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Evaluating soil parameters using numerical optimisation

Détermination des paramètres de comportement du sol à partir de méthodes d'optimisation numériques

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ABSTRACT

Evaluating parameters for complex constitutive models can be a laborious and time consuming task, particularly where data from a number of different tests and test types are available. This paper explores the use of mathematical optimisation techniques to derive parameters for a given constitutive model based on the stress-strain/load-displacement response measured in both laboratory and in-situ tests. An example is presented involving triaxial, plate-load and pressuremeter tests conducted at different elevations in a clay deposit for which the strength and stiffness increase (non-linearly) with depth. Finite element models representing each test are used to calculate the constitutive model's response with a given set of parameters. Individual norms, which measure differences between the actual and calculated load-displacement response, are then calculated for each test. Individual norms for each test are combined to form a global norm, or objective function, for all tests. Direct search methods are then employed to vary the constitutive model parameters with the aim of minimising the objective function. Methods for forming both individual norms and objective functions are discussed and the performance of a commercially available search method is assessed. It is concluded that mathematical optimisation can assist in the practical application of constitutive models, as it promotes a systematic treatment of data and removes much of the subjectivity involved in parameter selection. It also provides a rational basis for extrapolating from the known test data to predict the unknown field response. This is because precisely the same assumptions are made when interpreting soil test data as those that are made when undertaking calculations to predict the performance of real structures (i.e. the assumption in the constitutive model).

RÉSUMÉ

L'évaluation des paramètres des modèles de comportement peut s'avérer une tache lourde et laborieuse, notamment lorsqu'ils sont dérivés de nombreux essais de différentes natures. Cet article présente une technique d'optimisation mathématique développée afin de déduire les paramètres d'un modèle de comportement donné à partir des courbes contraintes déformations issues d'essais de laboratoire et d'essais in situ. Un exemple est présenté, basé sur des essais triaxiaux, des essais de plaques et des essais pressiométriques réalisés à plusieurs profondeurs dans un massif d'argile dont la résistance au cisaillement et le module de déformation augmentent de manière non linéaire avec l'état de contrainte. Une modélisation par éléments finis des différents essais été réalisée afin d'évaluer la réponse du modèle de comportement en fonction des paramètres du modèle. Une norme individuelle, caractérisant la différence entre la courbe contrainte déformation mesurée et la courbe calculée, a été calculée pour chaque tests. Les normes individuelles de chaque essai ont été ensuite combine afin de former une fonction objective pour tous les tests. Des méthodes de cisaillement direct ont alors été utilisées pour faire varier les paramètres du modèle de comportement afin de minimiser la fonction objective. Les méthodes disponibles utilisées pour déterminer les formes individuelles et les fonctions objectives sont discutées et évaluées. Il apparaît que les techniques d'optimisation mathématiques peuvent être utilisées avec succès pour déterminer les paramètres des modèles de comportement, car elles permettent un traitement systématique et objectif des données d'essais. Elles permettent également une approche rationnelle d'extrapolation de la réponse (inconnue) d'un massif de sol à partir des données (connues) d'essais de caractérisation du fait que les hypothèses faites pour interpréter les résultats d'essais sont les mêmes que celles faites pour déterminer la réponse du sol à une sollicitions donnée.

Keywords : constitutive model, optimisation, back analysis

1 INTRODUCTION

A key step in any geotechnical analysis is the derivation of parameters describing the mechanical behaviour of the soil. As no universally accepted constitutive model for soil exists, the number and type of parameters that must be determined depends on the constitutive model that will ultimately be used in the design process. It is essential for consistency that the assumptions adopted (or model used) when interpreting site investigation data to derive parameters are compatible with the model that will be used the in the geotechnical analysis. As constitutive models become more complex, it becomes increasingly difficult to derive their parameters analytically. It is therefore convenient to set up models of the actual site investigation tests themselves. Soil parameters can be varied until a suitable match between the test and model data is achieved. This provides a consistent link between the parameters inferred from the test data and those used in the geotechnical design. However, this process can be very time consuming and it is difficult to treat all data in an ordered and systematic way, particularly where data from different test types are available.

