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Mathematical characterization of the compression to 100,000 kg/cm² of thirty-nine substances

Caractérisation mathématique de la compression sous 10.000 MPa de trente-neuf substances

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ABSTRACT

A very simple theoretical equation given by the Principle of Natural Proportionality for the compressibility of geomaterials: $V/V_0 = [(i + \sigma)/i]^{-\gamma}$ is applied to the experimental data obtained by P.W. Bridgman in thirty-nine substances from 0 to 100,000 kg/cm². Independently of polymorphic transitions, the compression response of the natural solid substances presented in this paper is described by this simple theoretical equation with constant natural internal pressures i and constant natural coefficients of compressibility $\gamma < 1$.

RÉSUMÉ

Une équation théorique très simple donnée par le principe de la Proportionnalité Naturelle pour la compressibilité des géomatériaux: $V/V_0 = [(i + \sigma)/i]^{-\gamma}$ est appliquée aux données expérimentales obtenues par P.W. Bridgman pour trente-neuf substances soumises à des pressions allant de 0 à 10.000 MPa. Indépendamment des transitions polymorphes, la réponse à la compression des substances solides naturelles présentée dans ce document est décrite par cette équation théorique simple avec des pressions internes naturelles constantes i et des coefficients naturels de compressibilité constants $\gamma < 1$.

Keywords : compressibility, geomaterials, substances

1 INTRODUCTION

“The compression of 39 substances to 100,000 kg/cm²” is the title of a paper by P.W. Bridgman of the Lyman Laboratory of Physics of Harvard University, Cambridge, Mass., published in Proc. Am. Acad. Arts. Sci., Vol. 76, No. 3, pp. 55-69-March, 1948. In the present paper all the experimental data are reproduced by a very simple theoretical equation provided by the Principle of Natural Proportionality (Juárez-Badillo 1985).

2 GENERAL THEORETICAL EQUATION

The very simple general theoretical equation reads (Juárez-Badillo 1965)

$$\frac{V}{V_0} = \left(\frac{p}{p_0} \right)^{-\gamma} \quad (1)$$

where V = volume, p = total pressure, γ = natural coefficient of compressibility and (V_0, p_0) = initial point. In this case $p = i + \sigma$ where i = internal pressure and σ = external pressure. Initially the internal pressure i was not considered and the Equation 1 for clays was extended to all types of soils (Juárez-Badillo 1981) and to concrete (Juárez-Badillo 1985).

Grimer & Hewitt (1968) found for the relation between pressure and volume for water at 60°C, using Bridgman experimental data, the next expression

$$pV^6 = \text{constant} \quad (2)$$

where $p = \sigma + i$, σ = external pressure, $i = 3750 \text{ atmospheres}$. Equation 2 may be written in the form of Equation 1 with $\gamma = 1/6$.

Some of the substances present discontinuities of volume Δ_{tr} at polymorphic transitions; therefore Equation 1 is modified to

$$\frac{V}{V_0} = \left(1 + \frac{\sigma}{i} \right)^{-\gamma} - \Delta_{tr} \quad (3)$$

where Δ_{tr} will be taken into account after their occurrence. The authors suggest to the readers of this paper to read the paper by Bridgman for a complete information on the experiments he realized.

Equation 3 has already been applied to the compression to 40,000 kg/cm² of certain liquids and to the compression to 45,000 kg/cm² of fourteen substances using Bridgman experimental data founding that parameters γ and i remain constant, both of them, even in the liquids that present freezing under certain pressure and also when some liquids present a second polymorphic transition, and they also remain constant, both of them, in the solid substances, even in the substances that present polymorphic transitions (Juárez-Badillo & Hernández-Mira 2006 a, b).

To obtain the parameters γ and i it should be taken into account that V/V_0 decreases more rapid when γ increases and i decreases. Figure 1 illustrates how the compressibility curves change when γ and i change such that they have a constant final point. In this graph the curves rise, with higher values of V/V_0 , as the values of γ and i increases. The highest curve with $\gamma = 1$ lowers for smaller values γ and i .

