Variation of Friction Angle and Dilatancy For Anisotropic Cohesionless Soils

Variations de l'angle de Frottement et de la Dilatance pour les Sols Anisotropes Sans Cohésion

Cinicioglu O., Abadkon A., Altunbas A., Abzal M.
Bogazici University, Istanbul, Turkey

ABSTRACT: The goal of this paper is to investigate and quantify the variation of peak friction and dilatancy angles of anisotropic cohesionless soils as functions of the in-situ state of the soil. In this context, in-situ state of the soil is used as a broad term that encompasses the combined effects of the stress state, volumetric state, and stress history of the soil prior to any shearing. Accordingly, the parameters that define the in-situ state of soil are in-situ confining pressure (p'_i), relative density (I_D) and overconsolidation ratio (OCR), respectively. In order to quantify the influences of these parameters on the peak friction angle and dilatancy angle, a special testing program was designed that employs mainly CK_oD triaxial tests. These tests were conducted on reconstituted sand samples at different p'_i-I_D-OCR combinations. Analyzing the obtained results, two new functions are proposed that allow the calculation of the peak friction angle and dilatancy angle of anisotropic cohesionless soils. The greatest advantage of the proposed functions is that they use directly measurable or calculable parameters as input. Finally, using similar test data collected from literature, the proposed empirical equations are validated.

RÉSUMÉ : Le but de cette étude est de chercher et de quantifier les variations des angles de frottement maximum et de dilatance de sols anisotropes sans cohésion comme des fonctions de l'état in-situ du sol. Dans ce contexte, l'état in-situ du sol est utilisé comme un terme général qui entoure les effets combinés de l'état de contrainte, l'état volumétrique, et l'histoire des contraintes du sol avant tout cisaillement. Par conséquent, les paramètres qui définissent l'état in-situ du sol sont la pression de confinement, la densité relative et le taux de surconsolidation, respectivement. Afin de quantifier les influences de ces paramètres sur l'angle de frottement maximum et l'angle de dilatance, un programme d'essai spécial a été conçu qui emploie principalement des essais triaxiaux. Ces essais ont été effectués sur des échantillons de sable reconstituées selon différentes combinaisons. L'analyse des résultats obtenus conduit à deux nouvelles fonctions qui permettent le calcul de l'angle de frottement maximum et de dilatance de sols anisotropiques sans cohésion.

KEYWORDS: dilatancy, friction angle, sand, K_o-consolidation, granular material

1 INTRODUCTION

Dilatancy is a property that is unique to granular materials. However, for soils, manifestations of dilatancy depends on grain size and shape; In case of fine-grained soils, we can describe dilatancy as latent dilatancy since dilatant behavior manifests itself as a change in the pore water pressure. Though, in case of coarse-grained soils, dilatancy is physically evident and can be directly measured by conducting simple soil tests. Even though for both fine and coarse-grained soils dilatancy influences strength, only for coarse-grained soils it has an effect on the formation of shear planes, thus controlling the geometry of failure mechanisms. Due to this fact, dilatant behavior of coarse-grained soils draws much attention from the academia (Taylor 1948, Rowe 1962, De Josselin de Jong 1976, Bolton 1986, Schanz and Vermeer 1996, Chakraborty and Salgado 2010). Even in the face of this ever-continuing scientific interest in dilatancy, a practical function that renders the quantification of dilatant behavior is yet to emerge. There are milestone works towards understanding dilatant behavior as listed in the references; however the proposed functions are either impractical or conceptual. For example, one of the well-known functions for calculating dilatancy (ψ) was proposed by Bolton (1986):

$$(-d\varepsilon_v/d\varepsilon_1)_{max} = 0.3I_R = 0.3\left(I_D\left[Q - lnp_f'\right] - R\right) \qquad (1)$$

