Effect of Particle characteristics on K_0 Behavior for Granular Materials

Effet des caractéristiques particulaires sur le comportement des matériaux granulaires K_0

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ABSTRACT: The Jaky's K_0 equation is commonly used for the estimation of K_0 in practice, which is given as a function of the internal friction angle of soils. As the friction angle is a state-dependent variable, uncertain aspect still exists regarding the value of to be adopted for the Jaky's K_0 equation. In this study, the application of the Jaky's K_0 equation for granular materials is investigated with focus on the effect of particle characteristics, which are related to strength, on K_0 behavior. Particle shape and angularity of particles are considered for the investigation of K_0 behavior. Experimental testing program is established to measure K_0 under various mechanical and physical conditions of test materials. Sand particles and spherically shpaed glass beads with and without etched particle surfaces are employed into the experimental testing program.

RÉSUMÉ: L'équation K_0 de Jaky est couramment utilisée pour l'estimation de K_0 en pratique, ce qui est donnée en fonction de l'angle de frottement interne du sol. Comme l'angle de frottement est une variable dépendant de l'état, l'aspect incertain existe encore au sujet de la valeur qui sera adoptée pour l'équation K_0 de Jaky. Dans cette étude, l'application de l'équation K_0 de Jaky pour les matériaux granulaires est étudiée, en particulier par rapport à l'effet des caractéristiques des particules, qui sont liés à la résistance sur le comportement de K_0. La forme et l'angularité des particules sont considérées comme des enquêtes sur le comportement de K_0. Le programme d'essais expérimentaux est prévu pour mesurer le K_0 dans différentes conditions mécaniques et physiques des matériaux d'essai. Les particules de sable et de billes de verre en forme sphérique avec et sans surfaces des particules gravées sont utilisées dans le programme d'essai expérimental.

KEYWORDS: coefficient of lateral earth pressure, granular materials, internal friction angle, thin wall oedometer test

1 INTRODUCTION

In-situ stresses are important state soil variables that are necessary for geotechnical analysis and design. The Jaky's K_0 equation (Jaky 1944) is commonly used in practice to evaluate K_0 based on the internal friction angle of soils. In fact, it was analytically derived assuming equilibrium condition for a sand pile with the statically admissible stress state. From the analysis using various assumptions on the stress distribution, it was confirmed that the Jaky's K_0 equation is valid and sufficiently accurate for general geotechnical purposes (Michalowski 2005). According to the Jaky's K_0 equation, K_0 is given as a sole function of the friction angle of soils, while K_0 itself represents the state soil variable that defines the geostatic stress state before failure.

The friction angle of soils can be differently defined. Common definition adopted in geotechnical engineering includes the peak, critical state, and dilatancy friction angles. The peak friction angle ϕ'_p corresponds to the maximum peak strength, and is composed of the critical state friction angle ϕ'_c and the dilatancy friction angle ψ_p. The dilatancy friction angle ψ_p is state-dependent varying as a function of the confining stress and relative density. The critical state friction angle ϕ'_c is an intrinsic soil variable that is dependent only on the inherent soil characteristics such as mineralogy, particle shape, and angularity. From the Jaky's K_0 equation, the highest and lowest K_0 values would be obtained from the critical state friction angle ϕ'_c and the peak friction angle ϕ'_p, respectively.

In this study, the application of the Jaky's K_0 equation for granular materials is investigated focusing on the effect of various particle characteristics such as particle shape, surface roughness, and relative density on the ϕ'-K_0 correlation. The variation of K_0 calculated using different types of friction angles is analyzed. For this purpose, an experimental testing program is established to measure K_0 under various soil and stress conditions. The tested granular assemblies include natural sand particles and spherical glass beads with and without etched particle surfaces.

