At-rest earth pressure coefficient and Poisson's ratio in normally consolidated soils

Les coefficients de pression des terres au repos et de Poisson dans les sols normalement consolidés

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ABSTRACT

The paper deals with the at-rest earth pressure coefficient K_{0NC} and Poisson's ratio ν prediction for normally consolidated soils based on the connection between the effective friction angle ϕ' and its mobilized proportion ϕ'_{mob} in the process of one-dimensional compression. After considering the existing correlations between these two parameters, the comparison with experimental data leads to both the validation of the existing theoretical K_{0NC} equation coming from the BRICK model and the proposal of an equivalent empirical K_{0NC} equation, derived without any assumptions. The same connection between ϕ' and ϕ'_{mob} leads to interesting correlations, one theoretical and one empirical, between ϕ' and ν for normally consolidated soils. No laboratory data in terms of pairs (ϕ',ν) have been found at all in literature, and, therefore, experimental validation of these correlations is necessary.

RÉSUMÉ

Cet article concerne la prévision du coefficient de pression des terres au repos K_{0NC} ainsi que du coefficient de Poisson ν pour des sols normalement consolidés sur la base du lien entre l'angle de frottement effectif ϕ' et son aliquote mobilisée ϕ'_{mob} dans le procès de compression monodimensionnelle. Après avoir considéré le lien qui existe entre ces deux paramètres, la comparaison avec les données expérimentales nous conduit à la validation de l'équation théorique de K_{0NC} dérivée du modèle BRICK et à la proposition d'une équation empirique équivalente obtenue sans aucune hypothèse. Le même lien entre ϕ' et ϕ'_{mob} nous conduit à d'intéressantes corrélations - l'une théorique, l'autre empirique - entre ϕ' et ν pour des sols normalement consolidés. On n'a pas repéré de données expérimentales (ϕ',ν) en littérature, c'est pourquoi il faut valider ces corrélations d'un point de vue expérimental.

Keywords: at-rest earth pressure coefficient; normally consolidated soil; mobilized effective friction angle; Poisson's ratio

1 INTRODUCTION

The at-rest earth pressure coefficient K_0, i.e. the ratio σ'_h/σ'_v between the horizontal and the vertical effective geostatic stresses at a point in a semi-infinite soil mass bounded by a horizontal plane, is a fundamental parameter in soil mechanics.

For normally consolidated soils, where K_0 ($\equiv K_{0NC}$) represents a measure of the gravity force transmitted in the horizontal direction, it can be estimated through empirical (e.g. Brooker and Ireland 1965; Alpan 1967; Yamaguchi 1972; Massarsch 1979; etc…) or theoretical expressions (e.g. Jaky 1944; Rowe 1958; Hendron 1963; Burland and Roscoe 1968; Burland and Federico 1999). However, the most famous and used expression is a simplified form of the Jaky's equation, i.e. K_{0NC}=1-sinϕ' (Jaky 1948).

In the mentioned expressions, the stress ratio K_{0NC} is appropriately expressed, as a rule, as a function of the effective friction angle ϕ', i.e. of the ultimate or failure stress condition. Anyway, since this ratio represents stress conditions well below failure, another appropriate measure of the above-said conditions can be, in principle, the mobilized friction angle (ϕ'_{mob}) at this stress state.

As regards the Poisson's ratio ν, i.e. the ratio of the horizontal (ε_h) to the vertical strain (ε_v), although this elastic parameter may be readily obtained from tables for most materials, for soils it is somehow problematic. In fact, the experimental results in this regard vary widely and are rather inconclusive (Cernica 1982). Fortunately, the value of ν - which is also stress dependent, although with not great variation - has a relatively small effect upon engineering predictions (Lambe and Whitman 1969). Anyway, according to theory of elasticity, the coefficient of earth pressure at-rest K_0 depends solely on the value of the Poisson's ratio ν, i.e. $K_0=\sigma'_3/\sigma'_1=\nu/(1-\nu)$, so relationships can be indirectly established between the strength properties (namely: the effective friction angle ϕ') and the Poisson's ratio. The existing relationships found in the literature, few and empirical, are shown in the APPENDIX.

In the first part of the note, the available expressions of the mobilized friction angle ϕ'_{mob} during one-dimensional compression of soils are critically reviewed and validated with experimental data from literature. Thus, the predictive capabilities of K_{0NC} equations expressed as a function of ϕ'_{mob} are assessed and compared with the Jaky's one.

Following the results obtained expressing K_{0NC} in terms of ϕ'_{mob}, a theoretical and an empirical equation of Poisson's ratio ν as a function of ϕ' are derived in the final part of the paper.

