Two methods for estimating excess pore pressure in LEM

Deux méthodes pour estimer l'excès de pression interstitielle

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ABSTRACT: Two methods for estimating excess pore pressure in undrained effective stress LEM calculations are presented. The new methods take into account the excess pore pressure from undrained yielding of clay, which traditionally has been difficult to model without the use of FEM. The method ru' calculates yield-induced excess pore pressure based on an initial yield surface. The work-in-progress method MUESA also takes into account factors such as overconsolidation and non-triaxial stress states. The paper also discusses the implicit overestimation of shear strength in undrained effective stress calculations. It is suggested that failure state pore pressure is universally used in the calculations to obtain a theoretically more sound definition of the factor of safety (F). The new definition of F is also compatible with φ = 0 stability calculations. A calculation example is presented.

RÉSUMÉ: Cet article présent deux méthodes pour estimer l'excès de pression interstitielle de porosité dans les calculs LEM d'un espace fermé. Les nouvelles méthodes tiennent compte de l'excédent de pression interstitielle dans la production non drainée de l'argile, ce qui a toujours été difficile à modéliser sans l'utilisation d'éléments finis. La méthode ru' calcule le rendement induit par excès de pression interstitielle reposant sur une surface d'élasticité initiale. Le méthode de travail MUESA en cours tient également compte des facteurs tels que la surconsolidation ainsi que les états de contrainte non triaxiaux. Le document étudie également la surestimation implicite de la résistance au cisaillement non drainé dans les calculs de contraintes effectives. Il est suggéré que l'état des pressions interstitielles défaillantes est universellement utilisé dans les calculs dans le but d'obtenir une définition théoriquement plus solide du facteur de sécurité (F). La nouvelle définition de F est également compatible avec les calculs de stabilité φ = 0. L'article propose un exemple de calcul.

KEYWORDS: stability, pore pressure, modeling, LEM, clay, embankment

1 INTRODUCTION

In this paper two methods of modeling excess pore pressure in undrained effective stress LEM calculations are presented. The theoretical grounds leading to an overestimation of shear strength are discussed.

In Finnish practice embankment stability on clayey subsoils is commonly calculated using undrained shear strength (s_u) measured with vane shear testing. This often results in unrealistically low factors of safety, which underestimates the safety and causes problems in directing stability improvement measures. The problem mostly lies in uncertainties in determining of s_u.

In undrained effective stress analyses the uncertainty in parameters is generally smaller, but the problem lies in the estimation of pore pressure at failure state. In limit equilibrium (LEM) calculations the yield induced pore pressure is usually totally disregarded, which then results in an overestimation of safety. In finite element (FEM) analysis yield induced pore pressure is possible to account for, but it requires the use of quite advanced soil models and proper understanding of the influence of model parameters on the development of pore pressure.

It is thus recognized that there is a need for effective engineering tools that account for the yield induced pore pressure in a simple LEM calculation.

2 THEORETICAL BACKGROUND

The excess pore pressure response of soft clay is a complex process with many affecting factors such as the stress history of the clay, rate and direction of loading, structural anisotropy etc.

The excess pore pressure response of clay can (conceptually) be divided into two components Δp and Δp', which represent changes in total mean stress and effective mean stress, respectively (Figure 1).

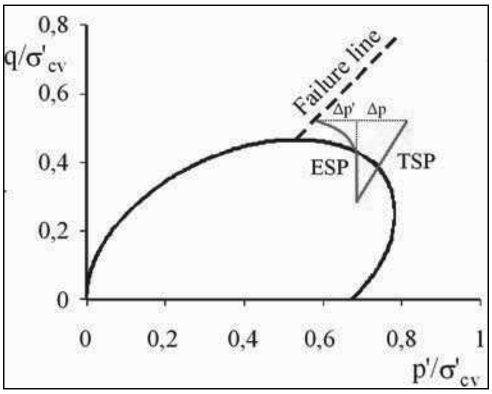

Figure 1. Components of excess pore pressure in (p', q) space.

The total stress component Δp represents excess pore pressure caused by a change in the total stress state of the soil element. As the clay is in undrained state and the total volumetric strains equal to zero (assuming isotropic elasticity), the excess pore pressure component is equal to change in total mean stress Δp.