$d\varepsilon_v$ and $d\varepsilon_1$ in Eq. (1) corresponds to the increments of volumetric strain and major principal strain, respectively. I_D is the relative density ranging from 0 to 1 and p'_f is the corresponding mean effective stress at failure. Q and R are empirical fitting parameters whose units are dependent on the unit used for p'_f. Accordingly, I_R is defined as the relative density index which yields p'_f dependent magnitude of I_D. Later Schanz and Vermeer (1996), relying on experimental results, improved Eq. 1:

$$sin\psi = -0.3I_R/(2 + 0.3I_R) = I_R/(6.7 + I_R) \qquad (2)$$

Recently Chakraborty and Salgado (2010) studied the values of the fitting parameters Q and R, especially for low confinement conditions. However, it is clear that the variables of Eq. 1 and Eq. 2 are defined for the moment of soil failure and this approach significantly reduces the practicality of the proposed equations. Hence, the goal of this study is to calculate dilatancy angle using parameters that correspond to the in-situ state of the soil. Previous studies have shown that dilatant behavior is affected by the confinement and compactness of the soil. Accordingly, confinement is defined by confining pressure (p') and compactness is defined by the relative density of the soil (I_D), as is the case in Eq. 1 and Eq. 2. In addition to the confinement and relative density, Vaid and Sasitharan (1991) showed that stress path affects the dilatant behavior. That is why, in this research the most ubiquitous stress path in nature is chosen for sample preparation which is the K_o consolidation. Even though stress path followed during sample preparation stage is confined to K_o consolidation, the influence of stress history is investigated by considering the overconsolidation ratio (OCR) as a third variable. Since all these can be achieved during a triaxial test, the tests conducted were K_o-consolidated and drained triaxial tests (CK_oD). In order to achieve different OCRs, the samples were unloaded under K_o conditions.

In the remainder of this paper, the results of the tests conducted are presented followed by the construction of the dilatancy equations. Following, the proposed equations are validated using data collected from the literature.

2 EXPERIMENTAL STUDY

The experimental approach in this study is to conduct sufficient number CK_oD tests at different p'_i-I_D combinations so that it would be possible to define the p'_i-ψ relationship for every 5% change in I_D. This is achieved for an I_D range within 0.35 to 0.95 by conducting 80 CK_oD tests. It is important to emphasize that p'_i and I_D are the in-situ (before shearing) mean effective stress and relative density values. The sand used in these tests is local sand called Silivri Sand. In order to have the same grain size distribution in all tests, this sand was sieved and prepared with the standard grain size distribution of Ottawa sand (Table 1).

Table 1. Properties of the test sand.

Sand	G_s	C_u	C_c	e_{max}	e_{min}
Silivri sand with Ottawa distribution (SP)	2.67	2.16	1.45	0.96	0.56

Samples were prepared by dry pluviation. Several tests were conducted at different OCRs (1,2,4,8) to consider the influence of unloading on dilation. Overconsolidated samples were unloaded under K_o conditions.

3 TEST RESULTS

3.1 *Dilatancy as a function of p'_i and I_D*

Dilatancy angle is calculated from the test results using the relationship proposed by Schanz and Vermeer (1996).

$$sin\psi = (d\varepsilon_v/d\varepsilon_1)/(2 - [d\varepsilon_v/d\varepsilon_1]) \tag{3}$$

The relationship given in Eq. 3 is preferred since it is specifically developed for triaxial testing conditions. As the goal is to investigate the uncoupled effects p'_i and I_D on ψ, test results are divided into several I_D ranges. In other words, p_i'-ψ relationships are defined separately for each 0.05 increment in I_D (i.e. a single p_i'-ψ relationship is defined for the tests with $0.65 \leq I_D < 0.70$, and this p_i'-ψ relationship is considered to be applicable for I_D=0.675). The reason for choosing the I_D increment to be 0.05 is because this much variation in I_D is within the measurement margin of error. So for each I_D range, the tangents of calculated ψ values (tanψ) are plotted against the corresponding end of consolidation (in-situ) mean effective stresses that are normalized with the atmospheric pressure (p_i'/p_a). The tanψ-(p'_i/p_a) relationships obtained for three different I_D ranges are shown in Figure 1 as examples. For all tanψ-(p_i'/p_a), the relationships that yield the greatest coefficient of determination (R^2) are used.

