil boundaries were roller supported and the bottom was pinned. The top surface was assumed to be free. A fully bonded foundation/soil interface was used to model the undrained behaviour.

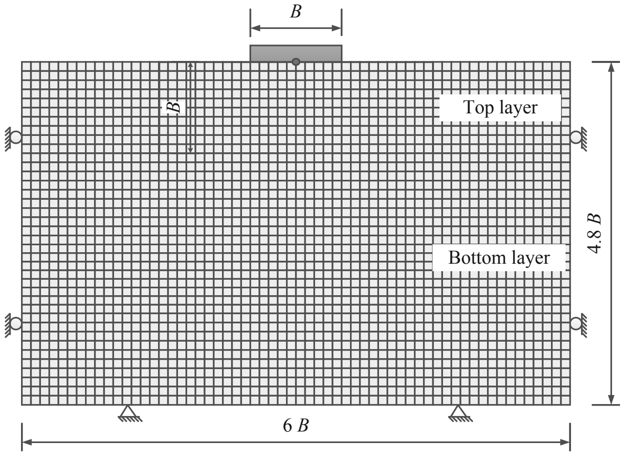



Figure 2. The FE model used

The soil domain was divided into 60 by 48 square zones of width $0.1B$, as shown in Figure 2. In each zone the soil properties were constant and defined by an undrained shear strength $s_u$ and Young's modulus $E = 500s_u$. However, these properties changed from zone to zone representing the spatial variability of the soil. For the majority of the soil domain a zone was represented by one finite element. However, in a region of size 3B by 1B close to the strip footing (as bounded by heavy lines in Figure 2) nine smaller finite elements per zone were used. These smaller elements, each with the same material properties, were required to improve the numerical accuracy of the solution. Therefore, in total there are 5280 finite elements in the mesh but only 2880 zones of spatially varying soil properties.

The spatially variable undrained shear strength $s_u$ of both top and bottom layer was modelled as a normally distributed random field with a mean $\mu_{s_u}$ and standard deviation $\sigma_{s_u} = COV \cdot \mu_{s_u}$. Consistent with the deterministic values of Merifield et al. (1999), the mean shear strength of the top layer was set as twice the bottom layer, with values of $\mu_{s_{ut}} = 20$kPa and $\mu_{s_{ub}} = 10$kPa assumed in this paper. The COV, vertical and horizontal correlation length $\theta_v$ and $\theta_h$ for both top and bottom layer vary systematically. Table 1 details the random variables assumed for the 12 cases presented.

For each case, 1000 realisations of the random fields of undrained shear strength $s_u$ were generated using the Local Average Subdivision algorithm (Fenton and Vanmarcke 1990; Fenton 1994). One of the 1000 realisations of the random field of case 1 ($COV_t = COV_b = 0.3$ $\theta_{vt} = \theta_{ht} = \theta_{vb} = \theta_{hb} = 0.1B$ see Table 1 for details) is illustrated in Figure 3.

## 3 RESULTS

### 3.1 *Deterministic Case*

The modified bearing capacity factor $N_c^* = Q_u/Bs_{ut}$ was defined in Merifield et al. (1999) as the ultimate bearing capacity $Q_u$ normalised by the footing width $B$ and top layer shear strength $s_{ut}$. Merifield et al. (1999) reported $N_c^*$ as 4.44 (lower bound), 4.82 (upper bound) and 4.63 (average) for the situation considered in Figure 2. A deterministic case was first conducted in this paper with uniform undrained strengths of 20 kPa and 10 kPa for the top and bottom layer, respectively. An $N_c^*$ of 4.66 was obtained. This good agreement implies that the FE analyses in this paper are reliable and comparable to the Merifield et al. (1999) analyses.