When clay is yielding in the undrained state there will also be a pore pressure component Δp' that is caused by the reorganizing of the clay structure. Normally consolidated or lightly overconsolidated soft clays usually have a tendency for volumetric compression while yielding. In the undrained state this must be offset by excess pore pressure (again due to the assumption of zero volumetric strains).

In constitutive soil models it is often assumed that for overconsolidated clays the stress-strain response is elastic, i.e.

p' is constant in undrained loading. The behavior changes when the initial yield locus is reached and the clay begins to yield. The direction of the undrained stress path is then determined by the tendency of the clay fabric to compress or dilate. The direction and shape of the stress path (and at the same note, the amount of excess pore pressure) depends on several factors such as the shape and size of the initial yield surface, initial stress state and the rate of loading.

According to Länsivaara (1999) a high strain rate results in less excess pore pressure than a comparably lower strain rate due to effects related to undrained creep. The result of this behavior is that high strain rates result in higher undrained shear strength than comparably lower rates.

2.1 *Failure pore pressure in LEM*

In conventional undrained c'-φ' calculations there is an inherent overestimation of shear strength when for any factor of safety F > 1. This is caused by the fact that for a given loading the mobilized excess pore pressure is used in calculating the equilibriums and shear strength. This approach disregards the further increase of excess pore pressure between the mobilized stress state and failure (which is not a problem in drained calculations).

Shear stress is traditionally defined in LEM as:

$$F = \frac{\tau_f}{\tau_e} \tag{1}$$

where τ_e is the equilibrium shear stress, τ_f the corresponding shear strength and F the factor of safety. This definition implies a very specific stress path that is highly unrealistic in undrained loading, for example under an embankment. (Tavenas et al 1980).

The corresponding definition of shear stress effectively compares the mobilized shear stress τ_e to an unrealistically large strength τ_f that can never be attained because of further pore pressure buildup between the mobilized state and failure (Figure 2). This pore pressure behavior is common for soft normally consolidated or slightly overconsolidated clays that generally exhibit compressive behavior.

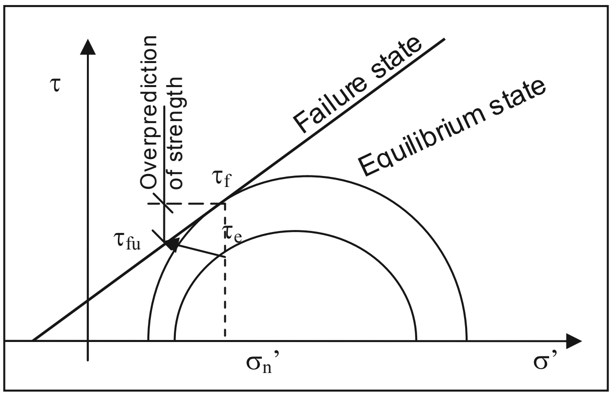

Figure 1. Overprediction of shear strength in typical undrained effective stress calculations. (Länsivaara et al 2011)

Consequently the definition of the factor of safety in undrained c'-φ' calculations is actually different from the factor of safety in φ = 0 calculations. In φ = 0 calculations the mobilized shear stress τ_e is always compared to a value of s_u that represents the shear strength τ_{fe} that can actually be mobilized for a given set of physical conditions (consolidation, rate and direction of shearing etc). The two otherwise conflicting definitions coincide at F = 1. As a conventional undrained c'-φ calculation for soft clays will overestimate shear strength when F > 1, one cannot even in theory expect the same result from a corresponding φ = 0 calculation (Leroueil et al 1990).

In light of this issue it is not enough just to accurately model the mobilized excess pore pressure, but the implicit overestimation of the factor of safety should also be taken into account if possible. At the least, the designer needs to be aware of these theoretical differences between the two methods and interpret the results accordingly.

