Using 3D numerical solutions for the simplified modelling of interaction of soil and elongated structures

Utilisation de solutions 3D numériques pour la modélisation simplifiée de l'interaction des sols et des structures allongées

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ABSTRACT: The problem of interaction of linearly deformable structure and linearly deformable soil is stated in a general form and then defined more precisely for an elongated structure that is rigid in transversal direction. Both loads acting on the structure and on the soil outside the structure (induced by of surface and/or subsurface construction, geological processes etc.) are considered. Numerical method for solution of corresponding equations is developed based on Galerkin boundary elements and numerically implemented. Examples of concentrated load and tunnelling effects on beam-like structure resting on half-space are considered.

RÉSUMÉ : Le problème de l'interaction d'une structure déformable linéaire et d'un sol linéaire est posé sous une forme générale puis défini plus précisément pour une structure allongée rigide dans le sens transversal. Les charges agissant sur la structure et sur le sol à l'extérieur de la structure (induite par une construction en surface ou en souterrain, par des processus géologiques, etc) sont considérées. Une méthode numérique pour la solution des équations correspondantes est développée sur la base des éléments de frontière de Galerkin et mise en œuvre numériquement. Des exemples de charge concentrée et d'effets dus au creusement de tunnels sont étudiées pour des structures assimilables à une poutre reposant sur un demi-espace.

KEYWORDS: half-space, deformable structure, soil-structure interaction, tunnelling effects, boundary elements, Galerkin method.

1 INTRODUCTION

Development of methods of soil-structure interaction with most adequate simulation of real conditions is one of the most important of research in structural mechanics and soil mechanics. The extensive literature and review of some problems may be found elsewhere (e.g., Gorbunov-Posadov e.a. 1984).

In recent years the researchers' attention is increasingly attracted to the study of soil mass effect due to natural or man-induced processes on above-surface and sub-surface structures. In such problems it is usually impossible to be restricted to conventional idealizations.

At the same time it is possible to choose a class of structures with elongated zone of contact with soil, when one zone dimension is significantly less than another: buildings with strip foundations, underground pipelines, transportation tunnels etc. Three-dimensional soil-structure interaction analysis in these cases may be simplified.

In such a way, analysis of solution for a beam on a half-space under concentrated load (Biot 1937) has led to the Winkler model that was used for calculation of beams on soil surface and pipes within it (Vesic 1961, Attewell e.a. 1986). Corresponding model has some known disadvantages and needs some development. Comparison of Winkler and half-space models for elongated structures was performed in the papers (Klar 2004, Fischer and Gamsjäger 2008).

At present time finite element is widely used for solving the problems of soil-structure interaction. However its application in case of domain with length and breadth of different orders of magnitude encounters additional difficulties in course of numerical implementation.

As a consequence, in case of elongated contact zones of deformable structures interacting with soil continuum another approach is needed that allows for geometrical features of the problems and makes it possible to develop a numerical calculation algorithm simple and providing sufficient accuracy.

2 SOIL-STRUCTURE INTERACTIOn

2.1 *Problem statement*

2.1.1 *The soil*

A problem of interaction between structure and linearly deformable soil is considered in discrete or continuum statement. In many instances, for example for tunnels and pipelines, the pressure exerted on soil is of small or moderate level; that makes possible disregarding nonlinearity. Effects both on structure and soil are permitted. The general form of the flexibility method for the linearly deformable soil is the following:

$$\hat{w}=\hat{C}\hat{p}+\hat{w}^*, \qquad (1)$$

where $\hat{w}$ = vector of displacements in the contact zone; $\hat{C}$ = flexibility matrix in discrete case or – corresponding operator in continuum case; $\hat{p}$ = vector of loads on the ground in the contact zone; $\hat{w}^*$ = vector of displacements in the contact zone due to the forces exerted on soil outside the structure under consideration.

The last value is non-zero when other structures are present or some geological processes are developing; it is supposed that corresponding loads on do not depend on the presence of the structure considered, i.e. back effect does not take place.

2.1.2 *The structure*

The .structure is supposed linearly deformable too. The equations of the stiffness method for it read

$$-p=Kw+p^*, \qquad (2)$$

where p = vector of loads transferred to the structure from its (strip) footing; K = matrix (or operator) of the stiffness of the structure, reduced to the nodes of footing axis; w = vector of displacements for the nodes at the footing axes; p^* = vector of loads transferred to the footing from the structure (dead loads, live loads on the floors, wind loads etc.).

