Note on the shearing strength of sand

Note sur la résistance au cisaillement des sables

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ABSTRACT

The shearing strength of sand is governed by interparticle friction, dilatancy and grain crushing, main variables being relative density and the stress level at failure. A model of hyperbolic type is used to describe the relationship between these variables. The description is simple, curves are smooth, have no sharp kinks and the analytical expression contains only mathematical operations of addition, and division. Examples of published triaxial test results on four sands are used for justification.

RÉSUMÉ

La résistance au cisaillement d'un sable est gouvernée par le frottement interparticule, la dilatance et le broyage des grains, les variables principales étant la densité relative et le niveau de contrainte à la rupture. Un modèle de type hyperbolique est utilisé pour décrire la relation entre ces variables. La description en est simple, les courbes sont lisses, sans arrête et leur expression analytique ne contient que des opérations mathématiques d'addition et de division. Des exemples tirés des résultats publiés de tests triaxiaux sur quatre sables sont utilisés pour la justification.

Keywords : sand, shearing strength, relative density, stress level relationship.

1 INTRODUCTION

The strength of any soil can be explained in terms of "cementation", interparticle bond or cohesion, interparticle friction, particle breakage, dilatancy, and rearrangement of particles. The cohesion term for non-cemented sands equals to zero. The experimental evidence clearly shows that the failure envelope is curved (Golder 1941, Taylor 1948, and many others in last 60 years). Most soils exhibit curved failure envelopes with the more pronounced curvature in the lower stress range and the angle of shearing resistance decreases as the level of normal effective stresses at failure increases.

Mechanical nonlinear models seem to be particularly appropriate for describing the failure, both in phenomenological as well as in numerical aspects. A dilatancy model of a "saw blade" type implies that the max. angle of the shearing resistance is a sum of two components; the first component is usually termed the angle of the shearing resistance at constant volume, sometimes the critical state angle, sometimes the angle of the physical friction, and the second is the angle of dilatancy. The dilatancy of all dense soils is gradually decreased by the rise of stress level, and at very high stresses the behavior is governed by the friction and particle breakage.

In densely packed granular materials shearing starts in a narrow band only a few grains thick. The interlocking resistance of the particles inside this band is overcome by grain breakage and dilation. The effects of dilatancy, which can be approximately estimated in cases of coarse grained soils, are more significant at lower stresses, as the contribution of dilatancy to the total shearing strength gradually decreases by the rise of stress level (Ladanyi 1960, Bishop 1971, Bolton 1986-87). If the normal effective stress acting on the failure plane is large enough, all dilatancy would be suppressed and the soil would shear at nearly a constant volume, though the component of particle degradation at certain high stress levels, might be significant portion of the total shearing resistance in terms of effective stresses.

The laws of failure described in terms of different dilatancy theories, although based on sound principles, usually involve the simultaneous description of the rate of the volume change at failure, or introduce some empirical constants and functions. It seems highly desirable to introduce some clear link to the mechanism of the dilatancy behaviour at failure and the stress level without excessive oversimplification. Bolton (1986), analyzing 17 sands proposed the consistent treatment of both density and the confining pressure for quartz sand in triaxial compression with empirical equations for the dilatancy contribution which are given as truncated straight lines in a semi logarithmic plot in terms of the mean effective stresses p'.

Materials composed of angular particles or under high stress level due to particle breakage have no constant critical state angle, as it depends on confining pressure and initial packing. This angle of the shearing resistance is mobilized at moderately high normal stress levels at which all dilatancy effects are suppressed. It is generally suggested that the angle of the shear strength resistance of non-cemented sand as obtained in a drained triaxial shear test consists of four components, coming respectively, from physical friction, degradation, reorientation and dilatancy of individual particles. In a model used here, four components are lumped in two components, namely a constant and the variable stress level dependant part. A model for the shearing strength of sand is used to describe the relationship between the angle of the shearing resistance, relative density and the stress level at failure. Four sets of published triaxial test results are used for justification and correlations related to the relative density and stress level at failure. The nonlinear failure envelopes in terms of effective stresses are described by a simple three parameter expressions of hyperbolic type. Expressions contain the basic angle of friction, contribution of max. dilatancy and the parameter related to sand density and the grain strength. The description is simple, curves are smooth, have no sharp kinks and the analytical expression contain only mathematical operations of addition, and division. Proposed model and correlations can be used in practice for an estimate of the shearing strength of various sands and for interpretation of shear strength tests.