A proposed workaround for the overestimation of shear strength is to universally use failure state pore pressure in the calculation even for F > 1, regardless of the actual mobilized pore pressure. This can be achieved in LEM if the effective stress path from the initial state to failure (and the corresponding excess pore pressure) can be approximated. In (σ'_n, τ) stress space this assumption places the effective normal stress to its value at failure, thus giving the ability to compare the shear stress with the shear strength at failure.

Since τ and F are co-dependent in LEM the mobilized shear stress τ in the "failure pore pressure formulation" will be slightly different from the corresponding "traditional" pore pressure formulation. Whether the difference will be positive or negative depends on several factors. According to studies with the method "MUESA" (section 2.3) the induced error on the factor of safety is small when compared to the overestimation of shear strength caused by the traditional use of mobilized pore pressure.

What the use of failure pore pressure does is effectively to substitute an "incorrect" effective normal stress to obtain a more realistic calculated shear strength. In LEM this can be considered an acceptable tradeoff as the main purpose of LEM is to obtain the factor of safety. If applied correctly, the shear strength and thus the factor of safety will have a more realistic value for F > 1 (when compared to the traditional approach of using mobilized pore pressure). At F = 1 the two different approaches coincide.

Two calculation methods for modeling the amount of yield-induced pore pressure in undrained effective stress LEM calculations are proposed in the following section. Both methods employ an anisotropic yield surface to describe the change of effective mean stress when clay is loaded to failure.

2.2 *Method 1: r_u'*

The method r_u' was developed for normally consolidated clays. It should be considered as a simple engineering tool to model yield-induced pore pressured for stability calculations of old embankments.

Finnish soft clays are usually only very slightly overconsolidated, mainly due to aging. Under old embankments the clays have generally become normally consolidated.

If failure occurs, excess pore pressure will thus have developed corresponding to a stress change from the initial in situ state at the K_{0NC} line to the failure state. The most critical event corresponds to slow loading or long loading time allowing for the yield induced pore pressure to develop. Excess pore pressure from yielding can now be simply calculated as the horizontal difference of the intersection of K_0-line with the initial yield surface, and the intersection of the failure line with the yield surface in the (p', q) stress space. For this a proper estimation of the yield surface is needed. It has been shown (Länsivaara 1995, Länsivaara 1999) that the initial yield surface can be estimated by knowing only the friction angle and the preconsolidation pressure of the clay.

This can further be utilized by applying a pore pressure parameter similar to the generally used r_u, with the exception that it now stands for yield induced pore pressure and should be applied to effective vertical stress. This pore pressure parameter is referred as r_u' and is defined as:

$$r_u' = \frac{u_{ey}}{\sigma_{v0}'} = \frac{1}{\sigma_{v0}'} f(\sigma_{v0}', \varphi) \tag{2}$$

where u_{ey} = yield induced excess pore pressure. The graphical solution for r_u' (Länsivaara 2010) is shown in Figure 3.

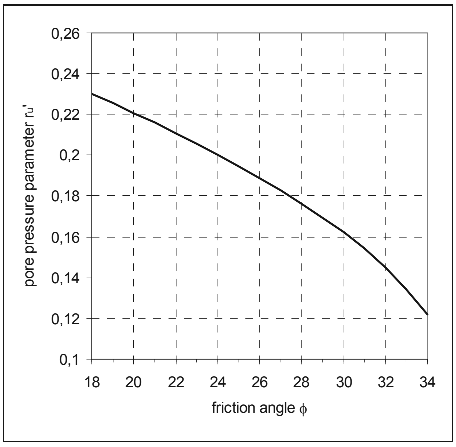

Figure 2. Effective stress pore pressure parameter r_u' as function of friction angle (Länsivaara 2010). The solution is valid for normally consolidated (K_0) clays.

This simple method is strictly valid only for active loading, and in the passive part of the failure envelope the pore pressure increase would be higher. However, as discussed by Länsivaara (2010) this error is compensated by the fact that next to the embankment the soil is at least slightly overconsolidated, which in turn leads to proportionally lower excess pore pressure.