2.1.3 *The footing*

Elongated footing may be supposed rigid in the transversal direction. It gives the possibility to express the displacements of the contact zone via the displacements of the footing giving the following expression:

$$\hat{w} = Aw \ . \tag{3}$$

The loads on the footings transferred from the soil are summed up according to the formula

$$p=B\hat{p}. \tag{4}$$

2.2 *General system of equation*

Eq. 1 and Eq. 3 give:

$$-\hat{C}\hat{p}+Aw=w^*, \tag{5}$$

while Eq. 2 and Eq. 4 give:

$$B\hat{p}+Kw=-p^*, \tag{6}$$

In further consideration system of Eq. 5 and Eq. 6 will be given concrete expression.

3 SOIL MODELLED BY LINEARLY DEFORMABLE HOMOGENEOUS HALF-SPACE

3.1 *Model substantiation and Galerkin method*

Small breadth of soil-structure contact zone leads to small depth of deformable soil layer that allows considering the soil mass as homogeneous continuum. Hence the computational domain may be supposed homogeneous half-space; its linearity was supposed earlier. Only normal loads on its surface are considered; tangential loads are zero.

Earlier one of the authors obtained (Kholmyansky 2007) the solution for three-dimensional problem about the system of rigid punches on the half-space obtained with the boundary element method and using Boussinesq solution.

Two specific variants of general numerical method of weighted residuals where compared: collocation method and Galerkin method (Finlayson 1972); the latter showed higher accuracy and was chosen for further work.

Efficiency of that approach was illustrated by the fact that the equilibrium of several hundreds of punches was considered without difficulties. This paper continues to use that approach for the discretization of Eq. 5.

3.2 *Operator discretization*

For discretization of the operator $\hat{C}$, that describes the flexibility of deformable foundation the simplest piecewise-constant basis functions are chosen. The footing-soil contact zone Ω divided into n boundary elements Ω_j.

Each basis function corresponds to boundary element; the function is unity for the points of the element and zero outside. That makes the pressure field piecewise-constant and equal to linear superposition of basis functions N_j:

$$\hat{p} = \sum_{j=1}^{n} p_j N_j \ , \tag{7}$$

where p_j = specific normal load on Ω_j.

As a consequence Galerkin method provides instead of Eq. 5 its discrete form

$$-\sum_{j=1}^{n} \frac{1}{m_i} c_{ij} p_j + a_j(Aw) = a_j(w^*)\,, \tag{8}$$

where $a_j(u)$ = average u over the boundary element Ω_j; m_j — area of Ω_j;

$$\begin{aligned} c_{ij} &= \left[N_i, \hat{C}N_j\right] = \iint_{\Omega} N_i(x,y)(\hat{C}N_j)(x,y)dxdy \\ &= \iint_{\Omega_i} N_i(x,y)(\hat{C}N_j)(x,y)dxdy \end{aligned} \ ; \tag{9}$$

[… , …] = scalar product of the two functions expressed by integral of their product over Ω.

If Ω_i — a rectangle ($x_1 \le x \le x_2$, $y_1 \le y \le y_2$), then in Eq. 9

$$\begin{aligned} \hat{C}N_i = \ & F(x_2 - x, y_2 - y) - F(x_2 - x, y_1 - y) \\ & - F(x_1 - x, y_2 - y) + F(x_1 - x, y_1 - y) \end{aligned} \ ; \tag{10}$$

$F(a,b)$ = displacement of the point of origin under the action of unit load on the rectangle with the abscissas of its corner points $x = 0$ and $x = a$ and with ordinates $y = 0$ and $y = b$:

$$F(a,b) = \frac{1-\nu^2}{E} \int_0^a \int_0^b (x^2 + y^2)^{1/2} dxdy \ ; \tag{11}$$

E, ν = deformation modulus and Poisson ratio of soil. The formula obtained is a form of the well known method of computation of half-space surface settlement by the superposition of rectangular loads (see Terzhagi 1943). Another form (Kholmyansky 2007) of well-known expression (see Terzhagi 1943) for $F(a,b)$ was obtained:

$$F(a,b) = \frac{1-\nu^2}{E} \left\{ |a| \operatorname{Arsh}(b/a) + |b| \operatorname{Arsh}(a/b) \right\}; \tag{12}$$

From this point on only uniform rectangular grids of boundary elements are considered. Integration for the computation of scalar product in Eq. 9 is performed numerically with the Gauss 2×2 cubature formula Гаусса.