This paper explores the use of mathematical optimisation techniques to calibrate constitutive models based on the stress-strain/load-displacement response measured from both laboratory and in-situ tests, including triaxial, pressuremeter and plate-load tests. The approach is illustrated using a simple "virtual" example, where the site investigation data are generated artificially using a numerical model with a given set of parameters. It is imagined that these parameters are not known and that they cannot be derived analytically from the individual tests; an optimisation problem is therefore formed to estimate them. The commercially available optimisation routine "fminsearch" was used, from the MATLAB optimisation

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toolbox (Coleman et al 1999). This was linked with a finite element program, developed by the authors. The optimisation process, illustrated in Figure 1, starts with an initial "estimate" of the parameters (for the example problem discussed later, the initial parameters were made to vary by up to 50% from the actual parameters). With the initial parameters, "model data" are generated from finite element models representing each test. The model data is compared with the (artificially generated) "test data". A positive scalar measure of the difference between the model and test data, for each test, is then calculated. These are referred to as individual norms. These individual norms are combined to form a single global norm, or objective function, which measures the match between model and test data for all tests. The calculation of individual norms and the formation of the objective function are discussed in section 4 of the paper.

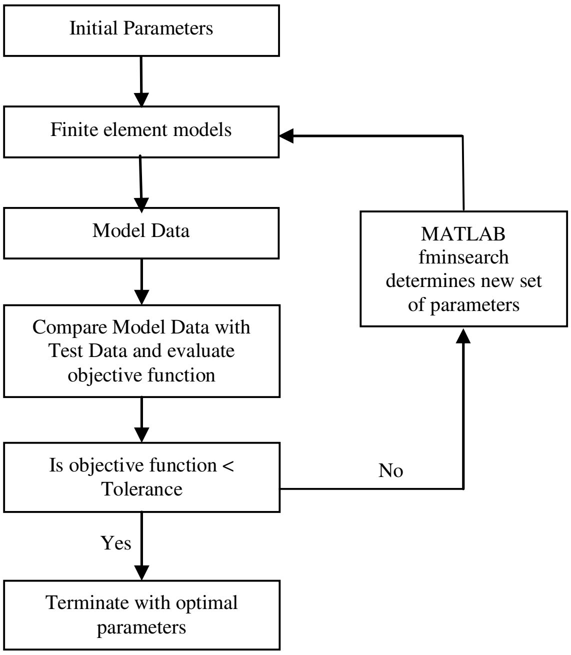

Figure 1: Optimisation process

Mathematical optimisation is commonly used for design in many disciplines. However, it has received surprisingly little attention in the field of geotechnical engineering. This is likely to change as soil constitutive models become more detailed and the number of parameters increase. The aim of this paper is to illustrate the use of a commercially available optimisation routine to derive parameters for a soil model using data from various test types. Different methods of combining individual norms to form the objective function are investigated.

2 TEST PROBLEM

In order to illustrate the approach, the following "virtual" test problem is presented, involving the characterisation of a 6m deep clay deposit. For simplicity, it is assumed that this clay deposit can be approximated as a linear elastic perfectly plastic material, obeying the Tresca failure criterion. The undrained strength (Su) is presumed to be independent of the mode of shearing and taken to vary with depth (z) according to:

$$Su = Su_0 + az^{\alpha} \tag{1}$$

where Su_0 is the undrained shear strength at the surface (z = 0), and a and α are parameters defining the variation of strength with depth. The clay stiffness is assumed to be isotropic, with a similar variation in Young's modulus (E) with depth:

$$E = E_0 + bz^{\beta} \tag{2}$$

where E_0 is Young's modulus at the ground surface and b and β define the variation of stiffness with depth z. Therefore a total of 6 parameters define the variation of strength and stiffness with depth. (A constant Poisson's ratio was used for the test data and all model data. Poisson's ratio was therefore not involved in the optimisation problem.)

It is imagined that a plate-load test (with a radius of 1m) at the ground surface, a pressuremeter test at a depth of 3m and a triaxial compression test at a depth of 5m have been performed. The load-displacement/stress-strain data for each of these tests, shown in Figure 2, was "artificially" generated with finite element models using the "Actual Values" given in Table 1.

Table 1: Actual profile parameters and initial estimates

Parameters	Actual Values	Initial Estimate
Su_0	20	30
a	4	6
α	1	0.6
E_0	5000	7500
b	1000	800
β	0.5	0.75

All finite element models employed 8-noded axisymmetric quadratic continuum elements with 4-point Gauss integration. The visco-plastic strain method was used and the Tresca failure criterion was assumed. The boundary conditions for each model are illustrated in Figure 3. All models used displacement controlled loading, with 12 increments, and radial pressure (P) vertical load and deviatoric stress required were computed. The radial pressure (P) is the net pressure above the initial lateral total stress, and the deviatoric stress is the difference between the stress in the z and r directions. The model for the pressuremeter test is shown in Figure 3a. A ratio of r_0/r of 50 was used, with a total of 96 elements. The finite element mesh for the plate-load test, shown in Figure 3b, comprised a total of 400 elements. The triaxial test, as illustrated in Figure 3c, was modeled using a single element.