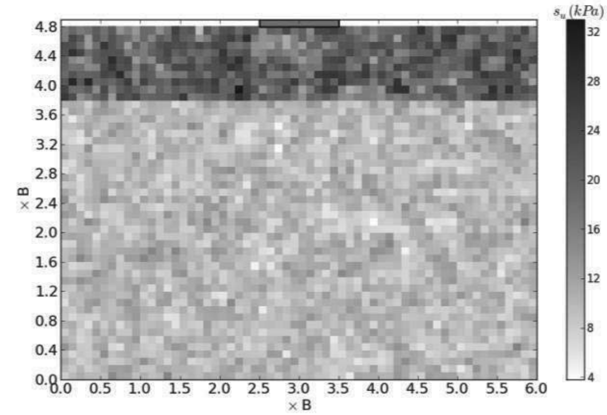



Figure 3. Example random field (for case 1)

Table 1. Calculation cases and summary results

| Case | Input parameters | | | | | | Analysis results | | |
|---|---|---|---|---|---|---|---|---|---|
| | Bottom layer | | | Top layer | | | | | |
| | $\Theta_{vb}$ | $\Theta_{hb}$ | $COV_b$ | $\Theta_{vt}$ | $\Theta_{ht}$ | $COV_t$ | $N^*_{cm}/N^*_c$ | $\sigma(N^*_{cr}/N^*_c)$ | $P_e$ |
| 1 | 0.1 | 0.1 | 0.3 | 0.1 | 0.1 | 0.3 | 0.93 | 0.02 | $5.0{\cdot}10^{-4}$ |
| 2 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.98 | 0.01 | $1.3{\cdot}10^{-3}$ |
| 3 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.3 | 0.95 | 0.02 | $5.6{\cdot}10^{-3}$ |
| 4 | 0.1 | 0.1 | 0.3 | 0.1 | 0.1 | 0.1 | 0.96 | 0.01 | $1.0{\cdot}10^{-4}$ |
| 5 | 0.1 | 0.1 | 0.3 | 0.1 | 10 | 0.3 | 0.94 | 0.07 | 0.144 |
| 6 | 0.1 | 0.1 | 0.3 | 0.1 | 1 | 0.3 | 0.93 | 0.05 | 0.057 |
| 7 | 0.1 | 0.1 | 0.3 | 1 | 10 | 0.3 | 0.93 | 0.18 | 0.277 |
| 8 | 0.1 | 0.1 | 0.3 | 1 | 1 | 0.3 | 0.89 | 0.12 | 0.133 |
| 9 | 0.1 | 10 | 0.3 | 0.1 | 0.1 | 0.3 | 0.92 | 0.04 | 0.022 |
| 10 | 0.1 | 1 | 0.3 | 0.1 | 0.1 | 0.3 | 0.92 | 0.03 | $4.0{\cdot}10^{-4}$ |
| 11 | 1 | 10 | 0.3 | 0.1 | 0.1 | 0.3 | 0.90 | 0.09 | 0.149 |
| 12 | 1 | 1 | 0.3 | 0.1 | 0.1 | 0.3 | 0.90 | 0.05 | 0.054 |

Note: $\Theta = \theta/B$

### 3.2 *Stochastic soil cases: variation of COV, constant $\theta$*

The mean undrained shear strength of the top layer $\mu_{sut}$ is used to defined a modified bearing capacity factor for the stochastic cases ($N_{cr}^*$) and where

$$N_{cr}^* = Q_{ur}/B\mu_{sut} \tag{1}$$

in which $Q_{ur}$ is the stochastic ultimate bearing capacity. The values of $N_{cr}^*$ of the 1000 realisations random field for each case were ordered and the sample median value denoted as $N_{cm}^*$. The standard deviation of the $N_{cr}^*/N_c^*$ for the 1000 random field realisations is calculated as $\sigma(N_{cr}^*/N_c^*)$. The values of $N_{cm}^*$ and $\sigma(N_{cr}^*/N_c^*)$ evaluated for all the cases presented in this paper are provided in Table 1. The histogram of $N_{cr}^*/N_c^*$ from the 1000 random field realisations for case 2 ($\theta = 0.1B$ $COV = 0.1$ see Table 1) is depicted in Figure 4, with $N_{cm}^* = 0.98\,N_c^*$ and $\sigma(N_{cr}^*/N_c^*) = 0.01$ . The empirical cumulative distribution functions for cases 1-4 are shown in Figure 5.