2 MOBILIZED FRICTION ANGLE DURING ONE-DIMENSIONAL COMPRESSION

During one-dimensional virgin compression of a soil, the effective stress ratio σ'_h/σ'_v developed as the vertical load is increased, is constant and equal to K_{0NC}. The Mohr's circles of stress drawn for each load increment are all tangent to a straight line (Figure 1), whose inclination angle is of course smaller than the (peak) friction angle ϕ' of the soil. This inclination angle represents the mobilized friction angle ϕ'_{mob} in the process of one-dimensional compression.

Note that the proper meaning of the ratio between ϕ'_{mob} and ϕ' is that of a mobilization factor rather than a safety factor.

Using the obliquity relations for the geometry of the Mohr's circle, the at-rest coefficient of earth pressure K_0 ($\equiv K_{0NC}$) can be geometrically related to the mobilized friction angle ϕ'_{mob} through the expression:

Proceedings of the 17th International Conference on Soil Mechanics and Geotechnical Engineering
M. Hamza et al. (Eds.)

doi:10.3233/978-1-60750-031-5-7

$$K_{0NC} = \frac{1-\sin\phi'_{mob}}{1+\sin\phi'_{mob}} = \tan^2\left(45° - \frac{\phi'_{mob}}{2}\right) \quad (1)$$

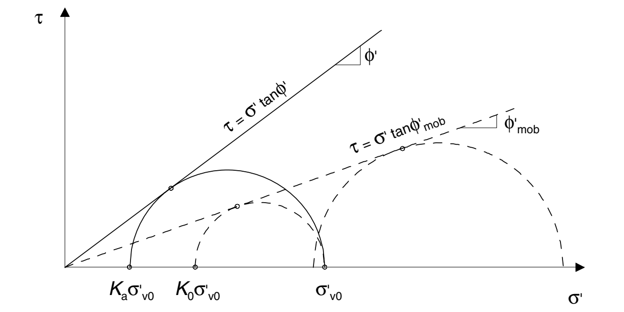

Figure 1. Mohr circles of stress for soils at failure and under one-dimensional loading conditions.

Equation (1) was originally introduced by Terzaghi (1923) and then discussed by Rowe (1954 and 1957). Although the mobilized friction angle ϕ'_{mob} in one-dimensional compression cannot be directly measured, it can be correlated with the effective friction angle ϕ' of soils. Some correlations between ϕ'_{mob} and ϕ' have been found in the literature.

Abdelhamid and Krizek (1976) correlated the Hvorslev angle of true friction ϕ_e - assumed, according to Rowe (1957), equal to the mobilized friction angle ϕ'_{mob} - with the friction angle ϕ', obtaining:

$$\phi'_{mob} \equiv \phi_e = 1.15(\phi' - 9°) \quad (2)$$

Equating the values of K_{0NC} coming from Jaky's (1948) simplified expression (K_{0NC}=1-sinϕ') to Equation (1), Bolton (1991) found:

$$\phi'_{mob} = \phi' - 11.5° \quad (3)$$

for ϕ' varying in the range 30° to 45°.

Using data collected from literature and relevant to sands of different relative density, Hayat (1992) found:

$$\phi'_{mob} = 0.67\phi' \quad (4)$$

Finally, a remarkable expression for ϕ'_{mob} is provided by Simpson (1992) using the BRICK model for the description of cohesive soil behaviour. The mobilized friction angle theoretically predicted by this advanced constitutive model during one-dimensional consolidation is:

$$\sin\phi'_{mob} = \frac{1}{\sqrt{2}}\sin\phi' \quad (5a)$$

In the range 20°~35°, this theoretical equation can be rewritten as:

$$\phi'_{mob} \cong 0.69\phi' \quad (5b)$$

quite similar to Equation (4).

The connection between ϕ' and ϕ'_{mob} leads to interesting correlations K_{0NC} - ϕ' and to an even more interesting theoretical link between the Poisson's ratio ν of a soil and its effective friction angle ϕ', as discussed in the following.

3 EXPERIMENTAL VALIDATION OF THE MOBILIZED FRICTION ANGLE EXPRESSIONS

A number of experimental data of K_{0NC} and ϕ', some of them rather "old", have been found in the literature. These data, reported by Federico *et al.* (2008), are relative mainly to reconstituted samples and have been obtained through a variety of experimental techniques characterized by different precision, especially as regards the control of the condition ε_r=0 during the consolidation phase. The K_{0NC} values have been substituted into Equation (1) and the derived mobilized friction angles ϕ'_{mob} have been plotted against the corresponding experimental friction angles ϕ', as illustrated in Figure 2.

