Coupled hydraulic and mechanical behavior of unsaturated soils: theory and validation

Comportement hydraulique et mécanique couplés des sols non saturés : théorie et évaluation

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ABSTRACT

Unsaturated soils are three-phase porous media consisting of a solid skeleton, pore water, and pore air. It is well known that the behavior of unsaturated soils is influenced heavily by matric suction. Soil water characteristic curves (SWCCs) are necessary to describe the hydraulic behavior (flow of water and air) in an unsaturated soil. SWCCs describe the relationship between the matric suction and the water content in unsaturated soils. SWCCs show hysteretic behavior depending on wetting/drying history of a soil. Recently geotechnical engineers have begun to realize that SWCCs also depend on the stress-strain history (mechanical behavior) of a soil. The hydraulic behavior of unsaturated soils, on the other hand, influences the mechanical behavior through matric suction. In order to predict the behavior of unsaturated soils, a comprehensive constitutive model for unsaturated soils is developed. The hysteresis in SWCCs is modeled using concepts that parallel the elastoplastic theory used to model stress-strain behavior of soils. Matric suction is used as the stress variable and volumetric water content is used as the strain variable in modeling the SWCCs. The model also captures the influence of stress-strain history on the SWCCs and the influence of SWCCs on the stress-strain behavior of soils through the use of an intergranular stress tensor and coupled elastoplastic hardening laws. The constitutive model is calibrated and validated using complex triaxial tests on Toyoura sand under both monotonic and cyclic loadings.

RÉSUMÉ

Les sols non saturé sont des milieux poreux composés de trois éléments consistant en un squelette solide, des pores d'eau, et des pores d'air. Il est bien connu que le comportement des sols non saturé est fortement influencé par la succion matricielle. Les courbes caractéristiques sol-eau (SWCCs) sont nécessaires pour décrire le comportement hydraulique (écoulement d'eau et d'air) d'un sol non saturé. Les SWCCs décrivent la relation entre la succion matricielle et la teneur en eau d'un sol non saturé. Les SWCCs mettent en évidence un comportement hystérétique qui dépend de l'historique du mouillage/séchage du sol. Récemment les ingénieurs géotechniques ont commencé à se rendre compte que les SWCCs dépendent aussi de l'historique de la contrainte-déformation (comportement mécanique) du sol. D'autre part, le comportement hydraulique des sols non saturé influence le comportement mécanique par la succion matricielle. Afin de prédire le comportement des sols non saturé, un modèle constitutif complet est développé. L'hystérèse des SWCCs est définie en utilisant des concepts équivalents à la théorie élastoplastique utilisée pour définir le comportement de la contrainte-déformation des sols. La succion matricielle est utilisée comme la variable de contrainte et la teneur en eau volumétrique est utilisée comme la variable de déformation pour de définir les SWCCs. Le modèle décrit également l'influence de l'historique de la contrainte-déformation sur les SWCCs ainsi que l'influence des SWCCs sur le comportement de la contrainte-déformation des sols en utilisant un tenseur de contrainte intergranulaire et les lois de durcissement élasto-plastique. Le modèle constitutif est calibré et validé en utilisant des essais triaxiaux complexes sur le sable de Toyoura qui a été soumis à des chargements monotones et à des chargements cycliques.

Keywords: unsaturated soils, stress-strain behavior, hysteretic SWCCs, elastoplasticity