2.3 *Method 2: MUESA*

The calculation method "MUESA" (Modified Undrained Effective Stress Analysis) is partly derived from the method "UESA" proposed by Svanø (1981). MUESA accounts for anisotropy, non-triaxial stress states and overconsolidation

In MUESA the amount of excess pore pressure is calculated based on stress changes in relation to the initial stress state (before the start of undrained loading). Excess pore pressure Δu is expressed as:

$$\Delta u = \Delta p - \Delta p' \qquad (3)$$

For a given slip surface, an initial stress state along a slip surface is calculated assuming K_0-conditions. This assumption is not very accurate for slopes but is reasonably valid for embankments on nearly horizontal soil. The initial stress state is defined as the state before undrained loading, such as a traffic embankment without external loading. The embankment with traffic load applied would then be the design stress state.

An initial assumption for the pore pressure (e.g. ground water + excess pore pressure) acting on the bottom of each slice is made. The limit equilibrium is then calculated in regular fashion (with the loading in place), using this initial pore pressure assumption. The desired LE method (e.g. Morgenstern-Price, Janbu's simplified etc.) can be used. The stress state (σ'_n, τ) resulting from the equilibrium is used to calculate the next assumption for Δu. The process continues iteratively until pore pressure converges. The value of the initial assumption (within realism) has no effect on the final result but good assumptions lead to fast convergence.

To calculate excess pore pressure its two components Δp and $\Delta p'$ need to be calculated. The component Δp can be calculated by using basic principles of continuum mechanics and the assumption of the Mohr-Coulomb failure criterion. The two compared stress points are the initial total mean stress p_0 and either mobilized or failure total mean stress (p_{mob} or p_f).

A three-dimensional stress space (with three principal stresses) is used to determine Δp so that non-triaxial stress states can also be considered ($p = p(\sigma_1, \sigma_2, \sigma_3)$). The principal stresses are easily derived using basic continuum mechanics.

The effective stress component $\Delta p'$ is derived from the yield surface formulation of the constitutive soil model S-CLAY1 (Wheeler et al 2003). S-CLAY1 is in good agreement with tests done on soft, lightly overconsolidated clays. The only parameters needed to define the initial yield surface are the friction angle φ' and the vertical consolidation pressure σ'_c.

To obtain $\Delta p'$ the effective mean stress at failure p'_f needs to be calculated. In the current formulation of MUESA it is assumed that the stress path follows the initial yield surface in the normally consolidated state (no volumetric hardening), and the stress path terminates at the intersection of the yield surface and the failure line. The assumption of no volumetric hardening can be regarded as the absolute maximum for the amount of excess pore pressure and minimum of shear strength and the calculation is thus on the safe side. The effective mean stress at failure p'_f is solved as the intersection of the S-CLAY1 yield surface and the Drucker–Prager failure surface in the principal stress space (σ_1, σ_2, σ_3).

In MUESA the use of failure pore pressure is considered as described in Section 2.1. The use of failure pore pressure is fairly simple as only the initial and failure stress states need to be considered, and the actual stress path in between can be disregarded. The method seems to be most sensitive to assumptions regarding anisotropy, especially in the passive end of the slip surface. Hardening will be implemented in the future.

3 CALCULATION EXAMPLE

A calculation example is a test embankment from Salo, Finland, where a full-scale railway embankment failure experiment was carried out in 2009 (Lehtonen 2011). An embankment on sensitive clay soil was quickly brought to failure, simulating a very heavy train coming to a standstill. Extensive pore pressure measurements were continuously conducted.

Table 1. Soil properties in the example.

Soil layers	γ [kN/m^3]	φ' [°]	c' [kPa]	s_u [kPa]	ds_u [kPa/m]	POP [kPa]	r_u'
Emb.	20	38	0				
Sand fill	19	35	0				
Dry crust	17			30			
Clay	15	25	0	12	1.5	20	0.2

The soil conditions and calculation parameters are give in Table 1. A small embankment is loaded with a 2.5 m wide rain load. The subsoil consists of a fill layer of sand, dry crust and a soft, slightly overconsolidated clay layer (POP = 20 kPa). Under the soft clay there are layers of clayey silt and moraine, but these are disregarded here as the slip surface is not located in them. Ground water level is near the bottom of the dry crust.

