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Thermo-mechanical constitutive modeling of overconsolidated saturated clays

Modélisation du comportement thermo-mécanique des argiles saturées et fortement consolidées

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ABSTRACT

Using modified Cam-clay model and implementing a new approach to achieve deformation formulas, a thermo-mechanical model for saturated clays is presented in this paper. Proposed model does not include the required formulation for simulating changes in volume and pore pressure of soil in isotropic loading conditions which means that the model only predicts soil's response against deviatoric loading at fixed temperatures up to 100ºC in conventional triaxial test condition. Results of modeling have been compared with some previous experimental data and validity of formulations is proved.

RESUME

Utilisation du modèle Cam-clay modifié et la mise en œuvre d'une nouvelle approche pour parvenir à des formules de déformation, un modèle thermo-mécanique pour les argiles saturées est présentée dans le présent document. Le modèle proposé ne comprend pas la formulation de simulation des variations de volume et la pression d'eau dans les pores des sols dans les conditions de chargement isotrope qui signifie que le modèle prédit que la réponse du sol contre déviateur de chargement fixé à des températures allant jusqu'à 100ºC dans les conditions triaxial. Résultats de la modélisation ont été comparés avec des données expérimentales antérieures et de la validité des formulations est prouvé.

Keywords: thermo-mechanical constitutive model, Cam-clay, saturated clay, deformation analysis.

1 INTRODUCTION

Thermal effects on various properties of soils have been studied by several researchers. There are many engineering problems in which soil will be encountered with change of temperature such as induced temperature around high voltage buried cables (Mitchel et al. 1982), change in temperature of soil during sampling and storing (Hight 1983), thermal treatment (Bels & Stanculescu 1958), induced temperature by accelerated motions of some landslides in the shear band (Voight & Faust 1992) and generated heat in buffer material around nuclear waste barriers (Lingnau et al. 1995). It has been shown that rising temperature up to temperatures 100ºC, highly affects the engineering behavior of soils (compressibility, yielding response and shear strength).

Several constitutive laws have been proposed for thermo-mechanical modeling of clays mostly within the framework of modified Cam-clay model and critical state concepts (e.g. Hueckel & Borsetto 1990; Robinet et al. 1996; Graham et al. 2001).

Based on suggested approach by Liu and Carter (2002) for modeling the evolution of normal consolidation line (NCL) and then extracting deformation formulas, similar technique has been employed in this research for taking into account changes in location of NCL because of increase in temperature.

It must be noted that only mechanical behavior of saturated clays at constant temperature, deviatoric loading and conventional triaxial test condition is considered in this model. For isotropic heating, several formulations have been presented before that can be applied (e.g. Laloui & Cekerevac 2003; Abuel-Naga 2005).

2 FORMULATION

For the purpose of modeling, some idealizations from the movement of NCL to lower values of void ratio due to the increase in temperature have been made:

1- The slope of critical state line (CSL) in deviatoric plane, q-p′, and also in compression plane, e-Lnp′, are not temperature dependant.

2- If the slopes of NCL and CSL are the same at ambient temperature, the slope of NCL will not be temperature dependant.

3- If the slopes of NCL and CSL differ at ambient temperature, the slope of CSL will always be smaller. By increase in temperature, the slope of NCL reduces so that at the maximum possible endurable temperature of soil, it will become equal to the slope of CSL. This idealization is illustrated in figure 1.

4- Reduction in void ratio by increase in temperature will not be more than the maximum difference between void ratio of NCL and CSL at ambient temperature thus it can be concluded that NCL at elevated temperatures will never be placed below CSL at ambient temperature.

5- Thermal volumetric strain of soil aggregates and water are neglected.

(Assumptions 1 to 4 are generally inferred from investigation of reported data by Graham et al., 2001; Ghahremannejad, 2003; Cekerevac & Laloui, 2004; Abuel-Naga, 2005)

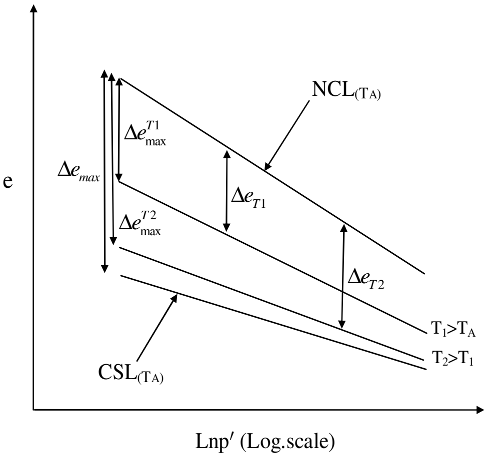

Figure 1: Idealization from the location of isotropic normal consolidation line at elevated temperatures

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From figure 1, for each temperature it can be written:

$$e_T = e_A - \Delta e_T \tag{1}$$

where e_T and e_A are void ratios of soil at elevated and ambient temperature, respectively and Δe_T is reduction in void ratio due to movement of NCL toward the critical state line associated with increase in temperature.