1 INTRODUCTION

Unsaturated soils are multiphase porous media consisting of a solid skeleton, pore water and pore air. The stress-strain behavior of unsaturated soils is elastoplastic. Soil water chracteristic curves (SWCCs) that relate matric suction (pore air pressure minus pore water pressure) and water content also show hysteretic behavior. Experimental evidence indicates that SWCCs (hydraulic) and stress-strain (mechanical) behavior of unsaturated soils are coupled (Cui and Delage 1996; Rampino *et al*. 2000; Geiser *et al.* 2006). Although many constitutive models for unsaturated soils have been developed in recent years, for example, Alonso *et al.* (1990), Wheeler *et al.* (2003), Li (2007a&b) and Khalili *et al.* (2008), some fundamental issues related to unsaturated soil modeling, such as the selection of stress-strain variables, are still being researched. Most of the available models for unsaturated soils are not complete in the sense that some do not account for the water content variation (e.g. Alonso *et al*. 1990; Farias *et al*. 2006), some of them can not describe the hysteresis of SWCCs (Thu *et al*. 2007) and some provide no information on unsaturated soil behavior under cyclic loading (Li 2007b). Furthermore, many models that predict coupled hydraulic-mechanical behavior (e.g. Wheeler *et al.* 2003) are restricted to isotropic stress states only. The dearth of information on the interactions between hydraulic and mechanical behavior forms a big gap in the constitutive modeling of unsaturated soils. This paper presents a comprehensive, coupled hydraulic-mechanical constitutive model to simulate the behavior of unsaturated soils in a general stress space.

2 SELECTION OF STRESS-STRAIN VARIABLES

Due to the success in modeling saturated geomaterial behavior using the effective stress concept, many researchers have attempted to extend the effective stress concept for unsaturated soils. Bishop (1959) is the first one to propose an expression for effective stress in unsaturated soils. The Bishop's expression

Proceedings of the 17th International Conference on Soil Mechanics and Geotechnical Engineering
M. Hamza et al. (Eds.)

doi:10.3233/978-1-60750-031-5-452

and many subsequent expressions either lack rigorous theoretical basis or have practical implementation limitations. Wei and Muraleetharan (2002a&b) established energy dissipation equations for the soil skeleton (D_s) and the pore water (D_w), as shown below, by considering the dynamic compatibility conditions on interfaces between phases and rigorous thermodynamic restrictions. Soil mechanics sign convention is used in these equations, i.e., compressive stresses are considered positive. These energy dissipation equations provide insight into proper stress and strain variables that can be used to describe the behavior of unsaturated soils.

$$D_s = n_s\left(\mathbf{t}_s - p_a\mathbf{I}\right) : \dot{\boldsymbol{\varepsilon}}^p + \xi\dot{\chi} \geq 0 \quad (1)$$

$$D_w = -s_c\dot{n}_w^p + \pi\dot{\gamma} \geq 0 \quad (2)$$

$$\boldsymbol{\sigma}' = n_s\left(\mathbf{t}_s - p_a\mathbf{I}\right) = \left(\boldsymbol{\sigma} - p_a\mathbf{I}\right) + n_w s_c\mathbf{I} \quad (3)$$

where n_s and n_w are volume fractions of solid component and water, respectively. s_c is matric suction, which is equal to the difference between pore air pressure p_a and pore water pressure p_w. $\mathbf{I}$ is the second order unit tensor. $\mathbf{t}_s$ is the intrinsic Cauchy stress tensor of solid component. $\boldsymbol{\varepsilon}^p$ is the plastic strain of solid component. n_w^p is the plastic volumetric water content; ξ and π are internal forces associated with χ and γ, which account for the hardening of the solid skeleton and the water phase, respectively. $\boldsymbol{\sigma}$ is total stress tensor. $\boldsymbol{\sigma}'$ is referred to as *the intergranular stress tensor*.

Based on these expressions, the appropriate stress-strain variables for unsaturated soils can be selected as the intergranular stress, which is conjugated with the strain tensor of the soil skeleton. As to the hydraulic behavior, the conjugated suction and volumetric water content will be adopted.

