Some notes concerning the dry density testing standards

Quelques remarques concernant les descriptions relatives aux essais de densité sèche

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ABSTRACT: The results of two doctoral programs concerning the dry density of sands are as follows: (i) the ratio of the minimum and the maximum dry density of sands is about constant, (ii) the dry density of the soil fractions slightly increases with the maximum grain diameter d_{max} being the diameter range doubled with increasing grain diameter. These results are used through a statistical analysis to show that the e_{max} test of the German DIN may be biased. The possible reason for the bias is arching due to the too small ratio of the diameter and the height of the mold. As a by-product of the research, the earlier finding that the maximum density of the specimens may be obtained indifferently using a Modified Proctor procedure or a vibrating table is extended to the DIN (implying that the density methods of the calibration chamber sands are basically equivalent to the DIN).

RÉSUMÉ : Deux thèses de doctorat ayant pour objet l'étude de la compacité des sables ont récemment formulé les importantes conclusions suivantes : (1) le rapport de la compacité minimale et maximale des sables est plus ou moins constant ; (2) l'étendue des fractions de diamètre croît avec le diamètre des grains, par conséquent la compacité des fractions croît également. En partant de ces conclusions, on a procédé à la comparaison (partiellement statistique) des valeurs de compacité minimales et maximales de cinq groupes donnés. Les résultats ont montré qu'une partie des normes actuelles conduisent à des erreurs déterminées dans la valeur de compacité minimale et ne conduisent pas à des erreurs déterminées dans la valeur de compacité maximale. L'explication en est, probablement, l'apparition de l'effet de voûte aux pots et vases de largeur différente, de hauteur constante..

KEYWORDS: sand, dry density, arching, Proctor, statistical test, coefficient of variation

1 INTRODUCTION

Two doctoral programs on the dry density of sands - where the soils used were indeed carefully chosen, artificial mixtures of natural grains (Lőrincz 1986, and Kabai, 1968) - ended with the two basic observations.

Kabai (1968 to 1974) found that the ratio of the minimum and maximum dry density was basically constant. Its value started to decrease as the soil became slightly plastic. He stated this on the basis of his experimental studies made on continuous, artificial mixtures of natural soil grains of the Danube river sand.

Lőrincz (1986) made a theoretical and experimental study on the grading entropy. He observed that the dry density of the sand fractions are increasing with grain diameter since the diameter range of the fractions are doubled with increasing grain diameter. (This result is paradox if the fractions are modelled by equal spheres.) He demonstrated this fact by his minimum dry density measurements.

The data bases of the foregoing two doctoral programs was compared first to the Calibration Chamber sand database, published by Lunne (1992) and Mayne and Kulhawy (1992) and, a significant difference was found on the databases (Imre et al, 2011) which was attributed to the dependence on the applied testing standard for density and to the geological origin.

Therefore, two additional data sets (one composed from Danube sands and one composed from Bochum sand) were measured in the University of Bochum. These were statistically analysed and it was found that the result of the minimum density measurement of the German Standard DIN 18126 (and some other standards not using the Proctor mold for the minimum dry density) may be biased and, as a result, the result of the minimum dry density test may be dependent on the grain diameter. The geological origin of the soils was indifferent.

2 MATERIALS AND METHODS

2.1 *Variables*

The results of the density tests can be expressed in terms of the void ratio e, the dry density ρ_d, the solid volume ratio s or its inverse, the so called specific volume v. The basic definitions are as follows:

$$s = \frac{V_s}{V} = \frac{1}{v} = \frac{1}{1+e} \tag{1}$$

$$e = \frac{V_v}{V_s} = \frac{1-s}{s} \tag{2}$$

where V is the total soil volume, V_s is the volume of solids and V_v is the volume of voids, $\rho_d = s\rho_s$.

2.2 *Density tests*

The minimum dry density testing methods are related to a funnel pouring device, differing in the mold size. The Proctor mold is used by Kabai (1978) and Lőrincz (1986). The diameter of the mold is 0,625 times the height for the DIN18126, 0,769 times the height for the ASTMD4254 and 0,879 for the Proctor mold (Fig 1).

Concerning the maximum dry density tests, the similarity of the Modified Proctor procedure and the Vibrating Table procedure is kown (Poulos and Hed, 1974). Kabai used an 8 mm thick steel plate was applied on the top of the sample, using the results of Leussink and Kutzner (1962), to prevent the local loosening of the sample.

2.3 *Soils*

The paper deals with 4 sets of density tests. The means (x) and the standard deviations (σ) of the variables s_{min} , s_{max} and s_{min} / s_{max} and e_{min} , e_{max} and e_{min} / e_{max} were determined.