In order to consider the reduction in slope of NCL at elevated temperatures Δe_T is assumed to be stress dependant as proposed by equation (2):

$$\Delta e_T = \omega \Delta e_{\max} \left(\frac{p_0'}{p'} \right)^n \tag{2}$$

where p_0' is the mean effective stress at beginning of test and n is a dimensionless parameter. $\Delta e_{\max}$ is the maximum difference between void ratio of NCL and CSL at ambient temperature which is shown in figure 1. The following equation is proposed for determination of ω:

$$\omega = 1 - \exp\left[\chi \left(1 - \frac{T}{T_A} \right) \right] \tag{3}$$

where T_A and T are ambient and elevated temperatures, respectively. χ is a model parameter which will be determined by equation (4):

$$\frac{\Delta e_{\max}^T}{\Delta e_{\max}} = 1 - \exp\left[\chi \left(1 - \frac{T}{T_A} \right) \right] \tag{4}$$

where $\Delta e_{\max}^T$ is the maximum reduction in void ratio at elevated temperature in respect to ambient temperature as illustrated in figure 1. From this figure it can be deduced that $\Delta e_{\max}^T$ will occur at the beginning of consolidation test.

From critical state theory, void ratio on the compression plane is defined as:

$$e_A = N - \lambda \ln\left(p_c'\right) + \kappa \ln\left(\frac{p_c'}{p'} \right) \tag{5}$$

where N is the specific volume at unit pressure and ambient temperature, λ is the slope of isotropic compression line, κ is the slope of unloading-reloading line, p_c' is the preconsolidation pressure and p' is the current mean effective stress.

Substituting equations (2) and (5) into equation (1) the following relation is achieved for determination of void ratio at elevated temperatures:

$$e_T = N - \lambda \ln\left(p_c'\right) + \kappa_T \ln\left(\frac{p_c'}{p'} \right) - \omega \Delta e_{\max} \left(\frac{p_0'}{p_c'} \right)^n \tag{6}$$

where κ_T is the slope of unloading-reloading line at desired temperature. Assuming that elastic volumetric strains are only caused by change in mean effective stress and knowing that plastic strains are resulted from change in preconsolidation pressure, it can be concluded that the strains caused by reduction in void ratio as a result of increase in temperature must be dealt as plastic strains and therefore the p' in equation (2) is replaced with p_c' when being substituted in equation (5). Dependency of κ_T to temperature is accepted as suggested by Graham et al. (2001):

$$\frac{\kappa_T}{\kappa_0} = 1 + C \ln\left(\frac{T}{T_0} \right) \tag{7}$$

where κ_0 is the slopes of unloading-reloading line at ambient temperature, T_0 and C is a model parameter.

Derivation of the equation (6) with respect to p' and p_c' will lead to equations (8) and (9) for calculating elastic and plastic strains, respectively.

$$d\varepsilon_v^e = \frac{\kappa_T}{1 + e_0} \frac{dp'}{p'} \tag{8}$$

$$d\varepsilon_v^p = \frac{\lambda_A - \kappa_T}{1 + e} \frac{dp_c'}{p_c'} - \frac{n \Delta e_T}{1 + e} \frac{dp_c'}{p_c'} \tag{9}$$

Elastic bulk modulus is assumed to be stress dependant as follows:

$$K = K_0 \left(\frac{p'}{p_0'} \right)^a \tag{10}$$

where P_0' and K_0 are effective stress and bulk modulus at the beginning of test, respectively. Also a is a model parameter. K_0 is defined with equation (11):

$$K_0 = \frac{\left(1 + e_0\right) p_0'}{\kappa_T} \tag{11}$$

therefore:

$$d\varepsilon_v^e = \frac{1}{K} dp' \tag{12}$$

A modification for considering the effect of shearing mechanism on plastic strains is accepted which has been originally proposed by Liu and Carter (2002):

$$d\varepsilon_v^p = \frac{\lambda_A - \kappa_T}{1 + e} \frac{dp_c'}{p_c'} - n \Delta e_T \left(\frac{M}{M - \eta} \right) \frac{dp_c'}{\left(1 + e\right) p_c'} \tag{13}$$

Shearing modulus is considered to be both temperature and stress-level dependant with equation (14):

$$G = G_0 \left(\frac{p'}{p_0'} \right)^b \left[1 + D \ln\left(\frac{T}{T_0} \right) \right] \tag{14}$$

where G_0 is the shearing modulus at ambient temperature. D and b are two dimensionless modeling parameters.