3 MODEL FORMULATION

3.1 *Elastic responses*

Elastic responses of soil skeleton and pore water are given as:

$$\dot{\varepsilon}_v^e = \frac{\dot{I}}{K}\;;\; \dot{\boldsymbol{\varepsilon}}_q^e = \frac{\dot{\mathbf{s}}}{2G}\;;\; \dot{n}_w^e = \frac{\dot{s}_c}{\Gamma^e} \quad (4.1)$$

$$K = K_0\left(\frac{I}{p_{ref}}\right)^{b_1}\;;\; G = G_0\left(\frac{I}{p_{ref}}\right)^{d_1}\;;\; \Gamma^e = const. \quad (4.2)$$

where I and $\mathbf{s}$ are the intergranular hydrostatic stress and the deviatoric stress tensor, respectively; ε_v^e and $\boldsymbol{\varepsilon}_q^e$ are the recoverable volumetric strain and deviatoric strain of soil skeleton; K , G and Γ^e are the bulk modulus, shear modulus and capillary elastic modulus, respectively. p_{ref} is a reference pressure. K_0 and G_0 are bulk and shear moduli of the soil skeleton when I equals p_{ref} . b_1 and d_1 are two model parameters. Γ^e is assumed to be a constant in this study, but Γ^e can be treated as a function of suction if test results warrant such a treatment.

3.2 *Yield surface*

The objective of this study is to develop a coupled model for unsaturated sands and silts. Therefore, following Manzari and Dafalias (1997) and Taiebat and Dafalias (2008), the yield surface is defined in such a way that isotropic and kinematic hardening effects are fully considered. The yield surface in general stress space is defined as:

$$f = \sqrt{(\mathbf{s} - I\boldsymbol{\alpha}):(\mathbf{s} - I\boldsymbol{\alpha})} - \sqrt{2/3}\,mI\sqrt{1-\left(I/I_0\right)^{\beta}} = 0 \quad (5)$$

where $\boldsymbol{\alpha}$ stands for the center of the yield surface. m is the isotropic hardening parameter. The coefficient $\sqrt{2/3}$ has been introduced for convenience of interpretation in the standard triaxial stress space. I_0 represents a high confining pressure, which should not be lower than the highest confining pressure experienced by a sand sample during loading. β is a model parameter, which can be set to 20 as a default value as suggested by Taiebat and Dafalias (2008) or calibrated from experimental results. A desirable property of this yield surface, especially for unsaturated silts, is its closed cap-like shape at the tip, where the stress level becomes close to I_0 . For clarity, a schematic illustrating the yield surface and the other three surfaces are given below:

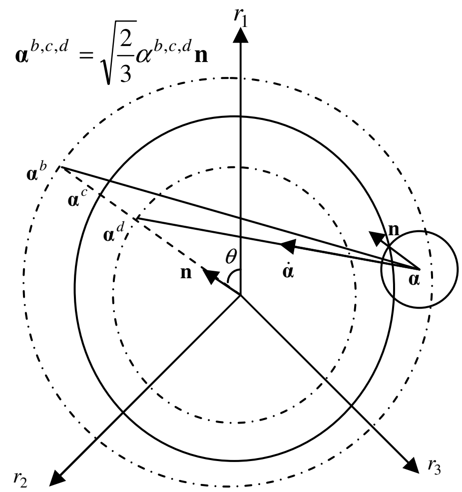

Figure 1. Model surfaces in general stress space (after Taiebat and Dafalias 2008)

3.3 *Dilatancy, critical and bounding surfaces*

Following Manzari and Dafalias (1997), definitions for the dilatancy, critical and bounding surfaces in triaxial stress space are given as:

$$M_c^b = \alpha_c^b + m\sqrt{1-\left(I/I_0\right)^{\beta}} = M_c^c + k_c^b < -\psi > \quad (6.1)$$

$$M_e^b = \alpha_e^b + m\sqrt{1-\left(I/I_0\right)^{\beta}} = M_e^c + k_e^b < -\psi > \quad (6.2)$$