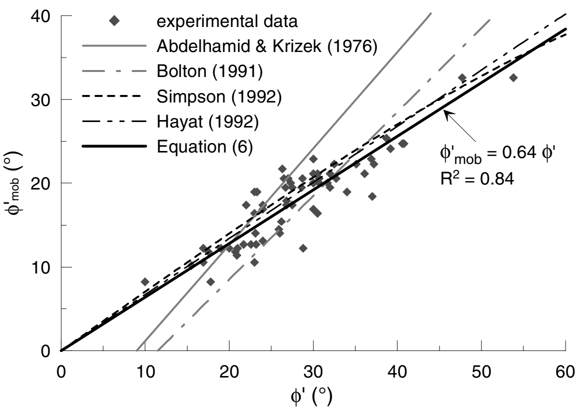

Figure 2. Correlation between ϕ'_{mob} and ϕ'.

The predictions of Equations (2), (3), (4) and (5a) have been reported in the same figure: the poor reliability of Equations (2) and (3) is evident, while the ϕ'_{mob} values theoretically predicted by Simpson's model are very close to the experimental data.

Nevertheless, a little better correlation between ϕ'_{mob} and ϕ' is given by:

$$\phi'_{mob} = 0.64\phi' \quad (6)$$

It is worth to note that Equation (6), as well as Equations (4) and (5b), are not significantly dissimilar from ϕ'_{mob}=(2/3)ϕ'.

The results shown in Figure 2 represent an experimental validation of the BRICK model predictions, although the use of the empirical Equation (6) gives slightly better results and does not require any theoretical assumptions in terms of soil behaviour.

Consequently, also the theoretical K_{0NC} equation obtained by Simpson (l.c.) substituting Equation (5a), coming from the BRICK model, into Equation (1), i.e.:

$$K_{0NC} = \frac{1-\frac{1}{\sqrt{2}}\sin\phi'}{1+\frac{1}{\sqrt{2}}\sin\phi'} \quad (7)$$

is validated. Anyway, being Equation (6) characterized by a slightly better statistical correlation with the experimental data, the corresponding K_{0NC} empirical expression, i.e.:

$$K_{0NC} = \frac{1-\sin 0.64\phi'}{1+\sin 0.64\phi'} \quad (8)$$

can be preferable from the practical point of view. Note that this empirical equation has been derived - as already said - without any assumptions.

The predictive capabilities of the above mentioned K_{0NC} equations, along with the one of Jaky, have been checked through the comparison between predicted and measured values. The comparison results are summarized in Table 1.

That the Jaky's equation predicts K_{0NC} quite well is surprising, since this equation was derived using questionable theoretical assumptions (Michalowski 2005).

Table 1. Comparison among pairs (R^2, s_d) relative to different K_{0NC} equations.

	Coefficient of determination, R^2	Standard deviation, s_d
Jaky (1948)	0.81	0.11
Simpson (1992)	0.74	0.09
Proposed Eq. (8)	0.82	0.08

4 CORRELATIONS FOR POISSON'S RATIO

The Poisson's ratio ν is defined as the ratio between the radial and the axial strain in an elastic material loaded uniaxially. More generally, it is the ratio between the strain in one coordinate direction (due to a stress in that direction) and the strain caused in the other coordinate directions by the same stress (Somerville and Paul 1983). Given the link between K_0 and ν in the theory of elasticity, the Poisson's ratio can be empirically correlated to the effective friction angle ϕ' equating the Jaky's K_{0NC} simplified expression to the at-rest pressure coefficient for elastic materials (e.g. Dysli 2001), i.e.:

$$K_{0NC} = 1 - \sin\phi' = \frac{\nu}{1-\nu} \quad \Rightarrow \quad \nu = \frac{1-\sin\phi'}{2-\sin\phi'} \qquad (9)$$

As discussed in the previous section, it is more appropriate to relate K_{0NC} to the mobilized friction angle ϕ'_{mob} through Equation (1). Therefore, adopting the same approach leading to Equation (9), it is possible to obtain a more general expression of the Poisson's ratio:

$$K_{0NC} = \frac{1-\sin\phi'_{mob}}{1+\sin\phi'_{mob}} = \frac{\nu}{1-\nu} \quad \Rightarrow \quad \nu = \frac{1-\sin\phi'_{mob}}{2} \qquad (10)$$