In the program of Kabai (1968 to 1974, Group I) 33 grading curve series (artificial mixtures made of natural soil grains of the Danube sands) were defined in terms of the coefficient of uniformity $C_{U,}$ and the grainsize parameters d_{max}, d_{10}. The grading curves can be seen in Imre et al (2009). The minimum and maximum dry density were determined. The results can be seen in Table 1.

The calibration chamber sand database (Group II, Lunne et al (1992), Mayne and Kulhawy (1992)) consists of 25 sands. It can be assumed that the minimum and maximum dry density tests were made by various methods, including the ASTM Standards (i.e. D 4253 Standard Test Methods for Maximum Index Density and Unit Weight of Soils Using a Vibratory Table and D 4254 Standard Test Methods for Minimum Index Density and Unit Weight of Soils) and the German Standard (i.e. DIN 18126 for the maximum and minimum dry density). The results are shown in the Table 2.

In the ongoing testing program two additional data sets were produced with the German DIN (Groups III and IV) using 24 Danube sands 182 German sands, respectively. In Group III some Danube sands were used, the composition can be seen in Imre et al (2011). in Group IV some German commercial sands were used. the composition can be seen in Szalkai (2012). The results are shown in the Tables 3 and 4. The artificial mixtures were made of soil grains with 7 fractions of a diameter range of d_{min} = 0.06 mm and d_{max} = 8 mm.

Concerning the data of Lőrincz (1986), some artificial mixtures of mold sand grains) were defined in terms of grading entropy for various purposes, to elaborate some particle migration criteria and filtering law, to test the existing filtering laws, some grading curves can be seen e.g. in Imre et al (2009). Only the minimum dry density was determined using the Proctor mold. Some fractions results can be seen in Figure 2 showing that dry density of the sand fractions are increasing with grain diameter.

2.4 *Statistical tests*

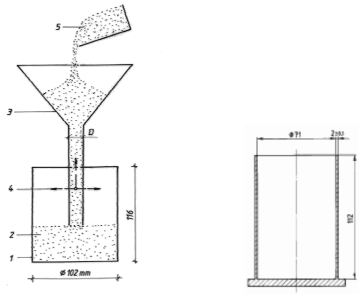
The means (x) and the standard deviations (σ) of the variables s_{min} , s_{max} and s_{min} / s_{max} for any two groups of data were compared using the standard parametric statistical tests (see e.g. in Réthàti, 1988).

The *F*-test was used to evaluate the null hypothesis that two data sets with normal distribution have the same variance. The Welch test and *t* tests were used to test the null hypothesis that two data sets with normal distribution have the same means with the data having possibly unequal and equal variances, respectively.

The Pearson correlation R is obtained by dividing the covariance of the two variables by the product of their standard deviations. The Pearson correlation is +1 in the case of a perfect positive (increasing) linear relationship (correlation) and, −1 in the case of a perfect decreasing (negative) linear relationship since the following relationship is met:

$$R(x, y) = 1 \leftrightarrow y = ax + b \quad (3)$$

The Pearson correlation is some value between −1 and 1 in all other cases, indicating the degree of linear dependence between the variables. As the Pearson correlation R approaches zero there is less of a linear relationship. The closer the coefficient is to either −1 or 1, the stronger the linear correlation between the variables.



(a) (b)

Figure 1. The molds (a) Proctor (d=100 mm, h=112mm) (b) DIN (d=71mm,h=112mm). Note the same height and different width.

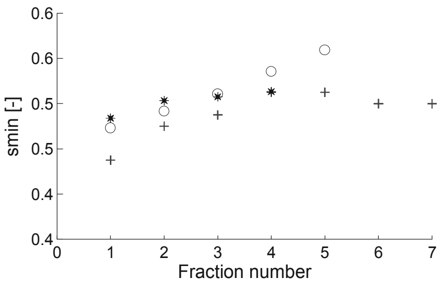


Figure 2. Fraction measurement of Lőrincz (1976), indicated by open circles, note the bias from fraction 3 (0.25 to 0.5 mm) in the new data.

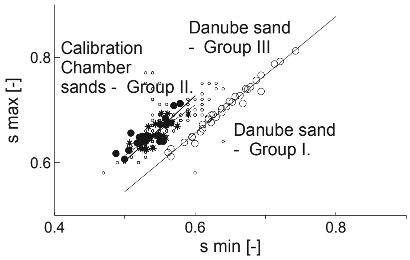


Figure 3. All data (Groups I to IV), $s_{min} - s_{max}$

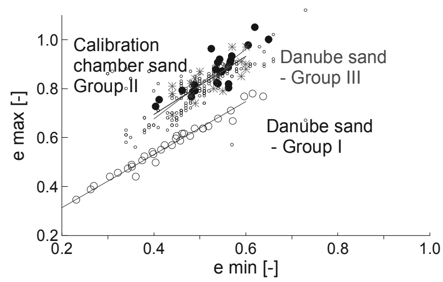


Figure 4. All data (Groups I to IV), e_{min} - e_{ma}

If the variables are independent, Pearson's correlation coefficient is 0, but the converse is not true, the correlation coefficient detects only linear dependencies. The parameters of the best fit linear function are as follows (E denotes expected value which can be approximated by the mean value):

$$a = \sigma(y)/\sigma(x), \quad b = E(y) - aE(x) \; [4]$$

3 INDIVIDUAL GROUP RESULTS

The results of the density tests for groups I to IV are summarized in Figures 3 to 6 where the small open circles are related to the Group IV. The results are presented in Tables 1 to 4 in statistical viewpoint.