To obtain γ and i a final experimental point is chosen and with an assumed value of γ , the corresponding value of i is obtained from the equation

$$i = \frac{\sigma}{e \left[\frac{1}{\gamma} \ln \left(\frac{V}{V_0} \right) \right] - 1} \quad (4)$$

and later on the values of γ and i are changed until a very good theoretical curve is obtained for the experimental points.

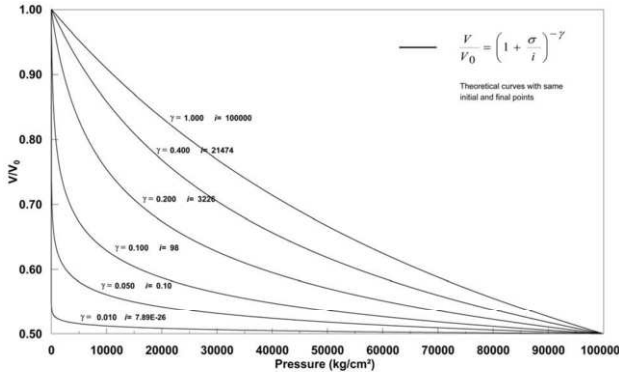


Figure 1. Comparison of different theoretical curves

It is important to note that the traditional linear bulk modulus K is given by

$$K = \frac{d\sigma}{-dV/V} = \frac{p}{\gamma} = \frac{i + \sigma}{\gamma} \quad (5)$$

and at the origin, $\sigma=0$

$$K_0 = \frac{i}{\gamma} \quad (6)$$

Also, notice that when $\sigma \rightarrow \infty$, $V \rightarrow 0$ and due to Δ_{tr} the value V tends to zero at a finite value of the pressure σ .

In the present paper Equation 3 is applied to the experimental data obtained by Bridgman on 39 substances. The 39 substances are: five alkali metals, 17 other elements (with two types of the element phosphorus: black and violet), 9 compounds: the sulfides, selenides and tellurides of zinc, mercury and lead, and quartz crystal and six glasses.

3 PRACTICAL APPLICATION

The values of γ and i for the different experimental curves were obtained from Equation 3, programmed in a computer together with the experimental data, such that the sum of the squares of the differences among the experimental and theoretical values were a minimum. Equation 4 was used in some cases to try to improve the results.

Some comments on the different figures follow.

Figure 2 presents the theoretical curves and experimental data for the alkali metals: lithium, sodium, potassium, rubidium and caesium contained in Table I of Bridgman's paper. "These are of especial theoretical interest because the structure of these elements is sufficiently simple to permit theoretical calculation of the volume changes under pressure".

Figures 3 and 4 present the compression for the elements: Be, C (Graphite), Mg, Al, Si, Phosphorus (Black and Violet), Ti, Mn, Ge, As, Zr, La, Ce, Pr, Nd, Th and U.

Figure 5 presents the compression of the 9 compounds: the sulfides, selenides and tellurides of zinc, mercury and lead.

Figure 6 presents the compression of Quartz Crystal and six glasses: Quartz Glass, Glass "A", Pyrex Glass, Glass "C", Glass "D" and Borax Glass.

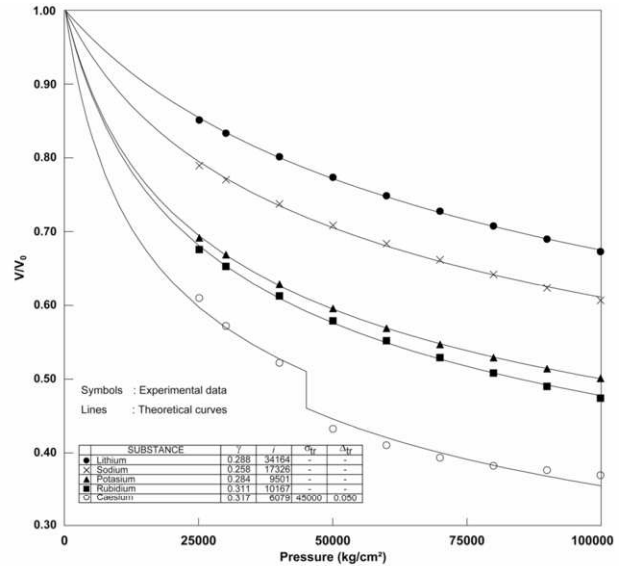


Figure 2. Compression of the alkali metals (Table I)