As it can be observed in Figure 1, tanψ-(p_i'/p_a) can be considered to be approximately a linear relationship. Therefore, it is defined using line equation as follows:

$$tan\psi = \alpha_\psi(p'_i/p_a) + \beta_\psi \tag{4}$$

Here in Eq. 4, α_ψ and β_ψ are unitless fitting parameters. The variations of α_ψ and β_ψ with I_D are plotted in Figure 2. It can be seen that the value of α_ψ is approximately constant and β_ψ varies linearly with I_D. However, in order to propose functions that would be applicable to all soils, α_ψ and β_ψ are defined as linear functions:

$$\alpha_\psi = a_\psi I_D + b_\psi \tag{5}$$

$$\beta_\psi = m_\psi I_D + n_\psi \tag{6}$$

Constants a_ψ, b_ψ, m_ψ, and n_ψ are fitting parameters. As a result, a general equation can be written with the form given below.

$$tan\psi = (a_\psi I_D + b_\psi)(p'_i/p_a) + (m_\psi I_D + n_\psi) \tag{7}$$

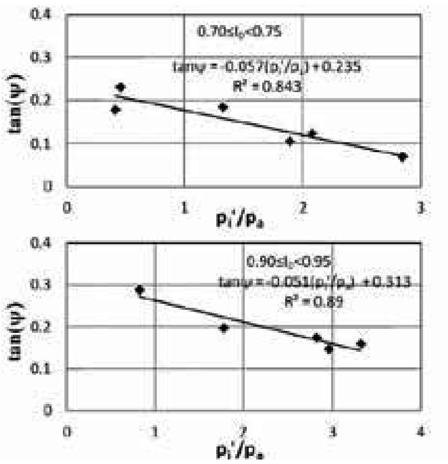

Figure 1. Tanψ-(p'_i/p_a) relationships for two different I_D ranges for Silivri sand.

However, for the soil tested, their values are given in Figure 2. According to Figure 2, a_ψ=0, b_ψ=-0.06, m_ψ=0.353, and n_ψ=0. Hence, the dilatancy equation for Silivri Sand with Ottawa distribution can be written as

$$tan\psi = b_\psi(p'_i/p_a) + m_\psi I_D = -0.06(p'_i/p_a) + 0.353 I_D \tag{8}$$

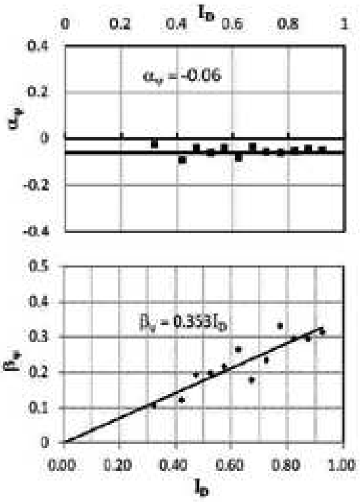

Figure 2. α_ψ-I_D and β_ψ-I_D relationships for Silivri sand.

When the test results are analyzed considering the influence of OCR, it is noticed that unloading has no effect on the dilatant behavior. Thus, OCR does not affect the proposed equations.

3.2 *Influence of dilatancy on peak friction angle*

Peak friction angle (ϕ') is a function of critical state friction angle (ϕ'_{crit}) and dilatancy which can be defined as in Eq. 9.

$$\phi' = \phi'_{crit} + r\psi \tag{9}$$

Here, the parameter r defines the proportion of dilatancy contribution to the frictional strength of the material. Up until now, researchers defined parameter r as a soil dependent constant. However, in this study the influences of I_D and p' on parameter r are also investigated. The same method of uncoupling the influences of I_D and p' is also used here. Accordingly, for each 0.05 increment of I_D, corresponding r-p' relationships are obtained. The results for $0.70 \leq I_D < 0.75$ and $0.90 \leq I_D < 0.95$ ranges are given in Figure 3 as examples.

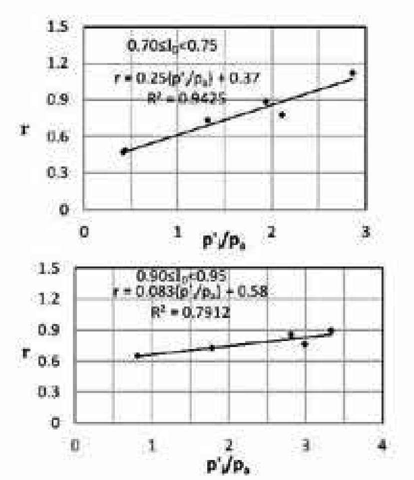

Figure 3. r-(p'_i/p_a) relationships for two different ID ranges of Silivri Sand with Ottawa grading.

As it can be observed in Figure 3, obtained r-(p'_i/p_a) relationships are approximately linear. Therefore, the relationships are defined using a line equation.

$$r = \alpha_r(p'_i/p_a) + \beta_r \tag{10}$$

Similar to Eq. 4, α_r and β_r are line-fitting parameters. Variations of α_r and β_r with I_D are given in Figure 4.