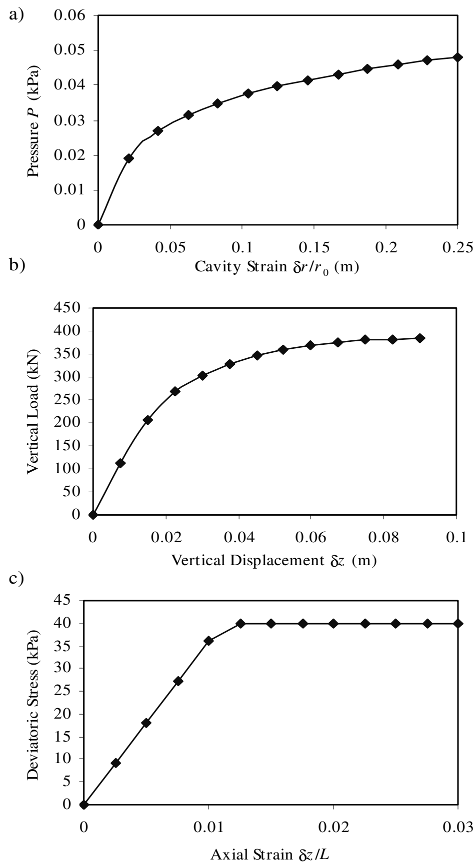

Figure 2: Test data (a) Pressuremeter Test; (b) Plate-load test; (c) Triaxial Compression test

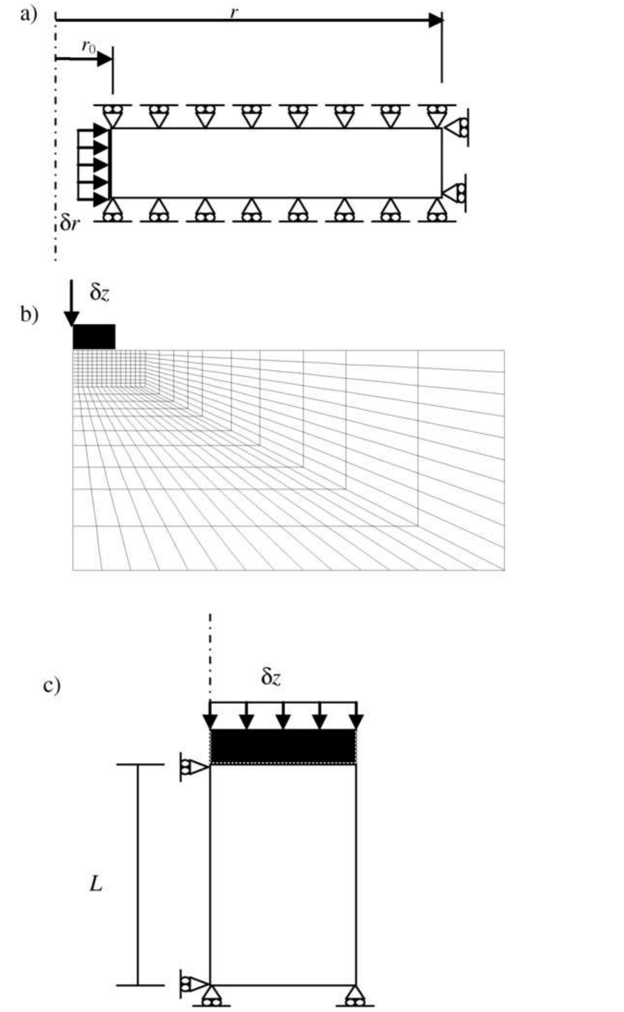

Figure 3: Finite element models representing boundary value problems a) Pressuremeter; b) Circular footing; and c) Triaxial test

3 OPTIMISATION ROUTINE

MATLAB includes an Optimisation Toolbox, which contains a library of optimisation functions. The test problem presented here was solved using the unconstrained non-linear minimisation function, "fminsearch". This function uses the simplex search method (Lagarias et al, 1998) to look for a minimum to a multivariable objective function. It is a direct search method that does not use numerical or analytical gradients to the objective function.