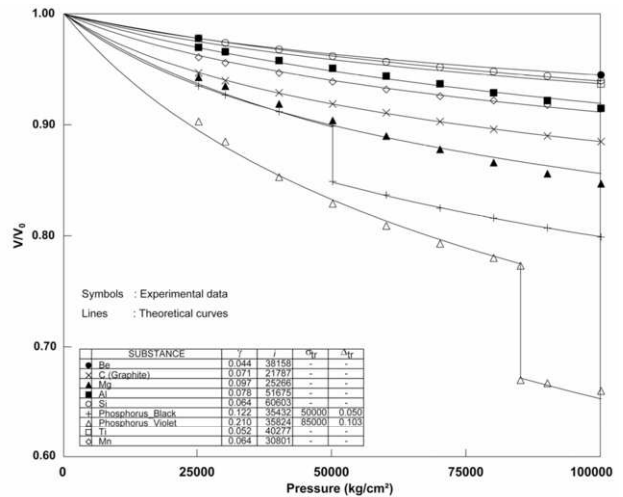


Figure 3. Compression of various elements (Table IIa)

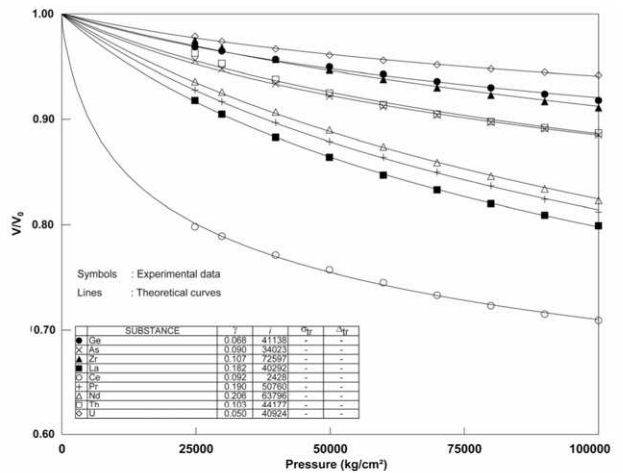


Figure 4. Compression of various elements (Table IIb)

From the above figures it may be observed that the values of γ were between 0.037 and 0.317 while the values of i were between 2,428 to 127,044 kg/cm². The Caesium (Figure 2) presented the $\gamma_{max}=0.317$ with $i=6,079kg/cm^2$. The minimum value of $\gamma_{min}=0.037$ is for the sulfide of mercury HgS (Figure 5) with $i=3,770kg/cm^2$. The maximum value of

$i_{max}=127,044\text{kg/cm}^2$ is for the telluride of lead PbTe (Figure 5) with $\gamma=0.271$ while the minimum value of $i_{min}=2,428\text{kg/cm}^2$ is for Ce (Figure 4) with $\gamma=0.092$. In general it may be observed that low values of γ are associated to low values of i and high values of γ are associated to high values of i .

