The Asaoka method revisited

Réexamen de la méthode d'Asaoka

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ABSTRACT

The Asaoka method is a useful tool for interpreting and extrapolating field settlement observations. The graphical procedure to estimate end-of-primary (EOP) settlement and coefficient of consolidation is simple, however, mathematical deduction of the method by Asaoka is not. The Asaoka method is deduced here using simple algebra for one-dimensional compression with and without vertical drains. The Terzaghi theory of one-dimensional consolidation, and the Barron equal strain theory of consolidation modified to include effects of smear, well resistance and vertical water flow through soil, are used. The Asaoka graphical procedure is applied to settlement observations at Skå-Edeby test area I with three spacings of sand drains and area IV without vertical drains. The Asaoka method is not recommended for predicting secondary settlement.

RÉSUMÉ

La méthode d'Asaoka est un moyen efficace pour interpréter et extrapoler les lectures de tassement. La construction graphique pour prédire la fin de la consolidation et estimer le coefficient de consolidation est simple; la dérivation mathématique de la méthode par Asaoka ne l'est cependant pas. La méthode d'Asaoka est dérivée ici de façon simple en utilisant les formulations algébriques de la consolidation unidimensionnelle avec ou sans drains verticaux. Les théories de Terzaghi et la théorie de Barron, modifiée pour inclure l'effet de colmatage, l'effet de puits et l'écoulement vertical dans le sol sont utilisées. La construction graphique d'Asaoka est appliquée au suivi des tassements au site expérimental de Ska-Edeby pour la parcelle I avec drains de sable et la parcelle IV sans drain vertical. La méthode d'Asaoka n'est pas recommandée pour la prédiction des tassements secondaires.

Keywords: Asaoka, settlement, coefficient of consolidation, vertical drains

1 INTRODUCTION

The Asaoka method is a useful tool for interpreting and extrapolating field observations of settlement (Asaoka 1978, Jamiolkowski et al. 1985). The graphical procedure for estimating the end-of-primary (EOP) settlement, S_{100} , and coefficient of consolidation (either c_v or c_h) is simple, as follows: (i) plot the observed settlements against the elapsed time, (ii) select a series of settlement S_1 , S_2 , S_3 , … S_j , S_{j+1} , …, respectively at times t_1 , t_2 , t_3 , … t_j , t_{j+1} , … such that $t_{j+1} - t_j$ = constant, (iii) plot S_{j+1} against S_j to obtain a straight line, and (iv) extrapolate the line to intersect a 45° line through the origin. The point of intersection defines EOP settlement, S_{100} , and the slope of the line is used to estimate c_v or c_h .

The mathematical deduction of Asaoka (1978) is not simple. Therefore, the Asaoka method is here deduced using simple algebra, for one-dimensional compression with or without vertical drains.

2 ONE-DIMENSIONAL COMPRESSION WITHOUT VERTICAL DRAINS

The Terzaghi settlement versus time relationship for time-independent loading and linear distribution of initial excess porewater presure, for average degree of consolidation greater than 50% is:

$$S = S_{100}\left[1-\frac{8}{\pi^2}\cdot\exp\left(\frac{-\pi^2 c_v t}{4H^2}\right)\right] \tag{1}$$

where S is settlement at elapsed time t, S_{100} is the EOP settlement, c_v is coefficient of consolidation for vertical water flow and vertical compression, and H is maximum vertical drainage distance.

Let $C = -(\pi^2 c_v)/4H^2$, and consider settlements at elapsed times t_j and t_{j+1} :

$$S_j = S_{100}\left[1-\frac{8}{\pi^2}\cdot\exp\left(C\cdot t_j\right)\right] \tag{2}$$

$$S_{j+1} = S_{100}\left[1-\frac{8}{\pi^2}\cdot\exp\left(C\cdot t_{j+1}\right)\right] \tag{3}$$