4 THE OBJECTIVE FUNCTION

As is typically the case, the optimisation problem sought to minimise a global norm, or single objective function (G) which is a measure of the difference between the test data and model data for all tests. The objective function is formed by first computing the individual norms, I, for each test. The individual norms were calculated by computing the minimum distance of each test data point to a straight-line fit between the two nearest model data points (d_{min}), as shown in Figure 4, and summing these distances for all tests points. I can be expressed as:

$$I = w \frac{1}{n+1}\left[\sum_{j=1}^{n} d_{min}{}^{j} \right] \tag{3}$$

where n is the number of experimental points and w is a weighting factor which assigns the relative importance of the particular set of data; scaling factors are required to allow for different dimensions of stress/strain or load/displacement in the respective test data (Wood et al 1992). In this paper, a weighting factor of unity was used for all tests. It should be noted that test data points are only included if the distance from the particular test point to model data i+1 is greater than the distance to model data i and i-1. This is done so that the individual norm does not increase if the experimental data extend further than the test data.

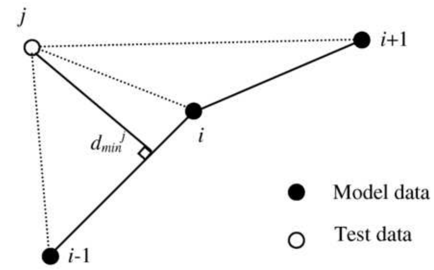

Figure 4: Evaluation of $d_{min}{}^{j}$

There are a number of possible ways to formulate an objective function from the individual norms, and some of these are discussed in Mattsson et al (2001), who recommend the following "combined" objective function:

$$G_{comb} = mI_{max} + \sum_{l=1}^{m} I^{l} \tag{4}$$

where I_{max} is the largest individual norm for all of the tests, I^{l} is the individual norm for the l^{th} set of test data and m is the total number of tests. This objective function accounts for all sets of test data, but is more heavily weighted by the worst test. An alternative approach is to use only the worst individual norm as the objective function i.e:

$$G_{max} = I_{max} \tag{5}$$

This is reasonable in the sense that the aim is to provide the best possible match to all tests. However, as Mattson et al (2001) point out, if the test with the worst individual norm is unaffected by certain parameters, then these parameters will not take part in the optimisation process.

A more targeted objective function may sometimes be possible by isolating aspects of the data that certain parameters control. For the simple example presented in this paper, the stiffness parameters (from Equation 2) control the slope of the lines from the origin to the first data point independently of the strength parameters (assuming that the first load step is small enough to avoid any plasticity). Conversely, the strength parameters control the final load or deviatoric stress reached in the plate-load and triaxial tests respectively, and also the slope at the end of the ln ($\delta r/r_0$) *vs.* pressure (P) curve for the pressuremeter test. These characteristics allow the following targeted objective function to be formed:

$$G_{targ} = \sum_{l=1}^{m} S^{l} + F^{f} + F^{t} + F^{p} \tag{6}$$

where S^{l} is the absolute difference between the initial gradients of the model and test curves, divided by the initial gradient of the test curve (for the l^{th} test); F^{f} is the absolute difference between the final vertical loads reached for the model and plate-load test data, divided by the final load for the plate test data. F^{t} is similar, but based on the triaxial data, and F^{p} is the difference between the slopes at the end of the ln($\delta r/r_0$) *versus P* curve for the model and test data, divided by the same slope for the test data.

5 RESULTS

Figure 5 plots the variation of each objective function, (normalised by its value using the "initial estimate" parameters; see Table 1) with the number of function evaluations during the optimisation process. In all cases, the optimisation routine failed to reduce the objective function to the specified tolerance of 1E-4. The optimisation process was therefore terminated after around 500 function evaluations, at which stage negligible changes in the objective function were computed.

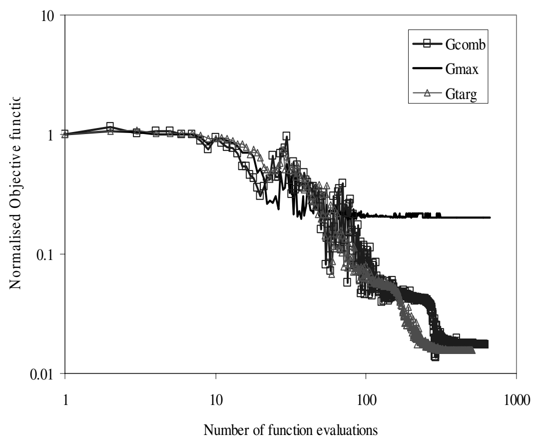

Figure 5: Objective function convergence

The final sets of parameters for each objective function are listed in Table 2. It can be seen that the optimisation process does not recover the exact parameters for any of the objective functions. However, the accuracy of the optimal parameters is best seen by plotting the *Su* and *E* profiles (using equations 1 and 2, respectively).

