An experimental study of clay based on a class of simple hypoelastic constitutive behavior

Une étude experimentale de l'argile fondée sur une classe de comportement hypoélastique simple

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ABSTRACT
The objective of this paper is to present a mathematically simple model to define the mechanical behavior of clay that can be utilized in determination of practical solutions for soil mechanics problems. Variable moduli models in the form of tangential stress-strain relationships according to the theory of hypoelsticity are used in this paper. The incremental constitutive relations are formulated directly based on the isotropic linear elastic formulation simply by replacing the elastic constants with variable tangential moduli which are functions of strain invariants. By examining an experimental stress-strain curve of a triaxial compression test, it has been demonstrated that the variable moduli, in this case, is a decreasing function of strain which starts at an initial value of E_0 and gradually reduces to zero at the ultimate deviator stress application. The stress-strain relationship, in this case, can be reduced to a simple hyperbolic from.

RÉSUMÉ
L'objectif de ce document est de presenter un modèle mathématique simple pour définir le comportement méchanique de l'argile qui peut être utilisé dans la détermination des solutions pratiques de méchanique des sols. Variables modules modèles sous les formes tangentiels des relations contrainte-déformation de la théorie d'hypoélasticité sont utilisés dans le present document. Les relations constitutives incrémentales sont formulées directement par remplacement des constants élastiques dans les relations de contraine-déformation d'élasticité linéaire et isotope avec les modules variables tangentiels qui sont fonctions des invariants du tenseur de déformation. En examinant une courbe expérimentale de contrainte-déformation d'un éssai de compression triaxiale, il a été démontré, dans ce cas, que le module variable est une fonction réduissante de déformation qui commence ā une valeur initiale de E_0 et progressivement réduit a zéro au contraint déviatorique ultime. La relation de contrainte-déformation, dans ce cas, peut être reduite a une simple forme hypérbolique.

Keywords : constitutive relations, hypoelasticity, hyperbolic, variable modulus

1 INTRODUCTION

Expansion of analytical capabilities in geotechnical engineering depends upon realistic and accurate modeling of soils constitutive relations. The models should be founded on a physical interpretation of the material response to applied stresses or strain in laboratory or field tests.

Many different stress-strain relationships have been developed for characterization of complex mechanical behavior of clay. They are mostly based on nonlinear elastic or elastoplastic theories. Examples of elastoplastic models are: Drucker-Prager type of plasticity cap models and boundary surface plasticity models of Defalias et al. Examples of nonlinear elastic models are hyperelastic models of Green and hypoelastic models presented by several authors, Kondner 1963, Kulhawy et al 1969, Duncan et al 1970, Wong and Duncan 1974, and Duncan et al 1978.

For most soils in the inclastic range, unloading follows a completely different path from that of loading. When unloading to the initial state of stress takes place, the strains are not recovered completely and a permanent strain remains. In order for elasticity-based models to be used they must account for unloading behavior. Hypoclastic models, in which path-dependent behavior can be accounted for seems to be appropriate models for characteristization of mechanical behavior of soils.

The variable moduli models may be considered as a special class of hypoelastic models in which the behavior is further restricted to be incrementally isotropic. Variable moduli models are computationally simple in comparison with other more sophisticated models. They are particularly well suited for finite element calculations. In this paper a variable moduli model in the form of tangential stress-strain relationship according to the theory of hypoelasticity is used for characterization of mechanical behavior of clay.

2 STRESS-STRAIN RELATIONSHIP

For isotropic hypoelastic models, the incremental constitutive relations are formulated directly based on the isotropic linear elastic formulation simply by replacing elastic constants by variable tangential moduli that are functions of stress andw/or strain invariants.

$$\overset{\circ}{\sigma}_{ij} = \lambda\overset{\circ}{\theta}\delta_{ij} + 2\mu\overset{\circ}{\varepsilon}_{ij} \tag{1}$$

Where $\overset{\circ}{\sigma}_{ij}$ and $\overset{\circ}{\varepsilon}_{ij}$ are stress rate and strain rate tensors respectively. λ and μ are parameters of Lamé which can be functions of stress or strain invariants, and

$$\overset{\circ}{\theta} = (\overset{\circ}{\varepsilon}_{11} + \overset{\circ}{\varepsilon}_{22} + \overset{\circ}{\varepsilon}_{33}) \tag{2}$$