3 FLOW RULE

As it has been stated by almost all the previous researchers, an increase in temperature will cause a reduction in void ratio. Hence it seems logical for the effect of temperature on flow rule to be inline with change in void ratio Δe_T caused by increase in temperature.

Graham et al. (2001) reported independency of flow rule to temperature which means independency of $d\varepsilon_v^p / d\varepsilon_s^p$ to temperature.

Experimental data of Abuel-Naga (2005) for soft Bangkok clay show the increase of plastic strain ratio, $d\varepsilon_v^p / d\varepsilon_s^p$, with increase in temperature while results of Ghahremannejad (2003) indicate a reduction in this ratio with increase in temperature.

Equation (15) is implemented for change of plastic strain ratio with increase in temperature to include all possible trends:

$$\psi = \frac{d\varepsilon_v^p}{d\varepsilon_s^p} = \frac{M^2-\eta^2}{2\theta_T\eta} \tag{15}$$

where θ_T is the model parameter dependent to change in void ratio caused by increase in temperature. As a result, θ_T is presented as a function of ω. According to the three sets of data reported by Graham et al. 2001, Ghahremannejad, 2003 and Abuel-Naga 2005 and observation of an acceptable consistency between predictions and experimental data, the relation between θ_T and ω is suggested as follows:

1- Independency of ψ to temperature (Graham et al. 2001) $\theta_T = 1$

2- Decrease of ψ with increase in temperature (Ghahremannejad 2003) $\theta_T = 1+\omega$

3- Increase in ψ with increase in temperature (Abuel-Naga 2005) $\theta_T = 1-\omega$

In present model, the associated flow rule is accepted to be valid at all temperatures hence yield surface and plastic potential surface are the same.

4 CALIBRATION AND VERIFICATION

Current model has six model parameters of χ, *n*, *C*, *D*, *a* and *b* in addition to five known parameters of original Cam-clay model. All 11 parameters can be determined with conventional triaxial test.

The comparison of modeling results with experimental data on soft Bangkok clay reported by Abuel-Naga (2005) is presented in this section. The tests, drained and undrained, have been conducted in temperatures 25, 70 and 90ºC with preconsolidation pressure, p'_c, of 300 kPa. Modeling parameters of soft Bangkok clay are χ=0.26, *C*=0 and *D*=1.22. Since evolution of NCL to lower void ratios is reported by Abuel-Naga (2005) as a completely parallel movement, the value of *n* has been considered as zero for this soil. Some researchers have assumed two parameters of *a* and *b* to be equal and have proposed a relation for their calculation as a function of OCR (Liu & Xing 2008). However such a thing is not investigated in present research and these parameters, (*a*, *b*), have been determined by calibration for OCR values of 1, 2, 4 and 8 as (-5, 0), (-12, -3), (0, -3) and (0, -3) respectively in drained state. For undrained condition, *a* and *b* have been considered to be 0 and -3, respectively for all OCR values. Definition of OCR in following figures is the same as conventional modified Cam-clay model.

Figures 2 and 3 show modeling results for deviatoric stress-axial strain and volumetric strain-axial strain for overconsolidated samples at temperature 70ºC.

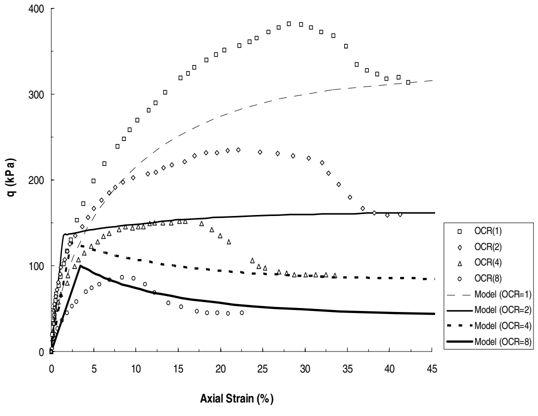

Figure 2: Model predictions for deviatoric stress-axial strain for OC samples of soft Bangkok clay in drained condition at 70ºC with p'$_c$=300 kPa (experimental data after Abuel-Naga, 2005)