$$M_c^c = \alpha_c^c + m\sqrt{1-\left(I/I_0\right)^{\beta}} \quad (6.3)$$

$$M_e^c = \alpha_e^c + m\sqrt{1-\left(I/I_0\right)^{\beta}} \quad (6.4)$$

$$M_c^d = \alpha_c^d + m\sqrt{1-\left(I/I_0\right)^{\beta}} = M_c^c + k_c^d\psi \quad (6.5)$$

$$M_e^d = \alpha_e^d + m\sqrt{1-\left(I/I_0\right)^{\beta}} = M_e^c + k_e^d\psi \quad (6.6)$$

where: superscripts *b*, *c* and *d* represent variables for the bounding surface, the critical surface and the dilatancy surface, respectively. Subscripts *c* and *e* stand for variables under compression and extension, respectively. M 's are stress ratios on the three surfaces and α 's are slopes. < > is the Macauley brackets. k_c^b , k_e^b , k_c^d and k_e^d are model parameters. The state parameter, i.e. $\psi = e - e_c = e - e_{cr} + \lambda\left(I/p_{ref}\right)^{\xi}$ is the difference between the current void ratio e and the critical void ratio e_c . e_{cr} , λ and ξ are model parameters.

3.4 *Lode angle effects*

With the definition of a modified Lode angle θ, all the variables defined in the triaxial space can be generalized to the multiaxial stress space. The Lode angle related variables are:

$$\mathbf{r}=\mathbf{s}-I\boldsymbol{\alpha}\,,\ \bar{\mathbf{r}}=\mathbf{r}/I=(\mathbf{s}/I-\boldsymbol{\alpha})\,,\ \mathbf{n}=\mathbf{r}/\sqrt{\mathbf{r}:\mathbf{r}} \tag{7.1}$$

$$\bar{J}=\left[\frac{1}{2}tr\left(\bar{\mathbf{r}}^2\right)\right]^{1/2},\ \bar{S}=\left[\frac{1}{3}tr\left(\bar{\mathbf{r}}^3\right)\right]^{1/3} \tag{7.2}$$

$$g(\theta,c)=\frac{2c}{(1+c)-(1-c)\cos 3\theta},\ \cos 3\theta=\frac{3\sqrt{3}}{2}\left(\frac{\bar{S}}{\bar{J}}\right)^3 \tag{7.3}$$

$$c=M_e^c/M_c^c\,,\ c_b=k_e^b/k_c^b\,,\ c_b=k_e^b/k_c^b \tag{7.4}$$

where $\bar{J}$ and $\bar{S}$ are the second and third stress invariants. θ is the Lode angle. c, c_b and c_d are the conversion factors between extension and compression quantities. $\mathbf{n}$ is the unit deviatoric stress ratio tensor. The stress ratios in the triaxial stress space can be generalized to the multiaxial stress space as:

$$\boldsymbol{\alpha}_\theta^b=\sqrt{2/3}\alpha_\theta^b\mathbf{n},\ \boldsymbol{\alpha}_\theta^d=\sqrt{2/3}\alpha_\theta^d\mathbf{n},\ \boldsymbol{\alpha}_\theta^c=\sqrt{2/3}\alpha_\theta^c\mathbf{n} \tag{8.1}$$

$$\alpha_\theta^b=g(\theta,c)M_c^c+g(\theta,c_b)k_c^b<-\psi>-m\sqrt{1-(I/I_0)^\beta} \tag{8.2}$$

$$\alpha_\theta^d=g(\theta,c)M_c^c+g(\theta,c_d)k_c^d\psi-m\sqrt{1-(I/I_0)^\beta} \tag{8.3}$$

$$\alpha_\theta^c=g(\theta,c)M_c^c-m\sqrt{1-(I/I_0)^\beta} \tag{8.4}$$

3.5 *Hardening laws*

The kinematic hardening parameter $\boldsymbol{\alpha}$ is assumed to be a function of the current stress state and plastic deviatoric strain of the soil skeleton. The suction is assumed to affect the kinematic hardening only through its contribution to the intergranular stress tensor. The isotropic hardening parameter m is not only a function of the stress state, but also plastic volumetric strain and irrecoverable volumetric water content.