In particular, using Equation (5a) for the mobilized angle of friction, a theoretical equation for ν results, i.e.:

$$\nu = \frac{1 - \frac{1}{\sqrt{2}}\sin\phi'}{2} \qquad (11)$$

whereas, using Equation (6), an equivalent empirical equation for ν is obtained, i.e.:

$$\nu = \frac{1-\sin(0.64\phi')}{2} \qquad (12)$$

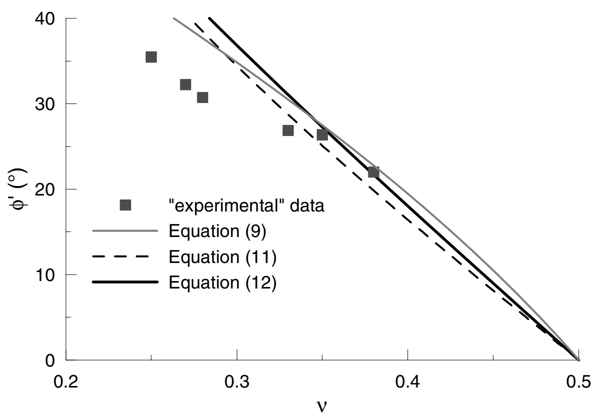

Figure 3. Poisson's ratio predictions as a function of ϕ' compared with "experimental" data.

No data sets in terms of pairs (ϕ',ν) have been found in the literature so far. However, some experimental data of ν were obtained by Wroth (1975) for several lightly overconsolidated soils and plotted against the corresponding plasticity index I_P. The related ϕ' angles have been obtained using the empirical correlation (Muir Wood 1990) between I_P and ϕ', i.e. $\sin\phi' = 0.35 - 0.1\ln I_P$, although the considerable scatter around the average line in this correlation gives the obtained ϕ' values the character of very rough estimates only.

Nevertheless, the trend of these ϕ' values plotted versus the experimental ν data is shown in Figure 3, where the predictions by Equations (9), (11) and (12) are also shown for comparison.

5 CONCLUSIONS

The link between the effective friction angle ϕ' and its mobilized proportion ϕ'_{mob} in the process of one-dimensional compression has been used for the prediction of the at-rest earth pressure coefficient and Poisson's ratio of normally consolidated soils. In the first part of the note, a brief review of the existing correlations between ϕ'_{mob} and ϕ', whose ratio has the meaning of a mobilization rather than a safety factor, has been presented. The comparison with experimental data has led to the validation of the theoretical K_{0NC} equation coming from the BRICK model developed by Simpson (1992) and to the proposal of an equivalent empirical K_{0NC} expression. The latter, derived without any assumptions, allows slightly better predictions.

The same connection between ϕ' and ϕ'_{mob} has allowed to derive two interesting correlations, one theoretical and one empirical, between the Poisson's ratio ν of a normally consolidated soil and its friction angle ϕ'. As no laboratory data sets in terms of pairs (ϕ',ν) have been found in the literature, an experimental validation of the link between ν and ϕ' is necessary.

AKNOWLEDGMENTS

This work has been supported by the Regional Agency for Technology and Innovation of Puglia Region (grant n° PS_119) and by "Fondazione Cassa di Risparmio di Puglia" (Progetto Integrato sulla Difesa Ambientale). The second author gratefully acknowledges the financial support of the Engineering Faculty of Taranto (Technical University of Bari).

APPENDIX

Differently from K_0, very little information is available in the literature for correlation studies with Poisson's ratio ν.

With reference to granular soils, Trautmann and Kulhawy (1987) suggested the following approximated relationship:

$$\nu \approx 0.1 + 0.3\phi_{rel} \qquad (13)$$

with:

$$\phi_{rel} = (\phi' - 25°)/(45° - 25°) \text{ and } 0 \le \phi_{rel} \le 1 \qquad (14)$$

ϕ_{rel} being a relative friction angle convenient to use for approximating the soil density state.

An alternative approach using a hyperbolic model for the initial tangent Poisson's ratio is described by Kulhawy *et al.* (1969).

Duncan *et al.* (1991) proposed the relationship:

$$\nu = \frac{4-3\sin\phi'}{8-4\sin\phi'} \qquad (15)$$

valid for a compacted fill only, assuming that the stress state of the soil in this particular state is half-way between an at-rest condition and failure.

REFERENCES

Abdelhamid M.S. & Krizek R.J. 1976. At rest lateral earth pressure of a consolidating clay. *Journal of Geotechnical Engineering Division, ASCE* 102(GT7): 721-728.

Alpan I. 1967. The empirical evaluation of the coefficient K_0 and K_{0R}. *Soils and Foundations* 7(1): 31-40.