In order to investigate the influence of changing the COV for both or one of the layers, the cumulative curves for cases 1~4 are shown in Figure 5. In these cases the horizontal and vertical correlation lengths of both top and bottom layers were kept constant as $\theta = 0.1B$. As shown in the figure, the average bearing capacity factor for all of the stochastic cases (represented by $N^*_{cm}$) is less than the deterministic case. This is consistent with the reports of Nobahar and Popescu (2000), Griffiths et al. (2002) and Cassidy et al. (2012). Further, when the COV of both layers is increased from 0.1 to 0.3 the average bearing capacity reduces from 0.98 to 0.93 and the normalised standard deviation increases from 0.01 to 0.02. This is as expected and is shown in the two extremity curves of Figure 5. Comparing the cases where the COV of only the top layer (case 3: $COV_b$=0.1, $COV_t$=0.3) and only the bottom layer (case 4: $COV_b$=0.3, $COV_t$=0.1) provides more insight into the mechanisms of failure. We can see from Figure 5 that the COV of the top layer has a more significant effect with case 3 trending towards case 1 where both layers are 0.3. Moreover, the similarity of the shapes of case 2 and case 4 as well as case 3 and case 1 implies that the top layer COV determines the variation (standard deviation) of the curves.

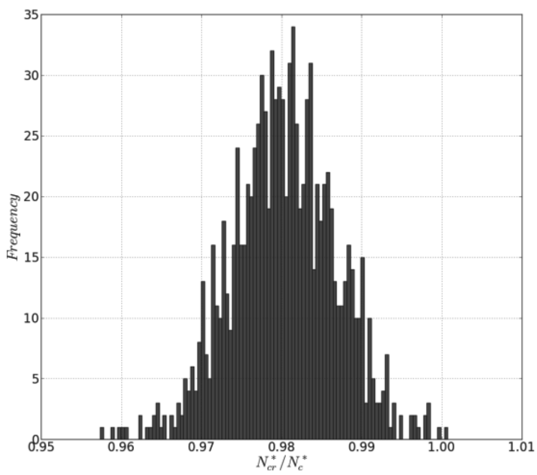



Figure 4 Histogram of $N^*_{cr}/N^*_c$ for case 2

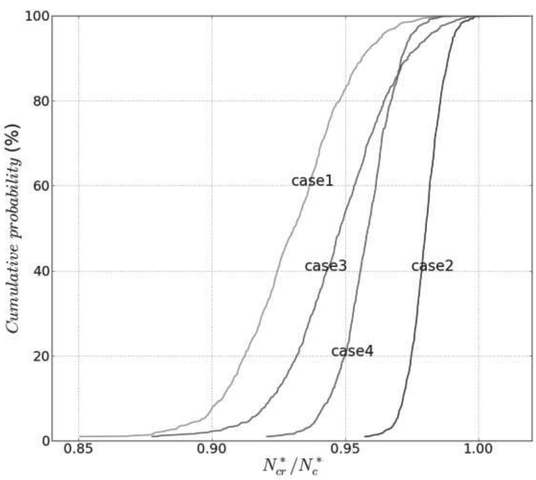
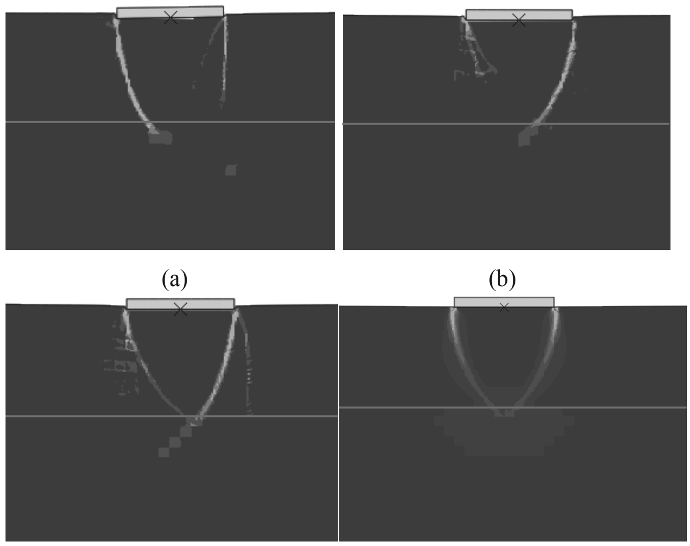