3.1 *Danube Sands data - Group I*

In the work of Kabai (1972) the minimum and maximum dry densities are presented for Danube sand mixtures. From these, the void ratios *e* and the solid volume ratios *s* were computed and their ratios were determined (Table 1). The mean of the s_{min} and s_{max} values are 0.640 and 0.700, with mean coefficients of variation of 0.070 and 0.072, respectively. The mean of the *s* ratio (s_{min} / s_{max}) is equal to 0.915 with a coefficient of variation *CV*=0.0116, indicating that the *s* ratio is practically constant. The mean of the e_{max} and e_{min} values are 0.569 and 0.436, with coefficients of variation of 0.192 and 0.234, respectively. The mean of the e_{min} / e_{max} is equal to 0.760 with a coefficient of variation *CV*=0.060.

3.2 *The Calibration Chamber sand data – Group II*

The mean of the s_{min} and s_{max} values are 0.536 and 0.652, with the coefficients of variation of 0.045 and 0.039, resp. (Table 2). The mean of the *s* ratio (s_{min} / s_{max}) is equal to 0.822 with a coefficient of variation *CV*=0.024. The mean e_{max} and e_{min} values are 0.870 and 0.536, with coefficients of variation of 0.097 and 0.110, respectively. The mean of the e_{min} / e_{max} is equal 0.617 with a coefficient of variation of *CV*=0.061. The mean of the e_{max} and e_{min} values are 0.810 and 0.496, with coefficients of variation of 0.114 and 0.159, respectively. The mean of the e_{min} / e_{max} is equal to 0.610 with a coefficient of variation *CV*=0.082.

3.3 *Danube Sands data – Group III*

The minimum and maximum dry densities for 24 Danube sand mixtures are shown in Table 3. The mean of the s_{min} and s_{max} values are 0.554 and 0.670, with mean coefficients of variation of 0.052 and 0.053, respectively. The mean of the *s* ratio (s_{min} / s_{max}) is equal to 0.827 with a coefficient of variation *CV*=0.025, indicating that the *s* ratio is practically constant.

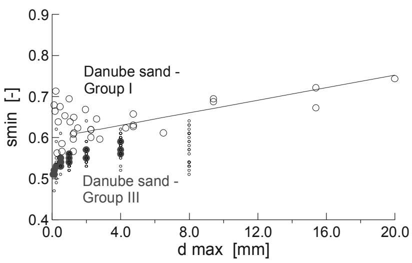


Figure 5. Groups I, III to IV data, s_{min} and maximum diameter. Note that the small open circles are 'below' the linear trend of Group I

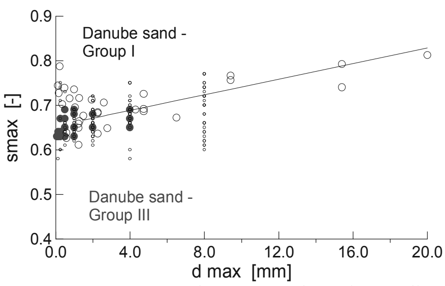


Figure 6. Groups I, III to IV data, s_{max} and maximum diameter. Note that the small open circles are 'centered' the linear trend of Group I

3.4 *Bochum Sand data – Group IV*

The minimum and maximum dry density results of the newly measured 182 Bochum sand mixtures are shown in Table 4.

The mean of the s_{min} and s_{max} values are 0.557 and 0.671, with mean coefficients of variation of 0.054 and 0.053, respectively. The mean of the *s* ratio (s_{min} / s_{max}) is equal to 0.831 with a coefficient of variation *CV*=0.044, indicating that the *s* ratio is practically constant.

The mean of the e_{max} and e_{min} values are 0.801 and 0.495, with coefficients of variation of 0.119 and 0.161, respectively. The mean of the e_{min} / e_{max} is equal to 0.619 with a coefficient of variation *CV*=0.125.

