Large-Scale Geotechnical Finite Element Analysis on Desktop PCs

Analyse par éléments finis de problèmes géotechniques de grandes dimensions sur ordinateur de bureau

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ABSTRACT: With the development of new hardware and software technologies, the trend of using three-dimensional finite element analysis in geotechnical engineering is growing recently. However, the solution of realistic large-scale problems still demands a significant amount of computational time and resources. The computational time can be even longer for the ill-conditioned systems when the stiffness of different elements differs by several orders of magnitude. In this paper, we demonstrate how the recent development of block diagonal preconditioning has effectively reduced the computational time of iterative solvers so that large-scale finite element analysis can be performed in a reasonable time on Desktop PCs using GeoFEA.

RÉSUMÉ : Avec le développement des nouvelles technologies matérielles et logicielles, l'analyse tridimensionnelle par éléments finis en géotechnique est de plus en plus utilisée. Cependant, la solution de problèmes de grandes dimensions réels exige toujours une quantité importante de temps de calcul et de ressources. Le temps de calcul peut être encore plus long pour les systèmes mal posés, lorsque la raideur des différents éléments diffère de plusieurs ordres de grandeur. Dans cet article, nous montrons comment le développement récent de préconditionnement diagonale par blocs a permis de réduire le temps de calcul des solveurs itératifs de sorte que l'analyse de problème de grandes dimensions par éléments finis peut être effectuée dans un délai raisonnable sur les ordinateurs de bureau utilisant GeoFEA.

KEYWORDS: Large-scale finite element analysis, iterative solvers, preconditioning, GeoFEA, Desktop PC.

1 INTRODUCTION

With the advancement of new hardware and software technologies (sophisticated finite element programs), fairly large-scale analyses are within the reach of geotechnical design offices and the emphasis of designs and analyses has been shifting from simple or empirical approaches to large-scale three-dimensional (3D) finite element modelling. 3D analysis is also useful in understanding the complex soil-structure interation problems. However, significant amount of time and large memory requirement for storage are the major challenges for 3D analysis because a large number of finite elements are required to represent the problem realistically. The resulting system of equations has, in general, the form:

$$Ax = b \tag{1}$$

where $A \in \Re^{N\times N}$ is known as coefficient matrix, $x \in \Re^{N}$ is the vector of unknowns, $b \in \Re^{N}$ is the force vector. N is the dimension of the linear system, that is, the degrees of freedom (DOFs) of the discretized mesh. Solution of this linear system (Eq. 1) is one of the most expensive computational parts in finite element analysis. For large linear systesms, Krylov subspace iterative method is popularly used to solve (Cipra 2000) them because of smaller memory requirement than direct solvers. However, for Krylov subspace iterative methods to be successful or efficient, preconditioning plays an important role.

In geotechnical engineering, consolidation is a general phenomenon, for which the coefficient matrix A can be severely ill-conditioned (Chan et al. 2001, Ferronato et al. 2001, Lee et al. 2002). Some effective preconditioners have been proposed in the past decade for Biot's (Biot 1941) consolidation equations; see, for example, Gambolati et al. (2011), Chen and Li (2011) for a brief review. Besides consolidation equations, highly heterogeneous soil profile or soil-structure interaction problems can further exaggerate the numerical instability of the solution. The recently proposed block diagonal preconditioners (Chaudhary et al., 2011, 2012) have shown to have effectively mitigated the ill-conditioning issues due to significant constrasts in stiffness as well as hydraulic conductivity of the materials in such problems.

This paper discusses the feasibility of 3D analysis with the implementation of these latest developments in preconditioned iterative solvers in GeoFEA, a commercial software package (http://www.geosoft.sg/). The results and how geometric idealizations can sometimes lead to erroneous results will be elaborated through using a case study of a basement excavation in Singapore.