Figure 5 Cumulative probability curves for varying COV of cases 1 to 4

The output samples of stochastic bearing capacity factor normalized by the deterministic value were analysed with the aim of estimating the frequentist probability of exceedence of unity, i.e., the probability that the stochastic bearing capacity factor exceeds the deterministic bearing capacity factor. This assessment is important in the context of engineering design, as it provides a measure of the unconservatism in using deterministic bearing capacity factors, i.e., in neglecting uncertainty and spatial variability. In only 6 cases out of the 12 analyzed, output samples resulted to be lognormal at the 95% confidence level using the Anderson-Darling test. Hence, estimating the probability of exceedence of unity from cumulative values of fitted lognormal samples would not allow confident assessement for all cases. Empirical cumulative distribution functions were calculated for each sample. The empirical probability $P_e$ of exceedence of unity for each case is noted in the rightmost column in Table 1.

The failure mechanisms of three selected realizations of case 1 ($\theta = 0.1B, COV = 0.3$) are shown in Figure 6. These represent the minimum, median and maximum $N^*_{cr}$ cases and are shown alongside the deterministic failure mechanism (uniform and mean parameter values). In all three cases the existence of the random field results in a non-symmetric failure mechanism, with the minimum bearing capacity case most unsymmetrical. With the increasing of bearing capacity, the failure mechanism tends to resemble the deterministic case. The importance of spatial variability in the top layer can be observed, with the majority of the failure mechanism residing in that layer. Further, with higher variability and potential for weaker zones the mechanism for lower bearing capacity is both more unsymmetric and shorter (pulling it further into the top layer).



(a) (b)

(c) (d)

Figure 6. Failure mechanisms from finite element analysis for (a) lowest, (b) median and (c) highest bearing capacity, and (d) deterministic uniform case (for clarity only a section of 3$B$ width and depth 2$B$ show)

### 3.3 *Stochastic soil cases: variation of correlation length in top layer*

With the top layer determined to play a more significant role in the problem configuration of this paper further concentration on the effect of top layer correlation length is discussed. The results for correlation length varying from 0.1B to 10B are presented as cumulative probability curves in Figure 7. These represent cases 5, 6, 1, 7 and 8 in Table 1. As for the $\theta_{vt}/B = 0.1$ cases (cases 5, 6 and 1), the largest bearing capacity corresponds to the largest horizontal correlation length $\theta_{ht} = 10B$ (case 5) while the minimum corresponds to $\theta_{ht}/B = 1$ (case 6). This is consistent with the observation of Griffiths et al. (2002) for the single layer case. In general, a large correlation length results in greater standard deviation of the bearing capacity, i.e. the foundation becomes more "non-uniform". The minimum bearing capacity occurs at $\theta_{vt}/B = \theta_{ht}/B = 1$.

### 3.4 *Stochastic soil cases: variation of correlation length in bottom layer*

Comparison of the results of case 9, 10, 1, 11 and 12 indicates the correlation length effect of the bottom layer. It again shows that increasing horizontal correlation length tends to increase the standard deviation of the bearing capacity factor (see Table 1 and Figure 6). However, the largest average bearing capacity corresponds to the minimum correlation length case (case 1: $\theta_{vt}/B = \theta_{hb}/B = 0.1$). This differs to what is occurring in the top layer. The maximum average bearing capacity corresponds to the largest correlation length case 11, which is consistent with the results of changing the correlation length of the top layer.