2 CORRELATION OF K_0

Jaky (1944, 1948) presented the well-known K_0 equation based on the stress analysis of a geometrically symmetric sand wedge, assuming a limit stress state. K_0 is defined as a function of the internal friction angle ϕ' of soils as given by:

$$K_0 = (1 - \sin\phi')\frac{1 + \frac{2}{3}\sin\phi'}{1 + \sin\phi'} \quad (1)$$

Where ϕ' = internal frictional angle of soils. Eq. (1) can be further simplified as a form that has been a norm in current practice, given as follows:

$$K_0 = 1 - \sin\phi' \quad (2)$$

It is indicated that the denser the sand, the higher the ϕ', resulting in lower K_0 values. The effect of stress history on K_0 is significant. K_0 for overconsolidated (OC) condition is greater than for normally consolidated (NC) condition (Wroth 1973). In order to reflect the effect of stress history on K_0, modifications into Eq. (2) have been proposed in a form of:

$$K_0 = K_{0,NC} \cdot OCR^{\alpha} \tag{3}$$

where $K_{0,NC}$ = K_0 for normally consolidated conditions, OCR = overconsolidation ratio, and α = exponential parameter. Mayne and Kulhawy (1982) and Mesri and Hayat (1993), for example, proposed $\sin\phi'$ for α.

Although the Jaky's K_0 equation have been widely used and modified to better reflect the actual stress state, uncertainties still exist, which has not been fully clarified yet. Examples are the effect of particle characteristics, which differ inherently for different materials, and the correlation to the friction angle ϕ' that is not constant but varies state-dependently.

3 TESTING PROGRAM

3.1 *Materials*

A series of laboratory tests were conducted to investigate and analyze the K_0 evolution of granular materials. Three different granular materials were adopted, which include Jumumjin sand, spherical glass beads (GB), and etched glass beads (EGB). The use of different granular materials was aimed to investigate the effect of inherent particle characteristics, such as particle shape, surface roughness, and angularity, on K_0 in more straightforward and systematic manner. Jumunjin sand particles were sieved, and the particles ranging between 0.425 mm and 0.85 mm (e.g., sieves #40 and #20) were collected and used in the test, to presumably minimize the effect of fines content and to make test samples equivalent in size hence directly comparable to glass beads. The grain size distribution of Jumunjin sand is shown in Fig. 1, in comparison with glass beads. Sand particles are uniformly distributed with the mean particle size D_{50} of around 0.56 mm.

The soda lime glass beads ($CaO\text{-}MgO\text{-}Na_2O\text{-}SiO_2$) with spherical particle shape and smooth surface ($D_{50} \sim 0.5$ mm) were selected as comparative materials. The surface roughness was controlled by etching clean, smooth glass bead surfaces with hydrogen chloride-based solution (HCl, pH=1±0.5). Glass beads were submerged in the hydrogen solution for 15 seconds and were thoroughly rinsed with deionized water followed by oven-drying at 80°C. This process produced uniformly etched glass beads with rough surfaces, while the same spherical shape as clean glass bead particles was maintained. Three samples were then subjected to the assessment of K_0 measurements.

The values of ϕ'_c were 37.1°, 26.7°, and 30.0° for Jumunjin sand, glass beads, and etched glass beads, respectively. The values of ϕ'_p varied, depending on the relative densities and particle characteristics considered in the tests.

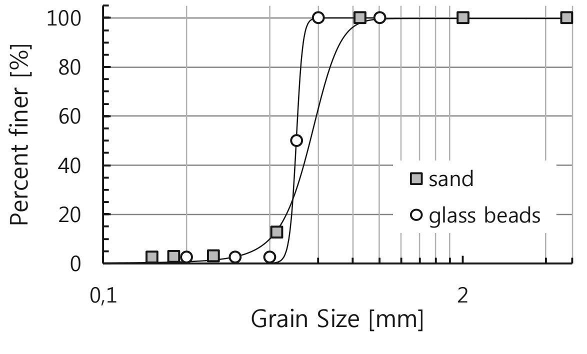


Fig. 1 Grain size distribution curves of test materials.