1 PRECONDITIONERS

The finite element discretization of the Biot's coupled consolidations equations is usually expressed in 2×2 block linear system (Smith and Griffiths, 1997):

$$\begin{bmatrix} K & B \\ B^T & -C \end{bmatrix} \begin{Bmatrix} \Delta u \\ \Delta p \end{Bmatrix} = \begin{Bmatrix} \Delta f \\ Cp_t \end{Bmatrix} \tag{2}$$

where K is solid stiffness matrix, C is fluid stiffness matrix, B is displacement-pore pressure coupling matrix, Δu is displacement increment, Δp is excess pore pressure increment, Δf is nodal load increment, and p_t is nodal pore-pressure at current time step. Chaudhary et al. (2012) observed that the performance of existing preconditioners based on above 2×2 block form of the coefficient matrix may deteriorate significantly for the problems with significant contrasts in material properties, such as in soil-structure interaction problems. They proposed to partition the solid stiffness matrix K such that the coefficient matrix A takes a 3×3 block form, which has more flexibility to construct optimal preconditioners for such problems. Comparing Eqs. (1) and (2):

$$A = \begin{bmatrix} K & B \\ B^T & -C \end{bmatrix} = \begin{bmatrix} P & L & B_1 \\ L^T & G & B_2 \\ B_1^T & B_2^T & -C \end{bmatrix} \text{ with } K = \begin{bmatrix} P & L \\ L^T & G \end{bmatrix}. \quad (3)$$

where P is the stiffness matrix corresponding to stiff materials, G is the (soft) soil stiffness matrix, L is the stiff material-soil connection matrix, and $B = [B_1, B_2]^T$. The proposed block diagonal preconditioners have the following form:

$$M_1 = \begin{bmatrix} P & 0 & 0 \\ 0 & diag(G) & 0 \\ 0 & 0 & \alpha\, diag(\hat{S}) \end{bmatrix} \quad (4)$$

$$M_2 = \begin{bmatrix} P & 0 \\ 0 & \mathrm{MSSOR}(H) \end{bmatrix} \text{ with } H = \begin{bmatrix} G & B_2 \\ B_2^T & -C \end{bmatrix} \quad (5)$$

where, $\hat{S} = C + B_1^T diag(P)^{-1} B_1 + B_2^T diag(G)^{-1} B_2$ is an approximation to the Schur compelement matrix and α is a non-zero parameter, which is set to -4 based on an eigenvalue theorem developed in (Phoon et al. 2002). Whether a material is considered stiff so that the use of above preconditioners can be advantageous is largely problem dependent. Our numerical experiences suggest that the material 1 would be considered stiffer than material 2 if the ratio of Young's modulus of material 1 (E_1) to material 2 (E_2) is greater than 300-400 for the use of above preconditioners.

In general, the linear system is more prone to numerical instability (even with direct solvers) if this ratio grows very large. However, the theoretical block diagonal preconditioner has turned this curse into an advantage (see the theorem in Chaudhary et al. 2012), which is the basis of these inexact block diagonal preconditioners. Thus, an even better performance can be achieved if the problem involves several orders of difference in stiffness properties. This is because the sensitivity of stiffness contrast of materials is effectively minimized when the submatrix P is solved directly (such as Cholesky factorization) in M_1 and M_2. The only limitation of the above preconditioners is that the size of submatrix P should be such that its direct factorization is not very expensive to compute with the available random access memory (RAM) of the computer. However, the memory demands of these preconditioners are still much more affordable than applying an incomplete LU factorization preconditioner to the entire A or even entire K, because the size of P is usually much smaller than that of A (or K) for most of the geotechnical problems. The preconditioner M_2 has an edge (up to about 2 times faster) over M_1 due to the modified symmetric successive over-relaxation approximation (Chen et al. 2006) of lower-right 2×2 submatrix of A. This is also helpful in minimizing the effect of contrasts in hydraulic conductivity of the materials. However, M_1 is easier to be implemented and parallelized than M_2. Both of these preconditioners have been implemented in GeoFEA.

2 CASE STUDY – BASEMENT EXCAVATION

2.1 *Site Condition*

Figure 1 shows the plan view of the excavation site. The 36 storey condominium housing development project has 2 levels of basement excavation for carparks at Ardmore Park, Singapore. As shown in Figure 1, there are three types of retaining systems used in this project: (1) Circular sheet pile cofferdam with concrete ring waling in WS-A, (2) Corner strut system in WS-B, WS-D and WS-E, and (3) Ground anchors in WS-C and WS-D. The ground surface was sloping downwards from North (RL 117.5m) to South (RL 112.5m). The depth of excavation varies due to sloping ground and was about 14m for the central cofferdam area (WS-A), and about 7.7m for outside it.