The main difference of the described discrete method from the well known Zhemochkin method (Zhemochkin and Sinitsyn 1947) is the fact that the Galerkin approach is used instead of collocation approach..

4 STRUCTURE MODELLED BY A BEAM

In case of narrow and stiff in transversal direction footing the displacements of points in contact zone $\hat{w}$ are determined by the displacements on the footing axis; elements of matrix A are units and zeros. Elements of matrix B are units and zeros too; matrix B is transposed to A:

$$B = A^T \, . \tag{13}$$

The structure model is supposed to be the Bernoulli-Euler beam. For the discretization of the well known ordinary differential equation of beam bending and computation of stiffness matrix K the method of real finite elements was applied (Karamansky 1981).

The essence of this method consists in concentration of bending ability in discrete points. If the beam with bending stiffness E_1I_1 is decomposed to the parts of length Δl the bending stiffness at each point is

$$r = E_1 I_1 / \Delta l . \qquad (14)$$

The stiffness matrix for a beam with free ends

$$K = \frac{r}{\Delta l^2}\begin{bmatrix} 1 & -2 & 1 & 0 & 0 & \dots \\ -2 & 5 & -4 & 1 & 0 & \dots \\ 1 & -4 & 6 & -4 & 1 & \dots \\ 0 & 1 & -4 & 6 & -4 & \dots \\ 0 & 0 & 1 & -4 & 6 & \dots \\ & & \dots & & & \end{bmatrix} . \qquad (15)$$

5 CALCULATION OF BEAMS ON ELASTIC HALF-SPACE

5.1 *Concentrated load*

The beam with plan dimensions 3×51 resting on the half-space with unit concentrated force in the centerpoint was calculated . Different beam stiffness values were considered; they were described by the flexibility index (Gorbunov-Posadov e.a. 1984):

$$t = \frac{\pi E a^3 b}{2(1-\nu^2)E_1 I_1} , \qquad (16)$$

where a, b = half-length and half-breadth of the beam. Contact zone boundary element decomposition was to unit squares. The calculation results are shown on Figure. 1 and demonstrate the effect of the beam stiffness in the longitudinal direction: the pressure diagram shows changes from the rigid punch type for $t = 10^{-3}$ to alternating-sign type for $t = 10^3$; for the intermediate value $t = 1$ there are maxima both in the center of the beam under the concentrated load and at the ends.

5.2 *Concentrated load*

For the calculation of bending of the beam that models an elongated structure we accept formula (Attewell e.a. 1986) for the settlement of the of the soil surface due to tunnelling with the account for the position of the tunnel face (the structure back effect on the structure is neglected):

$$\hat{w}^* = \frac{V_s}{\sqrt{2\pi} I} e^{\frac{-\eta^2}{2I^2}} \left\{ G\left(\frac{\xi - x_i}{I}\right) - G\left(\frac{\xi - x_f}{I}\right) \right\} , \qquad (17)$$

where V_s = volume of lost ground; $2I$ = width of the settlement trough between the inflection points; ξ, η = coordinates (ξ axis = projection of the tunnel axis on the soil surface, η axis is perpendicular to ξ axis); x_i and x_f — ξ coordinates of the tunnel initial and final points;

$$G(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha} e^{\frac{-\beta^2}{2}} d\beta = \frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{\alpha}{\sqrt{2}}\right) . \qquad (18)$$