$$\boldsymbol{\alpha}=\boldsymbol{\alpha}\left(\boldsymbol{\sigma}',s_c;\boldsymbol{\varepsilon}_q^p\right),\ m=m\left(\boldsymbol{\sigma}',s_c;\varepsilon_v^p,n_w^p\right) \tag{9.1}$$

$$\dot{\boldsymbol{\alpha}}=\sqrt{\frac{3}{2}}h\left[\frac{2}{3}\alpha_\theta^b\dot{\boldsymbol{\varepsilon}}_q^p-\dot{e}_q^p\boldsymbol{\alpha}\right]=<\Lambda>h\left(\boldsymbol{\alpha}_\theta^b-\boldsymbol{\alpha}\right)=<\Lambda>h\mathbf{b} \tag{9.2}$$

$$\dot{m}=\frac{\partial m}{\partial\varepsilon_v^p}\dot{\varepsilon}_v^p+\frac{\partial m}{\partial n_w^p}\dot{n}_w^p=c_v(1+e_0)\dot{\varepsilon}_v^p+c_m\left(\frac{s_c n_w}{p_{ref}}\right)^{\varpi}\dot{n}_w^p \tag{9.3}$$

$$h=h_0\frac{|\mathbf{b}:\mathbf{n}|}{b_{ref}-|\mathbf{b}:\mathbf{n}|},\ \dot{e}_q^p=\sqrt{2/3\dot{\boldsymbol{\varepsilon}}_q^p:\dot{\boldsymbol{\varepsilon}}_q^p}\,,\ b_{ref}=2\sqrt{2/3}\alpha_c^b \tag{9.4}$$

where Λ is the loading index. e_0 is the initial void ratio. c_v, c_m and ϖ are model parameters. The rate of change of the kinematic hardening parameter $\dot{\boldsymbol{\alpha}}$ depends on the distance between $\boldsymbol{\alpha}$ and its image on the bounding surface $\boldsymbol{\alpha}_\theta^b$. This is exactly what is stated by the bounding surface plasticity concept (Dafalias and Popov 1975). The added feature of h depending on $|\mathbf{b}:\mathbf{n}|$ was proposed by Dafalias (1986) and its performance was checked by Manzari and Dafalias (1997). Observations on the SWCCs show that $s_c n_w$ is an increasing function of suction. When suction increases, n_w^p also increases. The suction-induced hardening effects are accounted for in this way.

3.6 *Model for Hysteretic SWCCs*

To investigate the coupling effects between mechanical and hydraulic behavior of unsaturated soils, a model for SWCCs based on the bounding surface plasticity concept (Dafalias and Popov 1975) is proposed and the evolution of the bounding suctions (s_{c0w} and s_{c0d}) are given below:

$$\text{Wetting: } n_w=\frac{n_{ws}+n_{wr}\left(s_{c0w}/b_2\right)^{d_2}}{1+\left(s_{c0w}/b_2\right)^{d_2}} \tag{10.1}$$

$$\text{Drying: } n_w=\frac{n_{ws}+n_{wr}\left(s_{c0d}/b_3\right)^{d_3}}{1+\left(s_{c0d}/b_3\right)^{d_3}} \tag{10.2}$$

$$\text{Wetting: } \dot{s}_{c0w}=\Gamma_{vw}^p\dot{\varepsilon}_v^p+\Gamma_{0w}^p\dot{n}_w^p,\ \Gamma_{vw}^p=s_{c0w}\zeta v \tag{10.3}$$

$$\text{Drying: } \dot{s}_{c0d}=\Gamma_{vd}^p\dot{\varepsilon}_v^p+\Gamma_{0d}^p\dot{n}_w^p,\ \Gamma_{vd}^p=s_{c0d}\zeta v \tag{10.4}$$