The top 1.8m to 5.5m of soil is fill material with average SPT N-values estimated to be 3. This is underlain by residual soil derived from Bukit Timah Granite Formation. This residual zone is classified into GVI-1 and GVI-2 with a thickness ranging from 7.1 to 11.8m and 4.4 to 8.9m respectively. The residual zone is followed by zones of completely weathered Bukit Timah Granite Formation, GV-1 and GV-2. GV-1 ranges from homogeneous to non-homogeneous subsurface material with thickness varying from 3.1 to 10.0m. The SPT N-values lies between 24 and 51. The thickness of GV-2 zone is from 5.7 to 7.7m with SPT N-values lying between 53 and 94. The soil underneath consists highly weathered Bukit Timah Granite Formation GIV with SPT N-values well above 100blows/300mm. Most of the excavations are carried out in residual soil derived from Bukit Timah Granite Formation. Table 1 shows the soil properties used in the analysis.

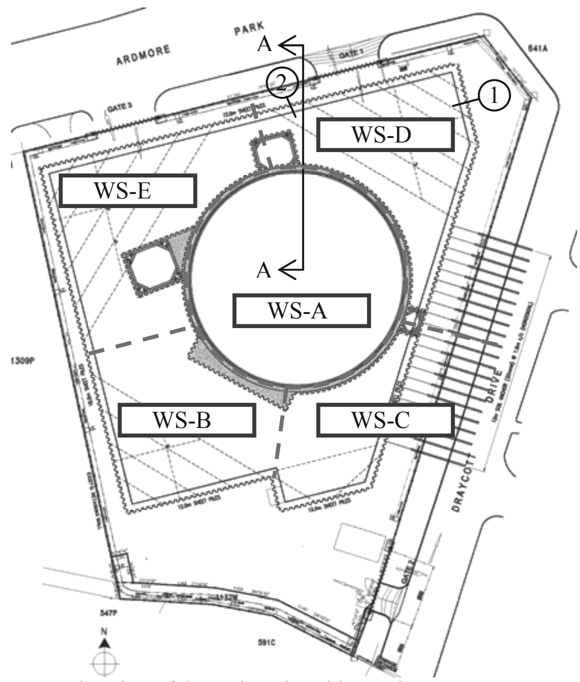

Figure 1. Plan view of the project site with strutting system

Table 1. Idealized soil profile used in the analysis.

Sub-layer	Fill	GVI-1	GVI-2	GV-1	GV-2	GIV	Hard Stratum
Depth (m)	0-14.3	1.8-20.6	9.7-27.9	12.8-35.6	18.9-37.7	26.1-43.9	25.5-41.2
SPT value	0-12	6-24	11-47	24-62	50-100	>100	>100
E, kN/m^2	9,000	15,000	38,000	80,000	155,000	650,000	950,000
c', kN/m^2	1	5	7	5	20	150	200
ϕ', degree	26	27	28	34	34	36	36
ν'	0.3	0.3	0.3	0.3	0.3	0.3	0.3
k, m/s	5×10^{-8}	2×10^{-7}	1×10^{-7}	1×10^{-7}	1×10^{-7}	1×10^{-6}	1×10^{-6}
OCR	1.5	1.2	1.2	1.1	1.1	1.1	1.1

2.2 *Finite Element Analysis*

The finite element analysis was conducted using GeoFEA version 9.0 (2012). The finite element mesh of the model is as shown in Figure 2. The geometry and ground profile were closely tried to replicate the real situation. The dimension of the model is 176m long by 141m width. The total number of elements used is 198,127 inclusive of 53,673 elements for structural elements. The structural elements are sheet pile, struts, walers, piles, etc. in this analysis, as shown in Figure 2b. This generated a total degrees-of-freedom (DOFs) of 855,645. Note that in the excavation analysis, the total DOFs changes in each stage due to excavation of soil (removal of elements) and installation of struts (inclusion of elements). Hence, the stiff DOFs (related to stiff materials) as well as total DOFs vary for different construction stages. The stiff DOFs range from 227,028 to 248,034 in different stages which are in the range of 30 to 40% of the total DOFs in respective stages. The element types used in the analysis are as follows:

a) Steel struts and walers were modeled using 3-noded linear elastic beam elements. A preload of 100kN was applied at each strut. This was achieved in GeoFEA by applying 100kN load at each connection point of struts with walers before installing the struts.
b) The sheet piles were modeled using 10-noded tetrahedron elements. The section modulus of the 0.3m wall is taken to that of equivalent to the section modulus (EI) of the FSP IV sheet pile, which has been used in the site.
c) All soil types were modeled using 10-noded tetrahedron elements. All soil types were modeled using Mohr-Coulomb models with associated flow rule.