Bolton M.D. 1991. *Geotechnical stress analysis for bridge abutment design*. Report 270, Transport and Road Research Laboratory, Crowthorne, UK.

Brooker E.W. & Ireland H.O. 1965. Earth pressure at rest related to stress history. *Canadian Geotechnical Journal* 2(1): 1-15.

Burland J.B. & Federico A. 1999. At rest earth pressure coefficient k0(NC) from Cam-Clay model. *Proc., 11th Asian Regional Conference on Soil Mechanics and Foundation Engineering*, Seoul, Korea, vol. 1: 15-18.

Burland J.B. & Roscoe K.H. 1969. Pore pressures in normally consolidated clay. *Géotechnique* 19(3): 335-356.

Cernica N. 1982. Geotechnical engineering. Holt-Sanders Int. Editions.

Duncan J.M., Williams G.W., Sehn A.L. & Seed R.B. 1991. Estimation earth pressures due to compaction. *Journal of Geotechnical Engineering, ASCE* 117(12): 1833-1847.

Dysli M. 2001. *Recherche bibliographique et synthése des correlations entre les caractéristiques des sols*. Ecole Polytechnique Fédéral de Lausanne-Laboratoire de mécanique des sols (LMS).

Federico A., Elia G. & Germano V. 2008. A short note on the earth pressure and mobilized angle of internal friction in one-dimensional compression of soils. *Journal of GeoEngineering* Vol. 3(1): 41-46.

Hayat T.M. 1992. *The coefficient of earth pressure at rest*. Ph.D. Thesis, University of Illinois at Urbana-Champaign, USA.

Hendron A.J. 1963. *The behaviour of sand in one-dimensional compression*. Ph.D. Thesis, University of Illinois at Urbana-Champaign, USA.

Jaky J. 1944. The coefficient of earth pressure at rest. *Journal of the Society of Hungarian Architects and Engineers*, Budapest, Hungary, 355-358 (in Hungarian).

Jaky J. 1948. Pressure in silos. *Proc. 2nd International Conference on Soil Mechanics and Foundation Engineering*, Rotterdam, Nederland, vol. 1: 103-107.

Kulhawy F.H., Duncan J.M. & Seed H.B. 1969. *Finite element analysis of stresses and movements in embankments during construction*. Contract Report S-69-8, US Army Engineer Waterways Experiment Station, Vicksburg.

Lambe T.W. & Whitman R.V. 1969. *Soil mechanics*. John Wiley & S.

Massarsch K.R. 1979. Lateral earth pressure in normally consolidated clay. *Proc., 7th European Conference on Soil Mechanics and Foundation Engineering*, Brighton, UK, vol. 2: 245-249.

Michalowski R.L. 2005. Coefficient of earth pressure at rest. *Journal of Geotechnical and Geoenvironmental Engineering, ASCE* 131(11): 1429-1433.

Muir Wood D. 1990. *Soil behaviour and critical state soil mechanics*. Cambridge University Press, Cambridge.

Rowe P.W. 1954. A stress-strain theory for cohesionless soil with applications to earth pressures at rest and moving walls. *Géotechnique* 4(2): 70-88.

Rowe P.W. 1957. Ce = 0 hypothesis for normally loaded clays at equilibrium. *Proc., 4th International Conference on Soil Mechanics and Foundation Engineering*, London, vol. 1: 189-192.

Rowe P.W. 1958. General report on papers in section I. *Proc., Conference on Earth Pressure Problems*, Brussels, Belgium, vol. 3: 25-30.

Simpson B. 1992. Retaining structures: displacement and design. *Géotechnique* 42(4): 541-576.

Somerville S.H. & Paul M.A. 1983. *Dictionary of geotechnics*. Butterworths, London.

Terzaghi C. 1923. Discussion of lateral earth pressure: the accurate experimental determination of the lateral earth pressure, together with a resume of previous experiments. *Transactions of the American Society of Civil Engineers*, vol. 86: 1525-1543.

Trautmann C.H. & Kulhawy F.H. 1987. *CUFAD–A computer program for compression and uplift foundation analysis and design*. Report EL-4540-CCM, vol. 16, Electric Power Research Institute, Palo Alto, California.

Wroth C.P. 1975. In-situ measurement of initial stress and deformation characteristics. *Proc., Specialty Conf. on In-Situ Measurement of Soil Properties*, Raleigh, North Carolina (New York: ASCE), vol. 2: 181-230.

Yamaguchi H. 1972. Some considerations on the earth pressure at rest. *Proc., 27th Annual Meeting of Japan Society of Civil Engineers*, vol. 3: 109-110 (in Japanese).