Table 1. Results of the statistical evaluation – Group

	s_{min}	s_{max}	$ratio_s$	e_{max}	e_{min}	$ratio_e$
X	0.640	0.700	0.915	0.569	0.436	0.760
σ	0.045	0.050	0.011	0.109	0.102	0.045
CV	0.070	0.072	0.012	0.192	0.234	0.060

Table 2. Results of the statistical evaluation – Group II

	s_{min}	s_{max}	$ratio_s$	e_{max}	e_{min}	$ratio_e$
X	0.536	0.652	0.822	0.870	0.536	0.617
σ	0.024	0.026	0.020	0.084	0.059	0.038
CV	0.045	0.039	0.024	0.097	0.110	0.061

Table 3. Results of the statistical evaluation - Group III

	s_{min}	s_{max}	$ratio_s$	e_{max}	e_{min}	$ratio_e$
X	0.554	0.670	0.827	0.810	0.496	0.610
σ	0.029	0.036	0.021	0.092	0.079	0.050
CV	0.052	0.053	0.025	0.114	0.159	0.082

Table 4. Results of the statistical evaluation - Group IV

	s_{min}	s_{max}	$ratio_s$	e_{max}	e_{min}	$ratio_e$
X	0,557	0,671	0,831	0,801	0,495	0,619
σ	0,030	0,036	0,037	0,095	0,080	0,077
CV	0,054	0,054	0,044	0,119	0,161	0,125

Table 5. Results of the statistical tests - s_{min}

	I	II	III	IV
I	+	-	-	-
II	-	+	+	-
III	-	+	+	
IV	-	-		+

Table 6. Results of the statistical tests - s_{max}

	I	II	III	IV
I	+	-	+	+
II	-	+	+	+
III	+	+	+	
IV	+	+		+

Table 7. Results of the statistical tests – ratio s_{min}/s_{max}

	I	II	III	IV
I	+	-	-	-
II	-	+	+	+
III	-	+	+	
IV	-	+		+

4 DISCUSSION, CONCLUSIONS

4.1 *The statistical features of the data bases*

The ratio of the minimum and the maximum dry density is statistically constant for sands since the coefficient of the variation is very small. The coefficient of variation of the *e* ratio is equal to 0.061 for the calibration chamber sands; 0.060, 0.082 and 0,125 for the Groups sands I and III to IV, respectively. The coefficient of variation of the *s* ratio is equal to 0.024 for the calibration chamber sands; 0.0116, 0.025 and 0,044 for the Groups sands I and III to IV, respectively.

This result implies linear correlations between the minimum and the maximum dry density data. The Pearson coefficient of correlation at the $s_{min} - s_{max}$ and e_{min} - e_{max} was equal to 0.99, 0,99 for the Danube sands I indicating that both the $s_{min} - s_{max}$ and e_{min} - e_{max} relations are linear. The Pearson coefficient of correlation at the $s_{min} - s_{max}$ and e_{min} - e_{max} was equal to 0.84, 0.85; 0.89 and 0.88 and; 0,64, 0,65; respectively for Groups II, III and IV – being smaller, still indicating linear correlation.

The smallest coefficient of variation values (and, the largest correlation values) of Group I sand data (produced by using the Proctor mold for maximum void ratio) can be attributed to the facts that the quality of testing on was good and the tested soils were homogeneous (having continuous distribution only).

4.2 *The information concerning the dry density ratio*

The results of the statistical tests indicate that the ratio s_{min}/s_{max} is about equal about to 0.76 for Group I where the Proctor mold is used for the minimum dry density test and the ratio s_{min}/s_{max} is about equal about to 0.62 for the remainder Groups where a different mold is used (i.e. the two newly tests sand databases – being tested with the German Standard – and the Calibration Chamber sands).

The result of the statistical tests (Tables 5 to 7) – show that the minimum dry density data are significantly different for Group I and the remainder groups but the maximum dry density results are similar.

The significant difference in the minimum dry density testing methods can be attributed to the fact that the molds have different sizes (Fig 1). This point needs some further (e.g. micromechanical) research.

4.3 *The dependence of the dry denisty on the diameter*

In accordance to the expectations, the measured s_{min} and s_{max} values slightly increase with the maximum grain diameter d_{max} in the tested sand databases (Figs 5 to 6).

However, an increasing bias with grain diameter - possibly due to arching – is found in the minimum density results. As it can be seen on Figure 2, the density increase is not possible to be reproduced by the DIN 18126 for fractions being larger than 0,5 mm, instead of this, even a decrease is experienced.

4.4 *Conclusions*

The different dimension of the mold of the various minimum dry density tests has some impact on the test results as follows. The smaller width and same height may cause some kind of arching which leads to smaller minimum dry density values and, this effect is the function of the grain diameter.

As a by-product of the research, an additional comment can be made. The result of the statistical analysis indicates that the equivalence of the Modified Proctor procedure and the Vibrating Table procedure (Poulos and Hed, 1974) can be extended to the DIN maximum dry density testing method.

Some further research is suggested, including the separation of the continuous and gap-graded mixtures and, on the investigation of the micromechanical features of the minimum dry density test.

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