The side boundaries are restrained laterally and the bottom boundary is fixed in all directions. The water table is set at RL 112.5m, lowest ground surface level in the model, so that no area is inundated. The sheet pile was assumed to be wished-in-place.

To model the whole excavation process, 41 increment blocks (or stages) were necessary besides the initial step (0) to compute the initial stresses. The excavation was carried out parts-by-parts as marked in Figure 1. The excavation was started from WS-A up to RL 101.5m, and followed to WS-D, WS-C, WS-B and WS-E up to 108m, respectively. The construction sequence consisted of alternate layers of excavation and installation of struts. Four layers of circular ring beam were installed within the circular pit as strutting system for the excavation from RL 115.5m to RL 101.5m. After excavating to the desired level, the pile cap and tower footing were constructed, 4th ring at RL 104m was removed, and backfilled up to RL 107.1m. Similarly, the excavation at WS-D was carried out to RL 108m with 2 levels of strut at corners and 2 levels of soil anchors inclined at an angle of 10° downward into the ground near WS-C. WS-C has only one level of strut at RL 110m as the excavation depth is shallower that other zones due to sloping ground. Note that, in actual construction, the soil anchors were replaced by raker system. Areas WS-B and WS-E both have two levels of corner struts at RL 113m and RL 110m, and RL 115m and RL 111m, respectively. Total pore pressures boundary conditions were set to zero on each exposed faces after excavation to represent a dried excavation pit. A surcharge of 2kPa was applied to each slope cuts as to represent the 10mm thick lean concrete.

All the stages were modeled with 5 load increments to account for nonlinear soil behavior except for the final excavation stage of WS-E, which was modeled with 20 load increments. This was decided to reduce the out-of-balance loads redistribution by the Newton-Raphson method resulting from equilibrating the external and internal forces. This gives a total of 221 load increments including in situ stress computation.

As linear elastic model was used for structural elements and Mohr-Coulomb model with associated flow rule was used for all soil types, the coefficient matrix *A* (Eq. 1) is symmetric indefinite. Hence, the symmetric quasi-minimal residual (SQMR) solver (Freund and Nachtigal 1994) was used in conjunction with M_1 and M_2 preconditioners for the solution. The solution with M_2-SQMR was completed in 48 hours and 11 minutes. Thus, the average solution time for each load increment was 13.1 minutes only. However, the average time for M_1-SQMR was about 20 minutes for each load increment. This is considerably faster, given the size and complexity of the problem. The solution was carried out on DELL XPS 8300 Intel® Core™ i7-2600 CPU @ 3.40GHz with 16GB RAM. Note that, there is no memory issue for the same sized problem on a PC with 8GB RAM as well. This shows that the large-scale simulations involving materials of strongly varying material properties are feasible for routine geotechnical analyses using above solvers in GeoFEA.

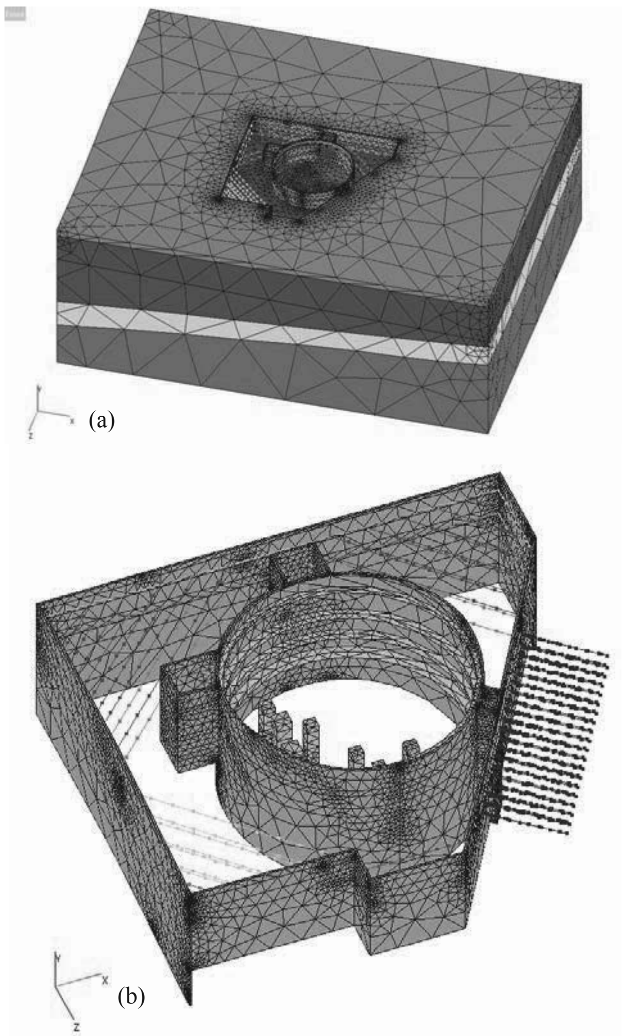

Figure 2. Finite element mesh: (a) Overall geometry, and (b) Structural elements and strutting system