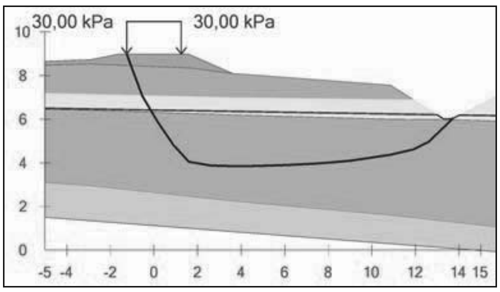

Figure 3. Soil geometry and slip surface used in the example. Soil layers from top down are embankment, sand fill, dry crust, clay and clayey silt.

A same slip surface that approximates the actual failure geometry in the experiment was used for all calculations (Figure 4). All calculations were made with the Morgenstern-Price method with interslice force function f(x) = sin (x).

In the r_u' calculation additional excess pore pressure from the loading was modeled to ensure realistic shear strength in the loaded slices. The MUESA calculation modeled the overconsolidation POP = 20 kPa in the clay layer, while the r_u' calculations assumed normally consolidated conditions as described in Section 2.1. In addition, a $\varphi = 0$ calculation was made using s_u values measured with vane shear testing.

Table 2 compares the factor of safety at 30 kPa external load, as well as the magnitude of the failure load.

It is seen that results (Table 2) with both new methods are plausible and in fairly good agreement both with the results of the $\varphi = 0$ calculation and actual observations from the experiment. It must however be noted that the $\varphi = 0$ calculation is still subject to inaccuracies and uncertainties of its own, but here the extensive soil investigations reduce the uncertainties involved.

Table 2. Calculation results.

	F for 30 kPa load	Failure load [kPa]
r_u'	1,43	74
MUESA	1,37	64
s_u	1,48	71

The calculated excess pore pressure levels (Figure 5) are reasonably close to what was measured at the site before failure (taking into account how LEM does not distribute excess loading). At failure excess pore pressures of 10...15 kPa were measured on the centre part and passive end of the failure zone, while much higher pressures were measured under the embankment.

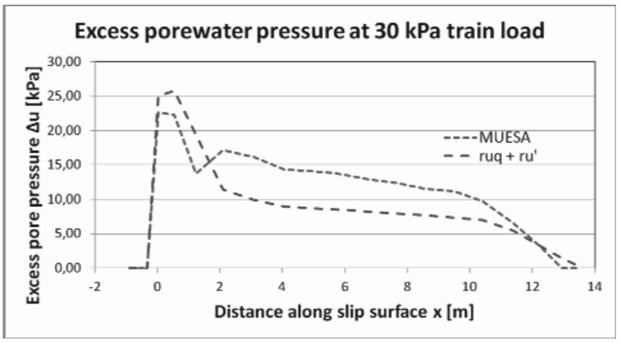

Figure 4. Calculated excess pore pressure levels for load q = 30 kPa

The test embankment eventually failed at a load of 87 kPa. Many factors lead to this high failure load, one of the most significant being the time-dependency of pore pressure increase. It has been estimated that the failure could well have occurred at a load of ca. 70 kPa, if enough time would have been given for pore pressure to develop at that load level.

MUESA in its current development state has a tendency to overestimate the excess pore pressure because volumetric hardening is not accounted for. This will be rectified in upcoming versions.

4 CONCLUSIONS

In this paper a concept of universally using failure pore pressure in undrained effective stress calculations is proposed. Two calculation methods for modeling excess pore pressure in undrained effective stress stability calculations are presented. The methods are intended as simple and effective calculation tools for basic design purposes.

The method r_u' offers a simple pore pressure parameter for modeling yield-induced excess pore pressure at failure.

The method MUESA is used to take into account the various factors that affect excess pore pressure in undrained c'-φ' calculations. It also makes it possible to universally use failure pore pressure. This makes the calculation theoretically comparable to $\varphi = 0$ calculations. MUESA is still in development stage, and will be refined further.

These new calculation methods offer an analytical approach to modeling excess pore pressure in LEM. While they cannot take into account all the factors that an ideal FEM calculation could, they can nevertheless be considered very useful and effective engineering tools.

5 ACKNOWLEDGEMENT

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