3.2 *Determination of K_0*

The various test devices have been proposed to measure K_0 that satisfy the radial strain limit, smaller than 5×10^{-5} (Okochi and Tatsuoka, 1984). The radial expansion of either membrane or thin-wall tube can be servo-controlled to maintain the zero lateral strain condition whereas the complex stress path and multiple measurement systems are involved. Alternatively, the strain gauges are attached to the thin-wall tube and the horizontal stress is directly related to the cell deformation within acceptable strain regimes (Zhu et al. 1995; Kolymbas and Bauer 1993).

The oedometer method to measure K_0 adopted in this study consists of a thin aluminum cylinder of 0.13 mm in thickness and 66 mm in diameter. A pair of strain gauges (120ohm, CEA-13-240UZ-120, Vishay) was attached at the middle height of the cylinder outside and another pair of dummy gauges was included for temperature compensation forming the full-bridge circuit. The sample height is about 40 mm with a height to diameter ratio equal to 0.61. Fig. 2 illustrates the experimental configuration and peripheral electronics. The voltage response of strain gauges was calibrated and correlated to the horizontal stress using a water-filled balloon inside the cylinder, assuming that the applied vertical stress is equal to the horizontal stress. The linear calibration factor was then obtained and given as follows:

$$\sigma = 0.0476 \cdot V - 4.8338 \tag{4}$$

where σ is the horizontal stress in the unit of kPa and V is the voltage out in the unit of mV.

Dry samples were placed in the thin-wall oedometer to achieve a target relative density and were subjected to loading-unloading-reloading cycles. The maximum loading reached 111 kPa and reloading ran up to 143 kPa beyond the preconsolidation stress. The voltage response was recorded every 1 sec and each loading step lasted 5 to 10 minutes. The horizontal stress for a given loading step was computed by averaging voltage response and by applying the calibration factor. The test specimens were prepared at different relative densities of D_R = 33% and 84% for Jumunjin sand and D_R = 56% and 80% and D_R = 55% and 81% for glass and etched glass beads, respectively.

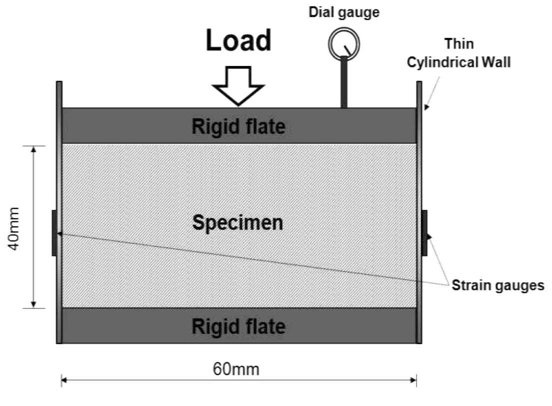


Fig. 2 Configuration of thin-wall K_0 test.

4 TEST RESULTS AND ANALYSIS

4.1 *K_0 for Different Test materials*

The changes of K_0 with σ'_v for the test materials measured from the thin-wall oedometer are shown in Fig. 3 during the whole loading cycles. As shown for the loose (LS) and dense (DS) sands with D_R = 33 and 84%, respectively, the K_0 values of the dense sand run below the loose one at all the loading stages. The dense sand provides the strong force chain along the

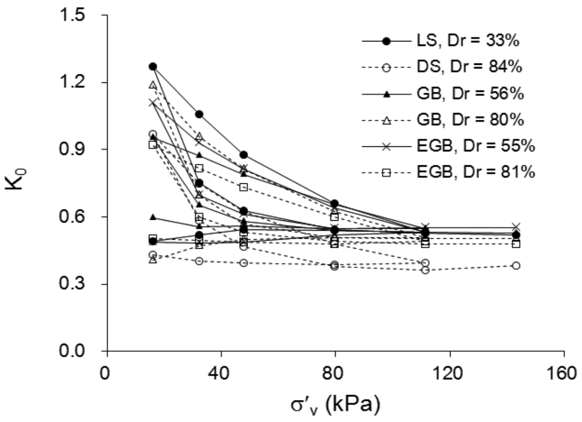


Fig. 3 Values of K_0 with σ'_v for test granular materials of Jumunjin sand (LS and DS) and glass beads with and without etching (GB, EGB).

vertical stress direction due to higher interlocking, which leads to lower K_0 during loading. The horizontally interlocked stress induces the increase of K_0 during unloading whereas the partial release of the horizontal stress during reloading makes the evolution of K_0 run between loading and unloading stages. Once the stress reaches the preconsolidation stress, K_0 remains constant as the normally consolidation condition prevails.