$$\text{Wetting: } \Gamma_{0w}^p=\frac{\partial s_c}{\partial n_w}=-\frac{1}{d_2}\frac{(n_{ws}-n_{wr})s_{c0w}}{(n_{ws}-n_w)(n_w-n_{wr})} \tag{10.5}$$

$$\text{Drying: } \Gamma_{0d}^p=\frac{\partial s_c}{\partial n_w}=-\frac{1}{d_3}\frac{(n_{ws}-n_{wr})s_{c0d}}{(n_{ws}-n_w)(n_w-n_{wr})} \tag{10.6}$$

where: b_2, d_2, b_3, d_3 and ζ are material parameters and v is the specific volume. n_{ws} and n_{wr} are saturated and residual volumetric water contents, respectively. s_{c0w} and s_{c0d} are suctions on the wetting and drying bounds, respectively. Γ_{0w}^p and Γ_{0d}^p are the capillary plastic moduli on the wetting and drying bounds, respectively. According to Dafalias and Popov (1976), capillary plastic modulus Γ^p is assumed as:

$$\Gamma^p=\Gamma^p(\delta,\delta_{in})=\Gamma_0^p(1+h\cdot\delta/<\delta_{in}-g\delta>) \tag{11}$$

where δ (in suction unit) is the distance between current suction and its corresponding bounding curve; δ_{in} is the value of δ at the initiation of yielding for each drying/wetting process. Γ_0^p in Eq. (11) depends on the suction path and it can be Γ_{0w}^p or Γ_{0d}^p when the path is wetting or drying. Details of the SWCCs model can be found in Liu and Muraleetharan (2006). The SWCCs model has also been implemented into a 1-D model (Muraleetharan *et al.* 2009; Miller *et al.* 2008) to investigate the coupled mechanical and hydraulic behavior of unsaturated soils under isotropic loading conditions.

4 MODEL PERFORMANCE

Very limited test results are available to investigate the coupling effects between the mechanical and hydraulic behavior of unsaturated soils. Generally, the cyclic drying-wetting process influences the elastoplastic behavior of unsaturated soils and the soil skeleton deformation introduces shift in the soil water characteristic curves. A series of tests on Toyoura sand (Verdugo and Ishihara 1996; Uchida and Stedman 2001; Unno *et al.* 2008) are used to verify the performance of the proposed unsaturated soil model. The model parameters were calibrated using both saturated and unsaturated triaxial tests and a SWCC measured during a drying test. Following model calibration an undrained strain controlled cyclic test (Unno *et al.* 2008) on an unsaturated Toyoura sample (initial degree of saturation = 84.6%) is predicted. Measured and predicted mean intergranular stress-deviatoric stress curves are shown in Figure 2.

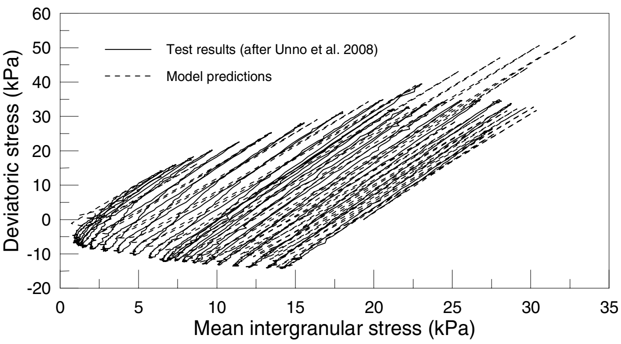

Figure 2. Measured and predicted stress paths in an undrained test

As shown in Figure 2, the mean intergranular stress decreases with cyclic loading. Under cyclic loading, the soil sample experiences compaction and this leads to gradual increase in pore air and water pressures as shown in Figure 3. The model predictions shown in Figures 2 and 3 compare well with the measured results.