Rewrite Eqs. 2 and 3 and divide:

$$\frac{S_{100}-S_{j+1}}{S_{100}-S_j} = \exp\left[C\,(t_{j+1}-t_j)\right] \tag{4}$$

make $t_{j+1} - t_j = \Delta t$ = constant, and let $C' = \exp(C\cdot\Delta t)$:

$$S_{j+1} = S_{100} - (S_{100} - S_j)\cdot C' \tag{5}$$

Because C ' is a constant, a plot of S_{j+1} versus S_j is a straight line. This is expected to intersect a 45° line through the origin when $S_{j+1} = S_j$. Thus from Eq. 5:

$$S_j = S_{100} - C'S_{100} - C'S_j \tag{6}$$

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or $S_j\,(1-C')=S_{100}\,(1-C')$ (7)

and $S_j=S_{100}$ (8)

Denote by β the slope of S_{j+1} versus S_j line defined by Eq.4, and rewrite:

$$\ln\beta = C\cdot\Delta t \tag{9}$$

Substitute for C and solve for c_v :

$$c_v = \frac{-4H^2}{\pi^2}\cdot\frac{\ln\beta}{\Delta t} \tag{10}$$

3 ONE-DIMENSIONAL COMPRESSION WITH VERTICAL DRAINS

The Barron (1944, 1948) equal-strain settlement versus time relationship, modified to include effects of smear, well resistance, and vertical water flow through soil (Hansbo 1981, Zeng and Xie 1989, Lo 1991) is:

$$S=S_{100}\left[1-\exp\left(\frac{-Ec_h\,t}{r_e^2}\right)\right] \tag{11}$$

where

$$E=\frac{n}{30}\left[\frac{2}{F(n,s)+2.5G}+\frac{4c_v/c_h}{(H/r_e)^2}\right]$$

$$F(n,s)=\frac{n^2}{n^2-1}\left(\ln\frac{n}{s}+\frac{k_h}{k_s}\ln s-\frac{3}{4}\right)+\frac{s^2}{n^2-1}\left(1-\frac{s^2}{4n^2}\right)+\frac{k_h}{k_s}\,\frac{1}{n^2-1}\left(\frac{s^4-1}{4n^2}-s^2+1\right)$$

$$G=\frac{\pi}{4}\,\frac{k_h\,\ell_m^2}{q_w}$$

and $n = r_e / r_w$, $s = r_s / r_w$, r_e = maximum horizontal drainage distance = 0.525·DS for triangular pattern, DS = vertical drain spacing, r_w = radius of vertical drain, r_s = radius of smear zone, k_h = horizontal permeability of soil, k_v = vertical permeability of soil, k_s = permeability of smear zone, c_v = coefficient of consolidation for vertical compression and vertical water flow, c_h = coefficient of consolidation for vertical compression and horizontal water flow, H = maximum vertical drainage distance through soil, ℓ_m = maximum drainage distance through vertical drain, and q_w = discharge capacity of vertical drain.

Because Eq. 11 has the same form as Eq. 1, one can readily deduce Eq. 8 also for one-dimensional compression with vertical drains. Thus we only derive the equation for c_h .

Consider settlements at elapsed times t_j and t_{j+1} :

$$S_j=S_{100}\left[1-\exp\left(\frac{-E\,c_h\,t_j}{r_e^2}\right)\right] \tag{12}$$

$$S_{j+1}=S_{100}\left[1-\exp\left(\frac{-E\,c_h\,t_{j+1}}{r_e^2}\right)\right] \tag{13}$$

Rewrite Eqs. 12 and 13, and divide to obtain slope of S_{j+1} versus S_j line:

$$\frac{S_{100}-S_{j+1}}{S_{100}-S_j}=\exp\left[\frac{-E\,c_h}{r_e^2}(t_{j+1}-t_j)\right] \tag{14}$$

Denote the slope by β and let $t_{j+1} - t_j = \Delta t$:

$$\ln\beta=\frac{-E\,c_h}{r_e^2}\cdot\Delta t \tag{15}$$

and

$$c_h=\frac{-r_e^2}{E}\cdot\frac{\ln\beta}{\Delta t} \tag{16}$$

Even though Eqs. 1 and 11 are solutions for instant loading, the Asaoka method is also applicable to time-dependent loading for construction time factors ($T_c = c_v\,t_c / H^2$ or $c_h\,t_c / r_e^2$) less than 1.0.