The effect of particle shape and angularity on K_0 was investigated by directly comparing the test results from Jumunjin sand and glass beads. In Fig. 3, the values of K_0 for the dense sand (DS) are compared with those for glass beads (GB) according to the vertical stress. The lower values of K_0 for the dense sand are manifest presumably due to the angularity effect. The particle surface roughness effect on K_0 values can be analyzed by comparing the test results from glass beads (GB) and etched glass beads (EGB). No marked difference of K_0 between GB and EGB is observed during loading, while GB exhibits higher K_0 values during unloading and reloading. The ratio between the artificially created surface dents (e.g., ~5 to 10 μm) and particle diameter ranges from 0.01 to 0.02 for EGB, which impose insignificant impact on K_0 during loading, while the particle geometry is predominant.

4.2 *Correlation to Strength*

The shear strength of granular materials can be described using different definitions of friction angle. The critical state friction angle ϕ'_c, the peak friction angle ϕ'_p, and dilatancy angle ψ are the typical examples. The inter-particle surface friction angle ϕ'_s can be regarded as another type of friction angle that contributes to the overall shear strength of granular materials.

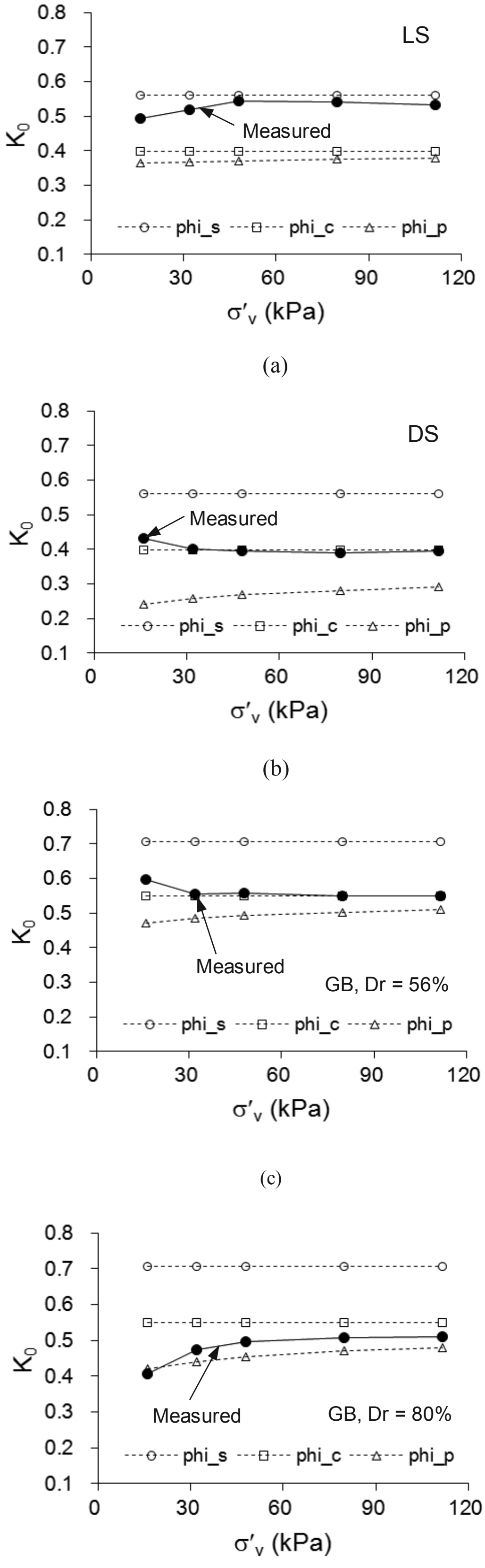
The confining stress within the specimen for the thin-wall oedometer tests continuously increases upon loading. The changes in confining stress result in changes in dilatancy and thus in the peak friction angle. Following Bolton (1986), the effect of confining stress and relative density on the peak friction angle can be evaluated using the following relationship:

$$\phi'_p = \phi'_c + R_D \cdot I_R \qquad (5)$$

where R_D = dilatancy ratio = 3 and 5 for triaxial and plane-strain conditions, respectively. The dilatancy index I_R is defined as:

$$I_R = I_D\left[Q - \ln\left(\frac{100\cdot\sigma'_{pm}}{p_A}\right)\right] - R \qquad (6)$$

where I_D = relative density as a number between 0 and 1; p_A = reference stress = 100 kPa; σ'_{pm} = mean effective stress at peak in the same unit as p_A; and Q and R = intrinsic soil variables.



(a)

(b)

(c)

(d)

Fig. 4 Measured and calculated K0 values with σ'_v for (a) loose sand; (b) dense sand; (c) medium etched glass beads; and (d) dense etched glass beads.

Using Eqs. (5) and (6), the variation of ϕ'_p during loading in the thin-wall oedometer tests for the test materials were obtained. The peak friction angle ϕ'_p decreased from 49.5° to 45.8° and 39.5° to 38.5° in the range of σ'_v from 16.0 to 111.5 kPa for the dense (DS) and loose (LS) sands, respectively. Using ϕ'_p, ϕ'_c and ϕ'_s for Jumunjin sand, K_0 was calculated and compared with the measured K_0 values in Fig. 4. The ϕ'_s was assumed equal to 26° and 17° for Jumunjin sand and glass beads, based on the values presented by Procter and Barton (1974) and Andrawes and El-Sohby (1973). Note that K_0 from ϕ'_c and ϕ'_s is constant as these are intrinsic soil variables. ϕ'_p produces the lowest range of K_0 values, while the upper bound is given by ϕ'_s. It is also noticed that the K_0 values measured during loading follows quasi constant, while decreases in ϕ'_p is certainly expected as indicated in Eqs. (5) and (6). From Fig. 4, it is seen that the measured K_0 values for the loose and dense sands are close to those calculated using ϕ'_s and ϕ'_c, respectively. This implies that the application of ϕ'_p is likely to produce underestimated K_0 values. For glass beads, the application of critical state friction angle ϕ'_c produces close match to the measured K_0 values. Similar results were observed for etched glass beads.

5 CONCLUSION

In this paper, the values of K_0 were investigated for different granular materials focusing on the effect of various particle characteristics. For this purpose, laboratory tests using the thin-wall oedometer were conducted to measure the values of K_0 under various test conditions. Sand particles, glass beads with and without etched particle surfaces were used in the testing program.

From the test results, it was observed that the effect of material density on K_0 was greater in OC stress state than in NC stress state, and in particular becomes more pronounced when unloaded. Regarding the effect of particle shape and angularity, the lower values of K_0 were observed from Jumunjin sand particles than from glass beads, due to the higher angularity and interlocking effects.

For sands, the values of the friction angle employed into the Jaky's K_0 equation to match the measured K_0 values were different for loose and dense sands. From measured and calculated K_0 values, it was found that the measured K_0 values for the loose sand were close to the calculated values using the inter-particle friction angle ϕ'_s. For dense sand, on the other hand, ϕ'_c produced close match to the measured K_0 values. This indicates that the application of the peak friction angle ϕ'_p is likely to result in underestimation of K_0. For glass beads, calculated K_0 values using ϕ'_c were in good agreement with measured results for both relative densities.

6 ACKNOWLEDGEMENTS

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