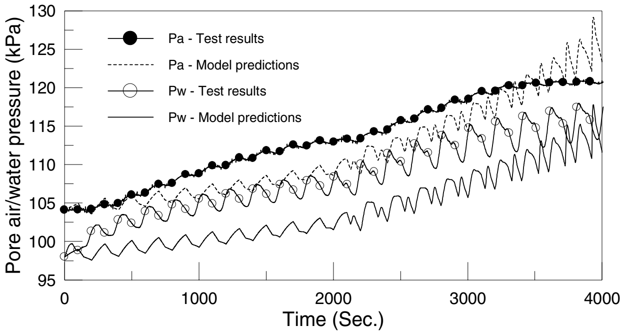

Figure 3. Measured and predicted pore air and pore water pressures

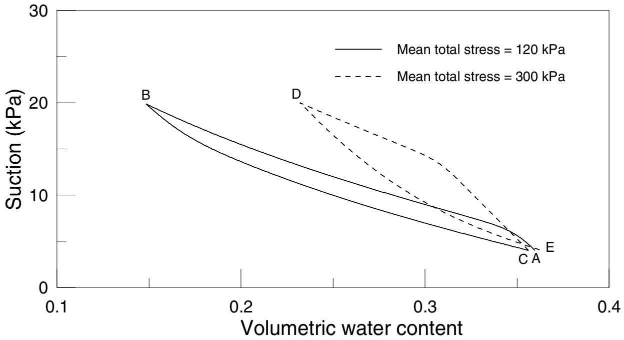

Figure 4. Influence of stress state on SWCCs

In order to study the influence of the stress state on SWCCs, a hypothetical model simulation is carried out using the calibrated model parameters and the results are shown in Figure 4. SWCCs obtained for two different mean total stresses are shown in Figure 4. The simulation was done as follows. The suction path A (4 *kPa*) → B (20 *kPa*) → C (4 *kPa*) is simulated keeping the mean total stress at 120 *kPa*. At point C, the mean total stress is increased from 120 *kPa* to 300 *kPa*. Then the suction is changed from C (4 *kPa*) → D (20 *kPa*) → E (4 *kPa*) while the mean total stress is kept constant at 300 kPa. The effects of soil deformation on the SWCCs can be clearly seen in Figure 4.

5 CONCLUSIONS

A constitutive model for unsaturated sands and silts in the general stress space is presented. A hysteretic SWCCs model, a new intergranular stress, and special hardening laws are proposed to describe the coupling effects between the hydraulic and mechanical behavior. Reasonable comparisons between model predictions and unsaturated cyclic triaxial test results on Toyoura sand are obtained. Suction affects the stress-strain behavior and the stress state of a soil has significant influence on the SWCCs.

ACKNOWLEDGEMENT

This work was supported by the U.S. National Science Foundation (Grant No. CMS-0301457) and this support is acknowledged.

REFERENCES

Alonso, E.E., Gens, A. and Josa, A. 1990. A constitutive model for partially saturated soils. Géotechnique, Vol. 40, pp. 405-430.

Bishop, A.W. 1959. The principle of effective stress. *Tek. Ukebl.*, Vol. 106, pp. 113-143.

Cui, Y.J. and Delage, P. 1996. Yielding and plastic behaviour of an unsaturated compacted silt. Géotechnique, Vol. 46, pp. 291-311.

Dafalias, Y.F. 1986. Bounding surface plasticity, I: mathematical foundation and hypoplasticity. J. Eng. Mech., ASCE, Vol. 112, pp. 966-987.

Dafalias, Y.F. and Popov, E.P. 1975. A model of nonlinearly hardening materials for complex loading. Acta Mech., Vol. 21, pp. 173-192.

Dafalias, Y.F. and Popov, E.P. 1976. Plastic internal variables formalism of cyclic plasticity. J. Appl. Mech., Vol. 43, pp. 645-651.