4 AN EXAMINATION OF THE ASAOKA METHOD

An examination of the Asaoka method, in terms of settlement observations at Skå-Edeby test field for test areas I with three spacings of sand drains and test area IV without vertical drains, is quite instructive because test areas I and IV were subjected to identical embankment loading, and the settlement observations at area I extend into secondary compression stage, Fig. 1 (Mesri et al. 1994).

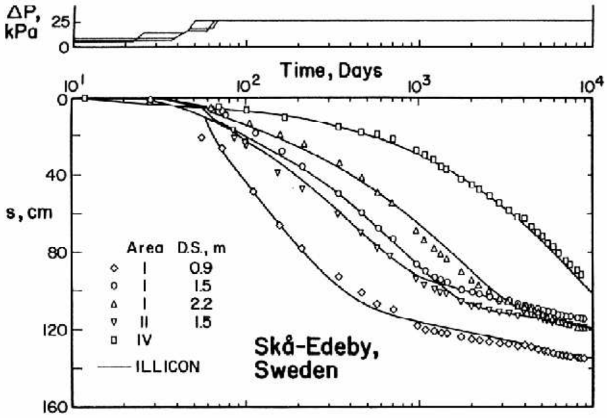

Fig. 1 Settlement observations at Skå-Edeby test field (Mesri et al. 1994).

Skå-Edeby test field constructed in the Spring of 1957 by the Swedish Geotechnical Institute in cooperation with the Swedish Board of Roads and Waterways, is situated on an island about 25 kilometers west of Stockholm, Sweeden (Hansbo 1960, Lo 1991, Mesri et al. 1994). The 70-m diameter test area I was divided into three sectors and 18-cm diameter displacement type, fully penetrating sand drains were installed in a triangular pattern at spacings of 0.9 , 1.5 , and 2.2 m. No vertical drains were installed in the 30-m diameter test area IV.

The 12-m thick compressible ground consists of soft clays with s_{uo}(FV) in the range of 10-20 kPa (see Fig. 36 of Mesri et al. 1994). A detailed settlement analysis has been previously carried out using the ILLICON procedure for all test areas at Skå-Edeby test field (Lo 1991, Mesri et al. 1994). The ILLICON predictions for three test areas with identical embankment load are shown in Fig. 1 ; however, the present paper is concerned with observed settlements at areas I and IV. The ILLICON analyses have shown that the sand drains at Skå-Edeby functioned without well resistance. Therefore, in the present examination of the Asaoka method, a discharge factor, $D = q_w / (k_h \ell_m^2) = 5$ is used (Mesri and Lo 1991, Mesri et al. 1994).

The observed settlements at areas I and IV are plotted in Figs 2 – 5. The EOP settlement according to the Asaoka method are listed in Table 1, and compared for area I with S_{100} determined using the Casagrande construction. Because for area IV EOP consolidation has not been reached for the observed settlement data in Fig. 1, only the Asaoka method could determine the EOP settlement.

Equation 10, together with H = 6 m, $\beta = 0.84$ from Fig. 5, and Δt = 1000 days, lead to a $c_v = 0.93$ m^2/year. This value compares quite well with $c_v = 1.04$ m^2/year computed using the Casagrande method applied at 50% settlement with the Terzaghi time factor of 0.197, and S_{50} = 53.5 cm determined with S_{100} = 107 cm by the Asaoka method.

In previous interpretations of field performance of vertical drains, in terms of the Barron theory or its modifications to include effects of smear, well resistance, and water flow through soil (Hansbo 1981, Lo 1991), values of coefficient of consolidation c_h , back-calculated using settlement or porewater pressure observations, have been found to depend on vertical drain spacing (Jamiolkowski et al. 1985, Mesri and Lo 1991, Holtz et al. 1991, Cao et al. 2001). This is mainly related to the increased influence with the decrease in drain spacing, of the disturbed smear zone, taken into account approximately only in terms of a decrease in k_h (Basu and Prezzi 2007). To overcome this shortcoming, the empirical correction factor of n/30 is introduced into the E defined by Lo (1991).