Parameters of Lamé are defined by the following relationships:

$$\lambda \quad \frac{E\nu}{(1+\nu)(1-2\nu)} \text{ and } \mu = \frac{E}{2(1+\nu)} \tag{3}$$

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Assuming that the Poisson's Ratio (ν) is constant and not a function of stress or strain, equation (1), in case of uniaxial stress application, reduces to

$$\overset{\circ}{\sigma}_{11} = E\,\overset{\circ}{\varepsilon}_{11} \tag{4}$$

and for triaxial tests we can reduce it to

$$\overset{\circ}{(\sigma_{11} - \sigma_{33})} = E\,\overset{\circ}{\varepsilon}_{11} \tag{5}$$

3 INCREMENTAL STRESS-STRAIN MODEL

Assume that the nonlinear stress-strain curve for clays obtained in triaxial tests Fig. (1) is a combination of incremental linear portions.

$$\frac{\partial(\sigma_1 - \sigma_3)}{\partial\varepsilon} = E \tag{6}$$

E is a variable modulus that can be a function of the current state of stress and/or strain. For the purpose of this paper, we assume E as a function of strain only E(ε).

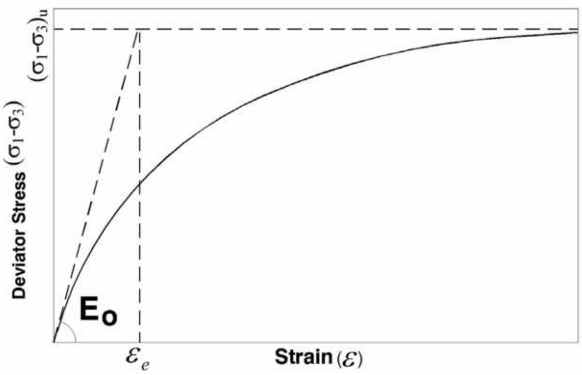

Figure 1. Typical stress-strain curve from triaxial tests.

By examining multiple curves of triaxial test results for clays, we conclude that E(ε) is a descending function of strain which ultimately reduces to zero. Therefore, a simple representation of E as a function of ε could be

$$E = f\left(\frac{1}{\varepsilon + \varepsilon_e}\right) \tag{7}$$

We use the Taylor Series to expand the function f to a series as follows:

$$E \quad f\left(\frac{1}{\varepsilon + \varepsilon_e}\right) = f(0) + \frac{1}{\varepsilon + \varepsilon_e} f'(0) + \frac{1}{2!}\left(\frac{1}{\varepsilon + \varepsilon_e}\right)^2$$

$$f''(0) + \ldots\ldots \tag{8}$$

$$E \quad f\left(\frac{1}{\varepsilon_e}\right) + \frac{1}{\varepsilon + \varepsilon_e} f'\left(\frac{1}{\varepsilon_e}\right) + \frac{1}{2!}\left(\frac{1}{\varepsilon + \varepsilon_e}\right)^2 f''\left(\frac{1}{\varepsilon_e}\right) +$$

$$\frac{1}{3!}\left(\frac{1}{\varepsilon + \varepsilon_e}\right)^3 f'''\left(\frac{1}{\varepsilon_e}\right) + \ldots\ldots \tag{9}$$

$$E \quad K_0 + K_1\left(\frac{1}{\varepsilon + \varepsilon_e}\right) + K_2\left(\frac{1}{\varepsilon + \varepsilon_e}\right)^2 + K_3\left(\frac{1}{\varepsilon + \varepsilon_e}\right)^3 + \ldots \tag{10}$$

Applying the boundary condition where for ε ∞, E=0 results in $K_0 = 0$

Substituting the value of E(ε from equation (10) into equation (6) and integrating results in:

$$(\sigma_1\text{-}\sigma_3 = K_1 \log(\varepsilon{+}\varepsilon_e + \frac{K_2}{\varepsilon_e})\frac{\varepsilon}{\varepsilon + \varepsilon_e} + \frac{K_3}{2}\frac{1}{(\varepsilon + \varepsilon_e)^2} \tag{11}$$

Applying the boundary conditions: ε=0, $(\sigma_1\text{-}\sigma_3) = 0$, and ε ∞, $(\sigma_1\text{-}\sigma_3)$ $\sigma_1\text{-}\sigma_3)_u$ results in:

$$K_1 \quad 0, \quad K_2 \quad \varepsilon_e(\sigma_1\text{-}\sigma_3 \;_u \;, \quad K_3 \quad 0 \tag{12}$$