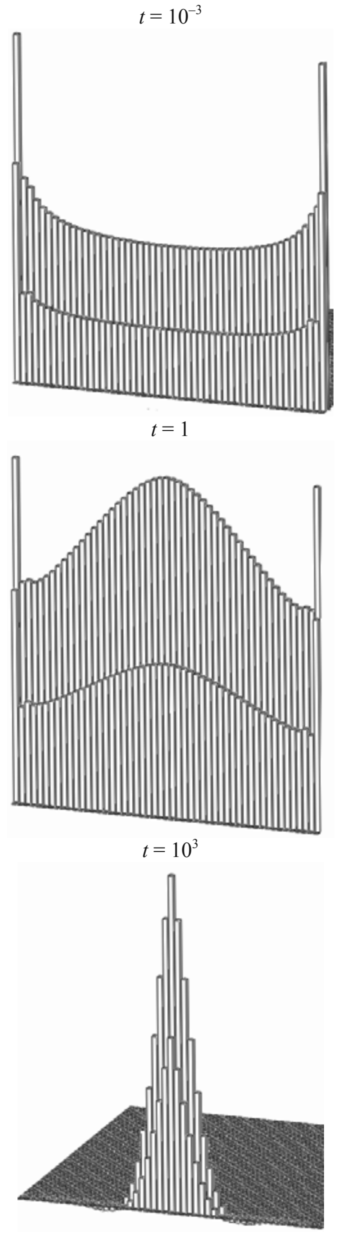

Figure 1. Contact pressure under the footing of the beam on the half-space with concentrated load for flexibility index $t = 10^{-3}$; 1; 10^3. The part of the diagram symmetrical about the axis of the structure is not shown.

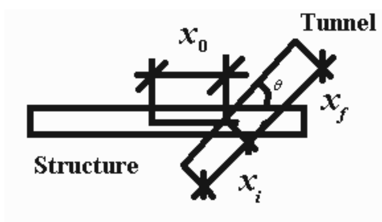

Figure 2. Relative position of the tunnel and the structure.

Consider different tunnel positions relative to the existing elongated structure, influenced by the tunnelling. The structure is the same as in the previous example; flexibility index is 1.

The half-width of the settlement trough is taken equal to the half-length of the structure. The general layout scheme of the structure and the tunnel with the corresponding parameters, describing their mutual position is shown on the Figure 2.

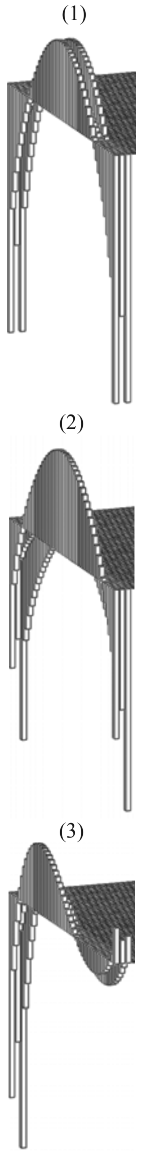

Рис. 3. Contact pressure due to tunnelling under the footing of the beam with flexibility index $t = 1$ on the half-space

Results are obtained for three cases (see Figure 3):

(1) Symmetrical layout with long tunnel perpendicular to the longitudinal axis of the structure: $x_0 = 0$; $x_i = 1000$; $x_f = 1000$; $\theta = 90º$.

(2) Tunnelling up to the structure axis perpendicular to the longitudinal axis of the structure. Parameters are the same as in case (1) except $x_f = 0$.

(3) Tunnelling with the angle $\theta = 60º$ to the longitudinal axis of the structure up to the end of the structure: $x_0 = 0$; $x_i = 25.5$; $x_f = -10$.

In all the cases there are no effects on the structure except soil induced effects, so contact stresses are self-equilibrated. In the case (1) there are two axes of symmetry, in case (2) — only one.

A similar (from the geomechanical viewpoint) approach to calculation of beam-like structures on soil deformed by subsurface works was developed in (Pushilin and Sheynin 2006), where the model was confined to the planar case. This approach allows allows to write down an ordinary differential equation for the beam deflection and formulate a finite-difference algorithm of its solution. Stresses in a beam-like structure on Winkler foundation whos deformations are induced by a nearby excavation are determined in, e.g. (Ilyichev e.a. 2006).

6 CONCLUSION

A numerical method was developed for the solution of soil-structure interaction problems of elongated deformable structures (rigid in transversal direction) on linear half-space. It was supposed that soil is deformed under the effect of additional outer sources, e.g. tunnelling. Numerical results were obtained for the most practically typical special case, when the structure may be modeled by a beam with finite bending stiffness in longitudinal direction and infinitely rigid and of finite breadth in transversal direction. The cases of load application both on the structure and on the soil continuum due to tunnelling were considered. Results for different beam stiffness and tunnel-structure layout are obtained and analyzed.

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