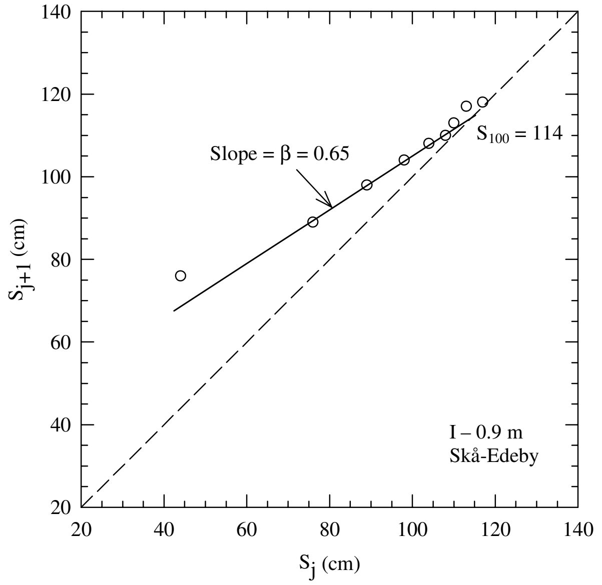

Fig. 2 Observed settlements at Area I with 0.9 m drain spacing

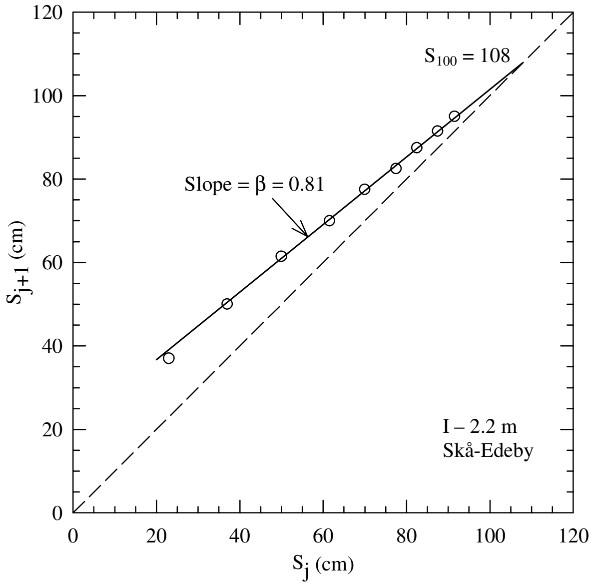

Fig. 4 Observed settlements at Area I with 2.2 m drain spacing

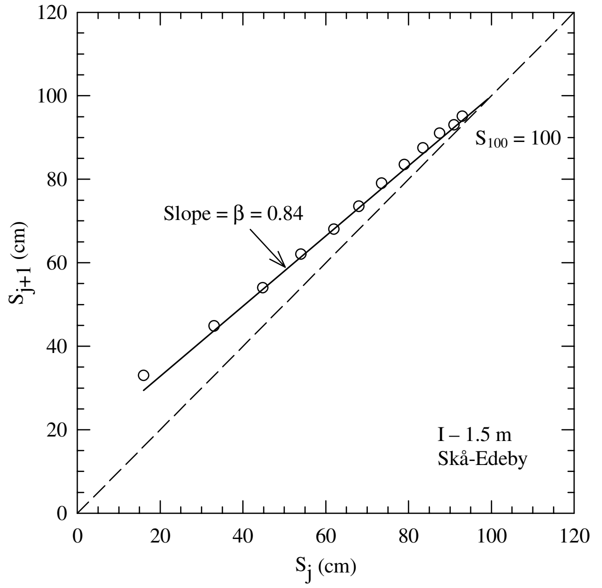

Fig. 3 Observed settlements at Area I with 1.5 m drain spacing

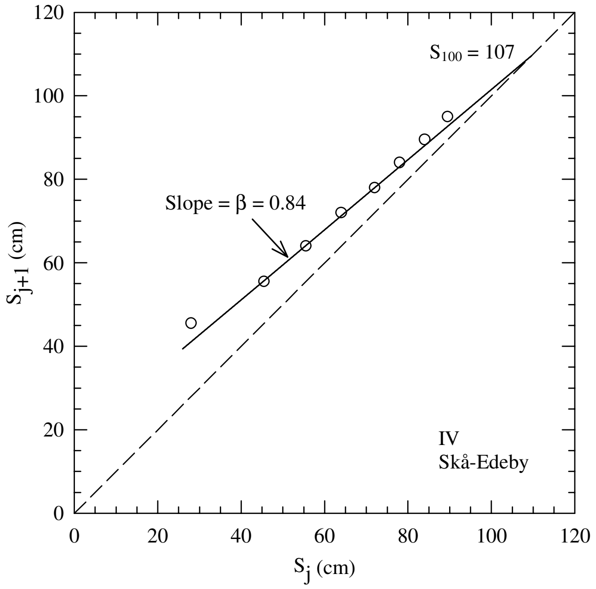

Fig. 5 Observed settlements at Area IV without vertical drains

In computing c_h using Eq. 16, based on previous detailed examination of subsurface conditions at Skå-Edeby test field (Lo 1991, Mesri et al. 1994), s = 2, $k_h / k_v = 1$, $k_s / k_v = 1$, and $q_w / (k_h \ell_m^2) = 5$, have been used. The values of c_h computed using Eq. 16, are listed in Table 1. These compare well with each other for the three different vertical drain spacing, and are close to the c_v back-calculated for area IV using both the Aaoka and Casagrande methods.

Table 1. Values of EOP settlement and coefficient of consolidation interpreted from field settlement observations

Test area	n	EOP settlement (cm)		Δt (days)	β	c_h or c_v (m^2/yr)
		Asaoka	Casagrande			
I – 0.9	5.25	114	113	100	0.65	1.35
I – 1.5	8.75	100	97	100	0.84	1.17
I – 2.2	12.83	108	104	200	0.81	1.14
IV	-	107	-	1000	0.84	0.93-1.04

5 SECONDARY COMPRESSION

Asaoka (1978) also proposed an approach based on the Voight model for interpreting secondary settlement. Secondary compression and associated settlement behavior of clays can not be approximated by the Voight model which predicts (i) a final secondary settlement, and (ii) secondary settlement directly proportional to external load; both are inconsistent with observed secondary compression behavior of soils. For example, a constant C_α with time leads to a S_{j+1} versus S_j curve that at large elapsed times in comparison to $\Delta t = t_{j+1} - t_j$ becomes parallel to the 45° line through origin suggesting, as expected, no final secondary settlement. Furthermore, the shape of the secondary settlement curve or behavior of $C_\alpha = \Delta e/\Delta \log t$ with time predicted by the Voight model does not represent the general secondary settlement behaviors that have been observed for a wide variety of inorganic and organic soils, and have been explained by the C_α/C_c law of compressibility (Mesri and Godlewski 1977, Mesri 1987, Mesri and Castro 1987, Mesri et al. 1994, 1997, Mesri and Ajlouni 2007). Therefore, the Asaoka method is not recommended for predicting secondary settlement.