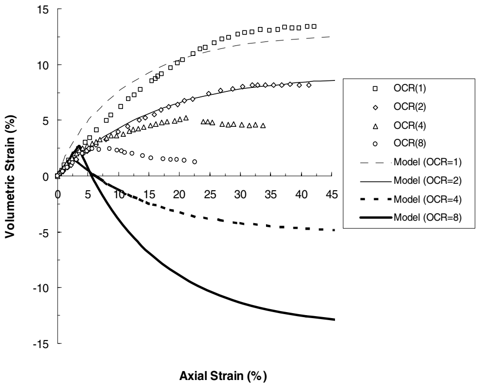

Figure 3: Model predictions for volumetric strain-axial strain for OC samples of soft Bangkok clay in drained condition at 70ºC with p'$_c$=300 kPa (experimental data after Abuel-Naga, 2005)

Although model predictions for pre-yield behavior of soil are not completely satisfactory, it simulates well the post-peak shear strength as shown in figure 2. The maximum shear strength in overconsolidated cases is simulated in strains smaller than those of experimental data but overall trend is agreeable. It's usually believed that peak shear strength increases by increase in OCR while counterintuitive data are shown in figure 1 which is due to identical initial consolidation pressure, p'_c, of 300 kPa in all tests. As OCR increases, initial confinement in start of shear loading reduces which results in lower peak shear strength. This behavior is successfully simulated by model.

Volumetric strains in OCR values of 1 and 2 are of excellent consistency with experimental data in figure 3 while by increase in OCR dilatancy predicted by model is far away from experiments. Although according to the critical state theory it seems reasonable that dilatancy occurs in high OCR values, tests conducted in higher temperatures do not show the same trend. Similar modeling results for undrained condition are presented in figures 4 and 5.

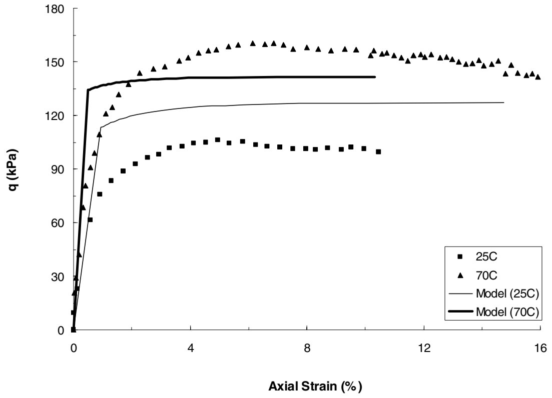

Figure 4: Model predictions for deviatoric stress-axial strain for OC samples of Bangkok clay with OCR=1.5 in undrained condition at temperatures of 25ºC and 70ºC with p'$_c$=300 kPa (experimental data after Abuel-Naga, 2005)

Predictions of pre-yield and peak strength in undrained condition illustrated in figure 4 are satisfactory. Final pore pressure in temperature of 70°C in figure 5 is totally simulated however convergence of curves in about 3% axial strain can not be based on reality. It seems that mutual interactions of parameters besides application of associated flow rule are responsible for this phenomenon.

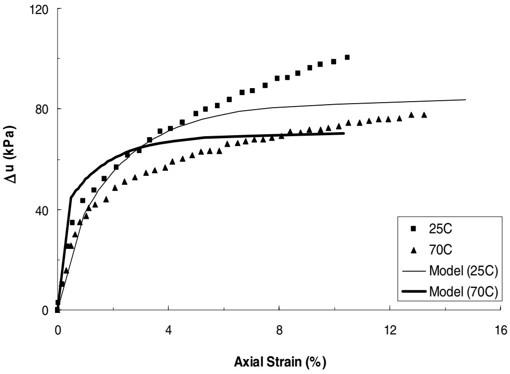

Figure 5: Model predictions for pore pressure-axial strain for OC samples of Bangkok clay with OCR=1.5 in undrained condition at temperatures of 25°C and 70°C with p'_c=300 kPa (experimental data after Abuel-Naga, 2005)

5 CONCLUSION

A new model for predicting thermo-mechanical behavior of saturated overconsolidated clays at temperatures up to 100°C is presented in this paper. The model is based on a new approach originally proposed by Liu and Carter (2002) for modeling behavior of structured soils using the results of experiment on reconstituted samples. However, present model deals only with reconstituted saturated clays at elevated temperatures. The model has six more parameters than modified Cam-clay model, all of which can be determined by conventional triaxial test. Simplicity of model makes it suitable for being applied in numerical simulations. All in all, overall predictions of model seem to be of relative accuracy for practical applications.

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