Table 2: Optimal parameters for each objective function

Parameters	G_{comb}	G_{max}	G_{targ}
Su_0	18.5329	16.3346	18.7131
a	6.08373	6.2323	6.5244
α	0.783382	0.754818	0.73205
E_0	5588.53	8006.64	4873.12
b	440.206	824.874	1158.12
β	0.669966	0.84214	0.438279

Figure 6 shows the *Su* versus depth (*z*) profile using the "Actual Values" and "Initial Estimates" from Table 1, along with the optimal parameters from Table 2. It can be seen that the optimisation process resulted in a reasonably good match to the actual *Su* profile for all objective function formulations, particularly when compared with the initial estimate profile.

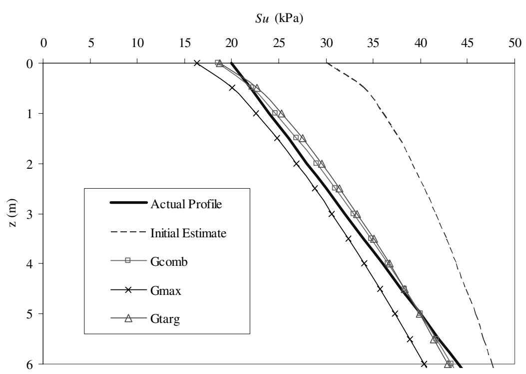

Figure 6: Strength profile

Figure 7 presents a similar plot for the stiffness (*E*) profile, where it can be seen that the G_{max} formulation for the objective function results in a poorer match to actual *E* profile than the initial estimate. However, other objective functions (particularly G_{targ}) provide an excellent match to the stiffness profile.

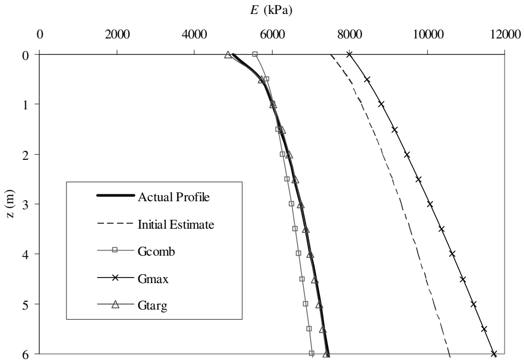

Figure 7: Stiffness profile

6 CONCLUSIONS

This paper uses a "virtual" example problem to demonstrate the application of numerical optimisation for the derivation of constitutive model parameters from plate-load, pressuremeter and triaxial compression test data. The example assumed the clay could be modelled as an isotropic elastic perfectly-plastic Tresca material, with a total of 6 parameters describing the variation of undrained shear strength and stiffness with depth, involved in the optimisation problem. Three methods for forming the global objective function were considered. Both the G_{comb} and G_{targ} formulations resulted in a good match to the actual profiles, while the G_{max}, which used only the worst individual norm, proved unreliable as it provided a poor match to the stiffness profile.

The numerical optimisation procedure illustrated in this paper can assist in the practical application of constitutive models, as it promotes a systematic treatment of data and removes much of the subjectivity involved in parameter selection. It also provides a rational basis for extrapolating from the known test data to predict the unknown field response. This is because precisely the same assumptions are made when interpreting soil test data as those that are made when undertaking calculations to predict the performance of real structures. This is because the same constitutive model (and its associated assumptions) is used to both interpret the soil test data and predict the performance of real structures.

REFERENCES

Mattsson, H., Klisinski, M. and Axelsson, K. 2001. "Optimization routine for identification of model parameters in soil plasticity". International Journal for Numerical and Analytical Methods in Geomechanics. Vol 25, 435-472.

Wood, D.M., Mackenzie, N.L. and Chan, A.H.C. 1992. "Selection of parameters for numerical predictions". Predictive Soil Mechanics, Proceedings of the Wroth Symposium St Catherine's College, Oxford

Lagarias, J.C., Reeds, J.A., Wright, M.H. and Wright, P.E. 1998. "Convergence Properties of the Nelder-Mead Simplex Method in Low Dimensions," SIAM Journal of Optimization, Vol. 9, Number 1, pp. 112–147.

Coleman, T.F., Branch, M.A., and Grace, A. 1999. Optimization Toolbox-for Use with MATLAB®, User's Guide, Version 2 MathWorks Inc.