6 CONCLUSIONS

The Asaoka method is a useful tool for interpreting and extrapolating field observations of settlement. The graphical procedure for estimating EOP settlement and coefficient of consolidation is simple, however, the mathematical deduction by Asaoka is not. The Asaoka method is deduced here, using simple algebra, for one-dimensional compression with or without vertical drains. The equation for coefficient of consolidation, c_h , includes effects of smear, well resistance, and vertical water flow through soil, as well as an empirical correction to avoid dependence of back-calculated c_h on vertical drain spacing. Even though Eqs. 1 and 11 are solutions for instant loading, the Asaoka method is also applicable to time-dependent loading for construction time factors less than 1.0. The most reliable values of EOP settlement and coefficient of consolidation, using the Asaoka method, are obtained with settlement observations in the range of 40% to up to at least 80% primary consolidation.

The EOP settlements determined by the Asaoka procedure for the three sectors of area I, with three different vertical drain spacings, are comparable to values interpreted using the Casagrande procedure. The EOP settlement estimated using the Asaoka method for area IV without vertical drains is quite similar to those observed for area I with vertical drains, in spite of the fact that EOP consolidation for area I – 0.9 m was reached in less than 2 years whereas the EOP settlement of 107 cm for area IV requires about 38 years. In other words, the settlement observations for area I and observed settlements for area IV extrapolated using the Asaoka method, support the concept of EOP settlement independent of the duration of primary consolidation (Mesri and Choi 1985, Mesri et al. 1995). The Asaoka method is not recommended for predicting secondary settlement.

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