Therefore, the stress-strain relationship reduces to a simple form:

$$(\sigma_1\text{-}\sigma_3) \quad \sigma_1\text{-}\sigma_3 \;_u \frac{\varepsilon}{\varepsilon + \varepsilon_e} \tag{13}$$

and

$$E \;\; \varepsilon = \frac{\partial(\sigma_1 - \sigma_3)}{\partial\varepsilon} \quad \sigma_1\text{-}\sigma_3 \;_u \frac{\varepsilon_e}{(\varepsilon + \varepsilon_e)^2} \tag{14}$$

This hyperbolic form of stress-strain relationship was discussed by Kondner (1963) and by Duncan et al (1969).

$$\text{For } \varepsilon \;\; 0, \quad E_0 \quad \frac{(\sigma_1 - \sigma_3)_u}{\varepsilon_e} \tag{15}$$

Substituting the value of (E_0)) into equations (14) results in

$$E \;\; \varepsilon \quad E_0\left(1 + \frac{\varepsilon}{\varepsilon_e}\right)^{-2} \tag{16}$$

Equation (13) can also be written in the form of

$$\sigma_1\text{-}\sigma_3 = E_0 \frac{\varepsilon\varepsilon_e}{\varepsilon + \varepsilon_e} \tag{17}$$

If we show the deviator stress (σ -σ) by the symbol Δσ then this equation can be reduced to

$$\varepsilon^{-1} = E_0(\Delta\sigma^{-1} - \Delta\sigma_u^{-1}) \tag{18}$$

Since the value of E_0 is not clearly defined from the curves of triaxial test results, we can not estimate directly the value of E_0 from the test results. But if we plot ε^{-1} as a function of $\Delta\sigma^{-1}$, the slope of the resulted straight line will give us an accurate value of E_0, as shown on Fig. (2).

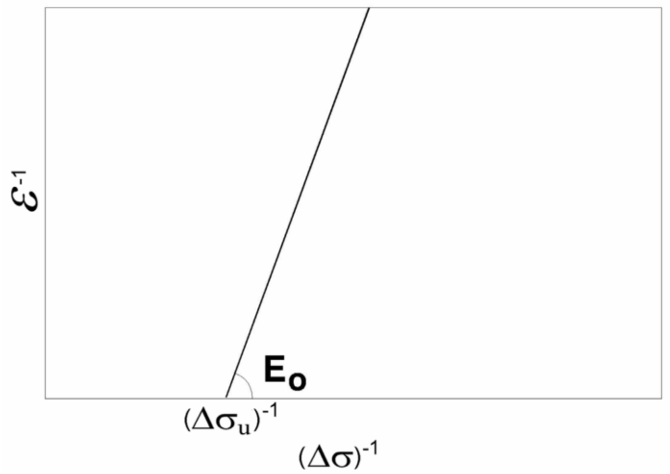

Figure 2. Typical plot of ε^{-1} vs. $(\Delta\sigma)^{-1}$ for back calculation of E_0.

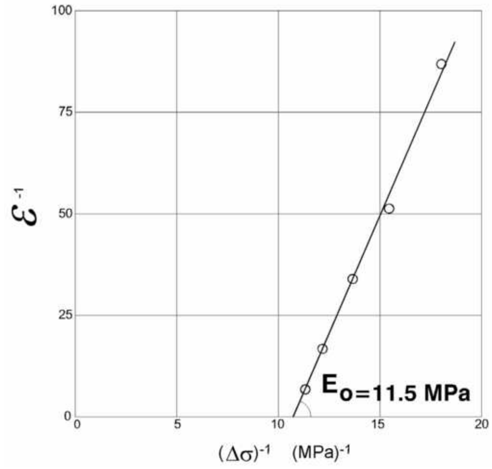

Figure 4. Example of back calculation of E_0.

4 FIELD EXPLORATION AND LABORATORY TESTING

Our exploratory program consisted of drilling, logging, and sampling of many borings to characterize the subsurface conditions and to collect clay samples from our project site located in Owens Valley in Central California. Undisturbed as well as remolded clay samples were obtained for laboratory testing. Our laboratory testing program was designed to classify the clay material and to characterize their stress-strain behavior. Laboratory tests performed included in situ density and moisture content (ASTM D-2937), Atterberg Limits (ASTM D-4318), Specific Gravity (ASTM D-854), Hydrometer (ASTM D-4221) and visual classifications. Test results are tabulated in Table (1). Ten samples were prepared for triaxial testing. The sizes of the cylindrical samples were 0.05m. in diameter and 0.15m. in height. Unconsolidated undrained triaxial compression tests (UU) were performed in accordance with (ASTM D-2850-95). Table 1 shows the values of confining pressure applied and the ultimate deviator stress obtained for each sample. Figure (3) shows an example of the triaxial compression test results. This curve represents the axial strain increases of a sample under increasing deviator stress. Ultimately, the sample fails and the axial strain continues to increase with no additional increase in deviator stress.