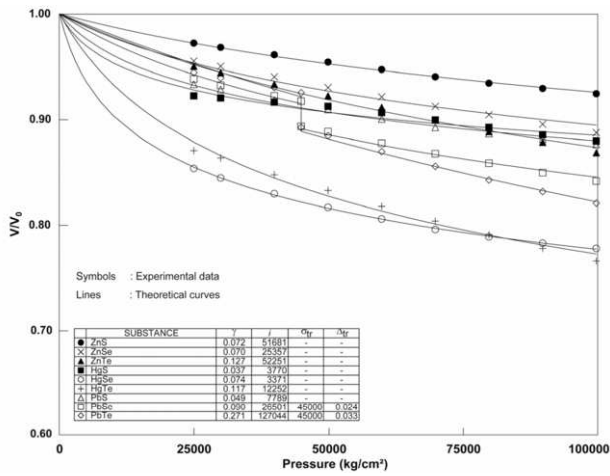


Figure 5. Compression of various compounds (Table III)

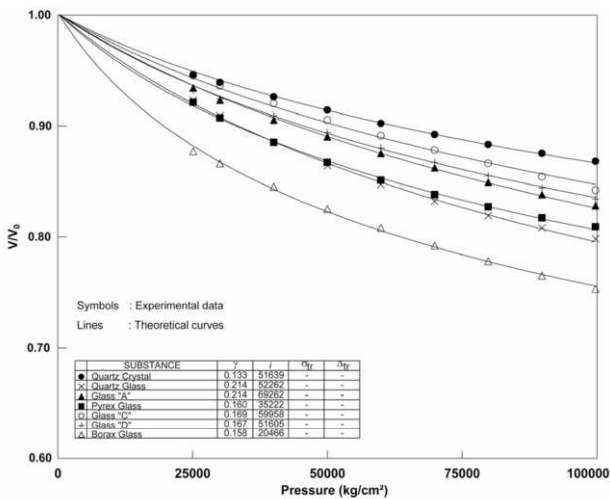


Figure 6. Compression of quartz crystal and six glasses (Table IV)

4 CONCLUSIONS

Independently of polymorphic transitions, the compression response of all natural solid substances is described by the Equation 1, with constant natural internal pressures i and constant natural coefficients of compressibility $\gamma < 1$.

The natural coefficients of compressibility γ varied between 0.037 and 0.317 and the values of the internal pressure i varied between 2,428 and 127,044 kg/cm².

Equation 1 is such that when the pressure σ tends to ∞ the volume tends to zero, but due to the polymorphic transitions, all liquids and solids tend to volume zero at finite values of the pressure σ .

REFERENCES

Bridgman, P. W. 1948. The compression of 39 substances to 100,000 kg/cm². Proc. Am. Acad. Arts. Sci. Vol. 76, No. 3, pp. 55 - 69.

Grimer, F. J. and Hewitt R. E. 1968. The form of the stress-strain curve of concrete interpreted with a diphase concept of material behavior. Building Research Station, Ministry of Public Building and Works, Great Britain, pp. 681 - 691.

Juárez-Badillo, E. 1965. Compressibility of Soils. V Symposium of the Civil and Hydraulic Engineering Department on Behavior of Soils under Stress, Indian Institute of Science, Bangalore, India, Vol. I, pp. A2/1 - 35.

Juárez-Badillo, E. 1981. General compressibility equation for soils. X International Conference on Soil Mechanics and Foundation Engineering, Stockholm, Vol. 1, pp. 171 - 178.

Juárez-Badillo, E. 1985. General volumetric constitutive equation for geomaterials. Special volume on Constitutive Laws of Soils. XI International Conference on Soil Mechanics and Foundation Engineering, San Francisco, Japanese Society of Soil Mechanics and Foundation Engineering, Tokyo, pp. 131 - 135.

Juárez-Badillo, E. and Hernández-Mira, S. 2006a. Mathematical characterization of the compression to 45,000 kg/cm² of fourteen substances. International Conference on New Developments in Geoenvironmental and Geotechnical Engineering, Incheon, Korea, pp. 154 - 161.

Juárez-Badillo, E. and Hernández-Mira, S. 2006b. Mathematical characterization of the compression to 40,000 kg/cm² of certain liquids. International Conference on New Developments in Geoenvironmental and Geotechnical Engineering, Incheon, Korea, pp. 162 - 170.