2.3 *Two-Dimensional Idealization*

Various types of idealizations are frequently made in the finite element analysis of many geotechnical problems in order to simplify the analysis. However, geometric idealization is often situational and less amenable to generalization compared to other idealizations such as numerical or material (Lee, 2005). The studied problem was also analyzed with two-dimensional plane strain and axisymmetric analyses along a section A-A, as shown in Figure 1, to compare the outputs.

2.4 *Comparison with Field Measurements*

Figure 3 shows the computed and measured wall deflection after the excavation of all zones was completed. The measured wall deflection profiles were obtained from inclinometers installed just behind the walls. The computed deflections are taken from the sheet pile sections around the locations of the inclinometers. As shown in Figure 3, the computed deflection profile from plane strain analysis is too far off from the measured profile, indicating its limitations for such geometries. The deflection profile from axisymmetric analysis is somewhat closer to the measured profile at section 2. However, the monitoring team has concluded that the localized large deflection at section 2 could be due to the presence of heavy vehicles park beside the location of the inclinometer near that section. Considering the uncertainties and complexities involved in the actual construction as well as in the analysis, the computed deflection profiles from 3D analysis and measured deflection profiles are in reasonable agreement. The 3D analysis (Figure 4) also shows that, that zone near section 2 is more critical in terms of deflection and more attention is required. In addition, the 3D analysis provides different deflection profiles at different locations, such as corner effects, which is difficult to achieve with simple idealizations.

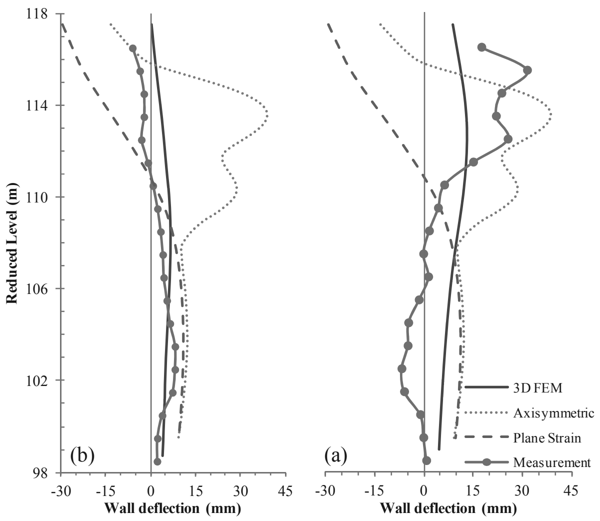

Figure 3. Deflection profiles of sheet pile wall: (a) at section 1, and (b) at section 2. Section 1 and 2 are as shown in Figure 1.

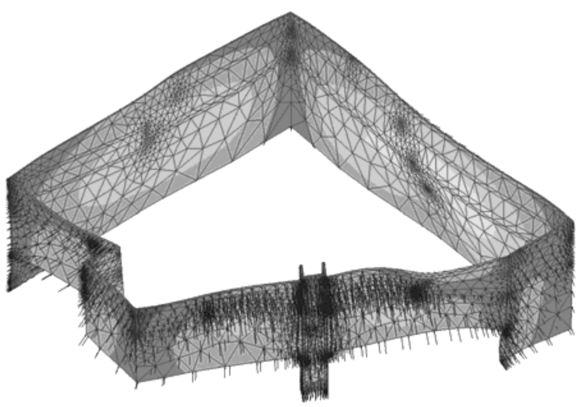

Figure 4. Deformed shape of the sheet pile wall with deflection vectors. (Deformed shape is scaled to 200 times).

3 CONCLUSION

The latest developments in preconditioning has led to significant improvement in computational times of iterative solvers and open up exciting possibilities of conducting large-scale 3D analyses on Desktop PCs. This will be helpful in simulating complex geotechnical problems with strongly varying material properties as simple idealizations may not be sufficient in many cases.

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