Farias, M.M., Pinheiro, M. and Neto, M.P.C. 2006. An elastoplastic model for unsaturated soils under general three-dimensional conditions. Soils Found., Vol. 46, pp. 613-628.

Geiser, F., Laloui, L. and Vulliet, L. 2006. Elasto-plasticity of unsaturated soils: laboratory test results on a remoulded silt. Soils Found., Vol. 46, pp. 545-556.

Khalili, N., Habte, M.A. and Zargarbashi, S. 2008. A fully coupled flow deformation model for cyclic analysis of unsaturated soils including hydraulic and mechanical hystereses. Computers and Geotechnics, Vol. 35, pp. 872-889.

Li, X.S. 2007a. Thermodynamics-based constitutive framework for unsaturated soils 1: Theory. Géotechnique, Vol. 57, pp. 411-422.

Li, X.S. 2007b. Thermodynamics-based constitutive framework for unsaturated soils 2: A basic triaxial model. Géotechnique, Vol. 57, pp. 423-435.

Liu, C. and Muraleetharan, K.K. 2006. Description of soil water characteristic curves using the bounding surface plasticity theory. Proc. 4th Int. Conf. Unsat. Soils, Geo-Institute, ASCE, Carefree, Arizona, Vol. 2, pp. 2432-2440.

Manzari, M.T. and Dafalias, Y.F. 1997. A critical state two-surface plasticity model for sands. Géotechnique, Vol. 47, pp. 255-272.

Miller, G.A., Khoury, C.N., Muraleetharan, K.K., Liu, C. and Kibbey, T.C.G. 2008. Effects of soil skeleton deformations on hysteretic soil water characteristic curves: experiments and simulations. Water Resour. Res., Vol. 44.

Muraleetharan, K.K., Liu, C., Wei, C.F., Kibbey, T.C.G. and Chen, L. 2009. An elastoplastic framework for coupling hydraulic and mechanical behavior of unsaturated soils. Int. J. Plasticity, Vol. 25, pp. 473-490.

Rampino, C., Mancuso, C. and Vinale, F. 2000. Experimental behaviour and modeling of an unsaturated compacted soil. Can. Geotech. J., Vol. 37, pp. 748-763.

Taiebat, M. and Dafalias, Y.F. 2008. SANISAND: simple anisotropic and plasticity model. Int. J. Numer. Anal. Meth. Geomech., Vol. 32, pp. 915-948.

Thu, T.M., Rahardjo, H. and Leong, E.C. 2007. Elastoplastic model for unsaturated soil with incorporation of the soil-water characteristic curve. Can. Geotech. J., Vol. 44, pp. 67-77.

Uchida, K. and Stedman, J.D. 2001. Liquefaction behavior of Toyoura sand under cyclic strain controlled triaxial loading. Proc. 11th Int. Offshore Polar Engng. Conf., Stavanger, Norway.

Unno, T., Kazama, M., Uzuoka, R. and Sento, N. 2008. Liquefaction of unsaturated sand considering the pore air pressure and volume compressibility of the soil particle skeleton. Soils Found., Vol. 48, pp. 87-99.

Verdugo, R. and Ishihara, K. 1996. The steady state of sandy soils. Soils Found., Vol. 36, pp. 81-91.

Wei, C., Muraleetharan, K.K. 2002a. A continuum theory of porous media saturated by multiple immiscible fluids: I. Linear poroelasticity. Int. J. Eng. Sci., Vol. 40, pp. 1807-1833.

Wei, C., Muraleetharan, K.K. 2002b. A continuum theory of porous media saturated by multiple immiscible fluids: II. Lagrangian description and variational structure. Int. J. Eng. Sci., Vol. 40, pp. 1835-1854.

Wheeler, S.J., Sharma, R.S. and Buisson, M.S.R. 2003. Coupling of hydraulic hysteresis and stress-strain behaviour in unsaturated soils. Géotechnique, Vol. 53, pp. 41-54.