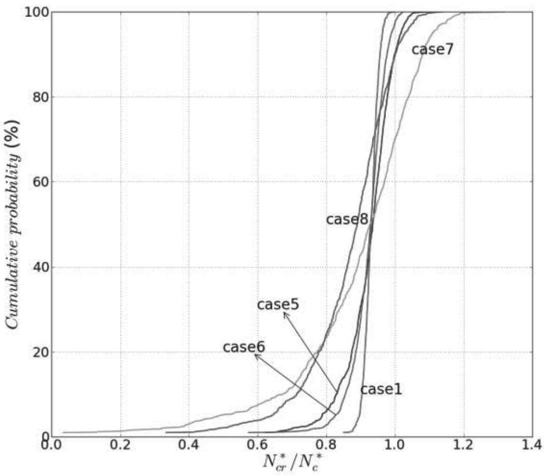



Figure 7. Cumulative probability curves for variation of correlation distance in the top layer (cases 1, 5, 6, 7 and 8)

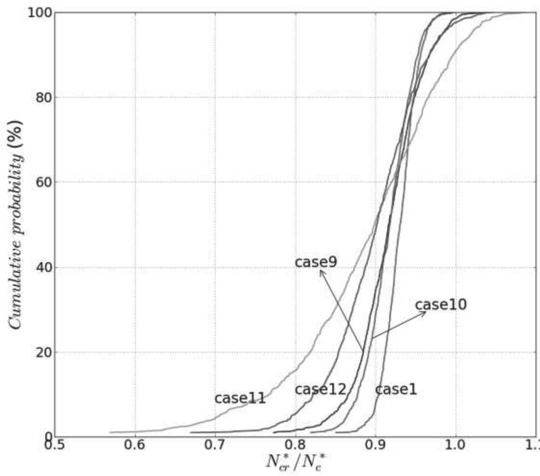



Figure 8. Cumulative probability curves for variation of correlation length in bottom layer (cases 9, 10, 1, 11 and 12)

## 4 CONCLUSIONS

In this study, finite element analysis of the vertical bearing capacity of a strip footing penetrating stiff-over-soft clay was conducted by taking the spatial variability of undrained strength into account. The results indicate that with high spatial variability in the undrained shear strength there is a significant reduction in the bearing capacity. Mean bearing capacity factors and statistical distributions were provided for 12 cases of $s_{ut}/s_{ub}$ = 2, COV = 0.1 and 0.3, and $\theta_v/B$ and $\theta_h/B$ = 0.1, 1 and 10. For the case of top layer thickness equal to the strip footing width presented it was shown that variation in the top layer had a greater effect on reducing the bearing capacity (when correlation distance was held constant). This was due to the unsymmetric bearing capacity shortening further into the top layer.

The empirical probabilities of exceedence of the deterministic bearing capacity factor in the stochastic case differ from case to case, ranging from on the order of $10^{-4}$ to 0.277, thus attesting for the influence of the magnitude of spatial variability and uncertainty on the effects of stochastic modelling. The maximum value observed for case 7 is well below a "central" value of 0.5; hence, overall, it is assessed that the deterministic case is significantly unconservative from an engineering standpoint.

The conclusions drawn in this paper may be specific for the geometery and soil conditions analysed. The 12 cases presented here, however, represent a small subset of 1600 cases analysed in a more ambitious numerical experiment. Cases of (i) $\mu_{s_{ut}}/\mu_{s_{ub}}$ = 4/3 and 2, (ii) COV = 0.1 and 0.3, (iii) $\theta_v/B$ and $\theta_h/B$ = 0.1, 1 and 10, as well as (iv) a gradient of increasing undrained shear strength with depth, and (v) a footing embedded to 0.5B into the top layer, make up the full programme. The results of the larger study will be published in due course.

## 5 ACKNOWLEDGEMENTS

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