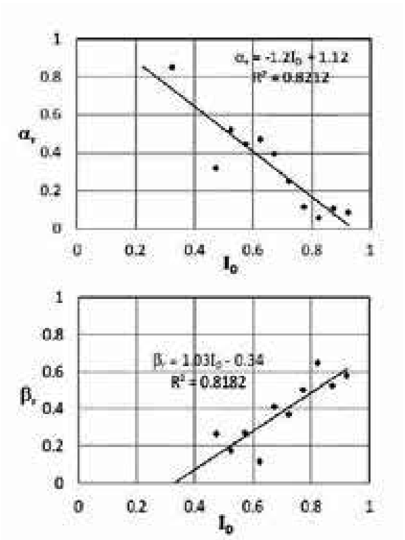

Figure 4. α_r-I_D and β_r-I_D relationships for Silivri sand.

The α_r-I_D and β_r-I_D relationships are approximately linear. Therefore, they are defined as

$$\alpha_r = a_r I_D + b_r \tag{11}$$

$$\beta_r = m_r I_D + n_r \tag{12}$$

Parameters a_r, b_r, m_r, and n_r are line-fitting parameters. Combining Eq. 10, Eq. 11, and Eq. 12, the overall function for calculating r is obtained.

$$r = (a_r I_D + b_r)(p'_i/p_a) + (m_r I_D + n_r) \tag{13}$$

For the Silivri sand with Ottawa grading, the parameters of Eq. 13 are as follows: a_r=-1.2, b_r=1.12, m_r=1.03, and n_r=-0.34. These values are obtained from Figure 4.

4 EVALUTION OF THE PROPOSED FUNCTIONS

The proposed equations (Eq. 8 and Eq. 13) were developed by investigating the results of the tests conducted on Silivri sand with Ottawa grading. Therefore, it is necessary to evaluate the proposed equations against data sets of different soils. However, it is very difficult to find a complete data set that provides sufficient number of p'_i-I_D-ϕ'-ψ combinations. Fortunately, Vaid and Sasitharan (1992) conducted a broad triaxial testing program on Erksak sand. Erksak sand has C_u=1.8, e_{max}=0.775, and e_{min}=0.525. Evidently, it is more uniform than Silivri sand with Ottawa grading.

The goal of their research was to identify the effects of stress path and loading direction on the strength and dilatancy of sands. Accordingly the researchers conducted tests with 10 different stress paths. One of the stress paths is the same as the tests of this program; Conventional drained triaxial compression test on consolidated sand. However the samples were isotropically consolidated. But, the data set of this test provided an invaluable source against which to evaluate the proposed set of equations.

Vaid and Sasitharan (1992) conducted their tests at three different relative densities and under several different confining pressures. All relevant tests, except the tests with p'_i>2000kPa, are used for the evaluation of the equations. The reason for discarding the results of the tests with p'_i>2000kPa is to prevent the possible grain-crushing mechanism from altering the results. As a result, again for each I_D, it is observed that tanψ-(p'_i/p_a) relationships are approximately linear (Figure 5). At this point, it is interesting to note that tanψ-(p'_i/p_a) relationships were even more linear when the tests with p'_i>2000kPa were considered.

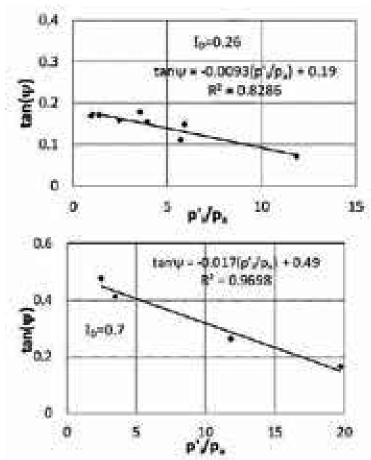

Figure 5. Tanψ-(p'_i/p_a) relationships for two different I_D values of Erksak sand.

For Erksak sand, the variations of α_ψ and β_ψ of Eq. 4 are obtained from Figure 6. Clearly the relationships have the same form as in the case of Silivri sand with Ottawa grading.