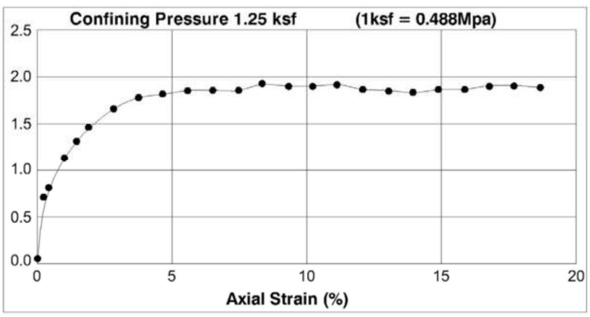

Figure 3. Example of an actual triaxial (uu) test result.

5 EXPERIMENTAL RESULTS

Plotting the curve of ε^{-1} as a function of $\Delta\sigma^{-1}$ shown in equation (18) by utilizing the numerical values of $\Delta\sigma$ and ε obtained from the curve of triaxial compression test results shown on Fig.(3) results in a straight line as shown on Fig.(4). The slope of this line will be equal to the value of initial modulus E_0. This procedure was applied to all the ten triaxial compression test results and the back calculated values of E_0 are shown on table (1).

Table 1. Back-calculated values of E_0 from ten triaxial test results.

Sample No.	Depth (m)	Soil description	Classification (USCS)	Density (KN/M^3)	Moisture contentl (%)	Confining pressure (MPa)	Ultimate deviatoric stress (MPa)	E_0 (MPa)
1	3.0-3.5	Olive gray silty clay (undisturbed)	CH	9.26	68	0.061	0.090	11.5
2	4.5-5.0	Gray silty clay (undisturbed)	CH	9.00	68.9	0.059	0.062	4.9
3	20-21.50	Gray silty clay (undisturbed)	CH	7.85	86.8	0.061	0.061	6.7
4	6.0-6.5	Oilve gray silt (undisturbed)	MH	6.49	112.7	0.068	0.137	10.0
5	1.0-1.5	Olive silty clay (remolded)	ML	11.68	35.3	0.024	0.292	16.0
6	0.5-1.0	Olive silt (remolded)	ML	9.80	13.5	0.024	0.093	24.0
7	4.5-5.0	Gray silty clay (undisturbed)	CH	9.90	56.4	0.056	0.056	5.6
8	3-3.5	Gray clay (undisturbed)	MH	8.52	11.8	0.059	0.073	11.8
9	1.5-2.0	Gray silty clay (undisturbed)	MH	11.21	12.6	0.056	0.102	16.0
10	1.5-2.0	Gray fat clay (undisturbed)	CH	7.55	87.7	0.102	0.128	6.7

6 CONCLUSIONS

- It appears that satisfactory results may be obtained for the values of initial modulus E_0 using a good estimate for a constant value of Poisson's Ratio (ν). In many practical applications, constant values for (ν) are usually considered.
- The hyperbolic models presented are mathematically simple and their parameters have direct relations to familiar physical quantities such as initial modulus E_0 and ultimate soil strength (σ_1-σ_3 $_u$.
- With the simple method presented in this paper, the parameters ε_e and E_0 can be easily determined from the standard Triaxial (UU) test results. The wide data base available for each type of soils is useful in assessing the reliability of the values obtained from laboratory test results.
- With the model parameters determined based on the behavior measured in Triaxial (UU) tests, reasonable results from the model are expected only for applications involving similar conditions such as finite element analyses of slope stability or dam foundations.
- In geotechnical problems, in which the stress history is close to proportional loading, the variable moduli model presented satisfies almost all the theoretical requirements and is probably satisfactory for practical use.
- The incremental form of the stress-strain relationship may be used very conveniently in incremental finite element analyses.
- The incremental models described can not account for the strain softening behavior after peak strength for overconsolidated clays.

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