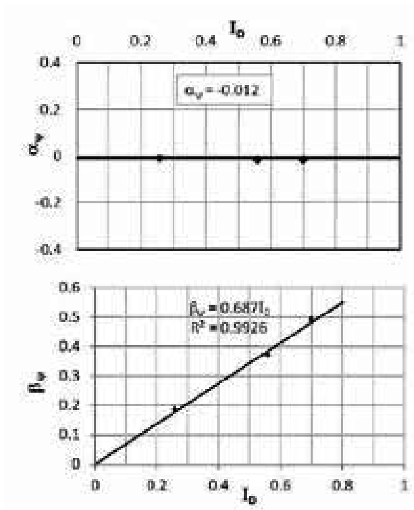

Figure 6. α_ψ-I_D and β_ψ-I_D relationships for Erksak sand.

Evidently, when the parameters of Erksak sand (a_ψ=0, b_ψ=-0.012, m_ψ=0.687, n_ψ=0) are inserted into Eq. 7, the following function is obtained.

$$tan\psi = -0.012(p'_i/p_a) + 0.687I_D \quad (14)$$

When Equations 8 and 14 are compared, it can be seen that for both soils the same form of tanψ-(p'_i/p_a) relationship is obtained.

The influence of dilatancy on the frictional behavior is also investigated. For the r parameter, the obtained α_r-I_D and β_r-I_D relationships are given in Figure 7.

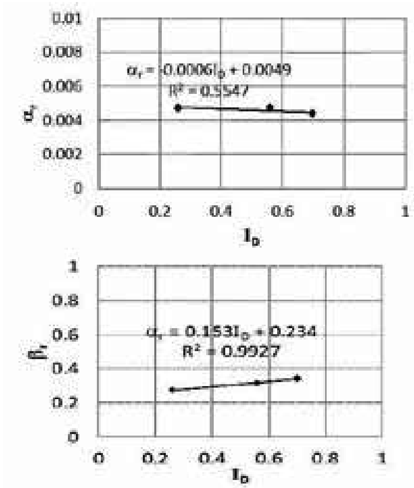

Figure 7. α_r-I_D and β_r-I_D relationships for Erksak sand.

As it can be observed from Figure 7, the same form of the r=$g(p'_i,I_D)$ function shown in Eq. 13 can also be defined for Erksak sand. Of course, the line-fitting parameters are clearly different but this can be attributed to the differences in the grain shape, size and distribution between the two sands. Erksak sand is obviously more uniform compared to Silivri sand with Ottawa grading. The difference between e_{max} and e_{min} is greater for Silivri sand than it is for Erksak sand. It might be proposed that the uniformities of sands control the influence of dilatant behavior on strength, but this proposition requires further testing on different sands with varying uniformities.

5 CONCLUSIONS

In this paper, dilatancy angle and its influence on friction angle are quantified for cohesionless soils. This is achieved by analyzing the results of an extensive triaxial testing program on K_o-consolidated cohesionless soils. The results are arranged in a way that allows the observation of the uncoupled effects of the influential parameters; I_D and p'_i. Moreover, it has been shown that OCR does not affect dilatant behavior. Even though the general form of the ψ=$f(p'_i,I_D)$ function is given in Eq. 7, the present data suggests a simpler version as shown in Eq. 8:

$$tan\psi = b_\psi(p'_i/p_a) + m_\psi I_D \quad (r.8)$$

The data from Silivri sand with Ottawa grading and Erksak sand, both support the Eq. 8 form of ψ=$f(p'_i,I_D)$ function. Here, b_ψ and m_ψ are soil dependent unitless constants. For now, there is not sufficient data to correlate the values of b_ψ and m_ψ to grain shape, grain size distribution, and mineralogy. However, it is believed that, as the corresponding constants for different soils are obtained, it would be possible to link b_ψ and m_ψ to mineralogy, grain shape, and grain size distribution characteristics. Similarly, the influence of dilatancy angle on the peak friction angle of the soil is defined. This influence is again a function of p'_i and I_D. As a result, peak friction angle can be calculated by using Eq. 9 and Eq. 13.

In order to obtain the constants for Eq. 8 and Eq. 13, it is sufficient to conduct 12 triaxial tests on clean cohesionless sands. The most important advantage of the proposed equations is that the dilatancy and peak friction angles are calculated using directly measurable and/or calculable soil parameters. This attribute significantly increases the practicality of the dilatancy and peak friction angle calculations. Once the required parameters are defined for a specific soil, it will be possible to calculate the variations in dilatancy and friction angle just by tracking the changes in stress state and volumetric state.

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