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2 MODEL "M", MENTAL FRAMEWORK

One micro-mechanical model and the derived family of failure envelopes described by author (Maksimović 1989, 1993, 1996) suitable for description of the nonlinear failure envelope in terms of effective stresses for most non-cemented soil types and rock joints, describes the secant angle of the shearing resistance as the function of the normal effective stress level. The module of the description given here as used in this paper is presented for completeness. The key factors considered by the model in fully drained conditions are friction, dilatancy and in an indirect manner, the grain breakage.

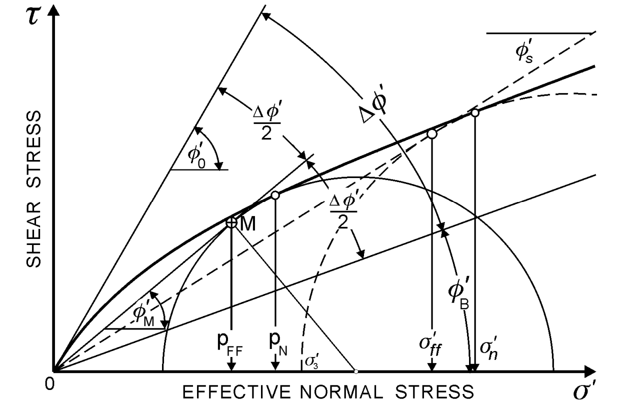

Figure 1. Mohr diagram and parameters.

The nonlinear failure envelope shown in Fig. 1, has a tangent in the origin inclined at the initial angle ϕ_0' which is the sum of the basic angle ϕ_B' and the maximum angle difference $\Delta\phi'$. The secant angle of the shearing resistance is the sum of two components, which are the "basic friction" ϕ_B' assumed to be a constant, and the contribution of the dilatancy and grain breakage which depends on the stress level as defined in the second term of eq. (2). In the case of the triaxial test results, the secant angle ϕ_S' of the tangent (Fig. 1) drawn from the origin to the Mohr circle at failure in terms of principal stresses is:

$$\phi_S' = \sin^{-1}\frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} \qquad (1)$$

and this secant angle varies with the effective normal stress σ_{ff}' according to the relationship:

$$\phi_S' = \phi_B' + \frac{\Delta\phi'}{1+\sigma_{ff}'/p_{FF}} \qquad (2)$$

The effective normal stress σ_{ff}' acting on the "triaxial plane" is:

$$\sigma_{ff}' = \sigma_3'\,(1+\sin\phi_S') \qquad (3)$$

The parameter p_{FF} is "normal pressure at max. obliquity for the median angle". It is the stress level at which the median angle of the shearing resistance is:

$$\phi_M' = \phi_B' + \Delta\phi'/2 \qquad (4)$$

Maximum angle difference $\Delta\phi'$ is the difference between the initial angle ϕ_0', when the normal stress tends to zero, and the asymptotic basic angle of friction ϕ_B', when the stress level tends to infinity. The median angle pressure p_{FF} is the stress level at which the angle of the shearing resistance is equal to the mean value of the initial and basic angle and corresponds to point M in Fig.1 and Fig.2. The second term in eq.(2) has a property that it decreases asymptotically towards zero with increasing normal stress, and the total angle of shearing resistance tends towards the lower constant limiting value defined here as a "basic angle of friction" ϕ_B'. In a semi-log plot, Fig.2, the point M (Median point) corresponding to p_{FF} is a point of central symmetry of the curve, it is an inflection point, and ϕ_B' and ϕ_0' are asymptotes. The initial angle ϕ_0' is an asymptote when the stress level tends to zero, and the basic angle is an asymptote when the stress level tends to infinity ϕ_B'. Neither of these two values can be directly measured. All expressions are dimensionally consistent. Values of the angles can be taken in degrees or radians and the description of stress must be in the same units as the median angle pressure.

It can be seen in Fig.2 that in the neighborhood of point M in certain interval a good approximation could be also obtained by using the linear relationship in a semi-log plot. This type of expressions might be locally applicable, though extrapolation of the straight line to low stress range would give an overestimate of the angle of the shearing resistance, and extrapolation in the very high stress range will obviously give an underestimate. This limitations are handled by Bolton (1986, 1987) by introduction of cut-off limits which generate two points of singularity, kinks which are not very handy in numerical applications.

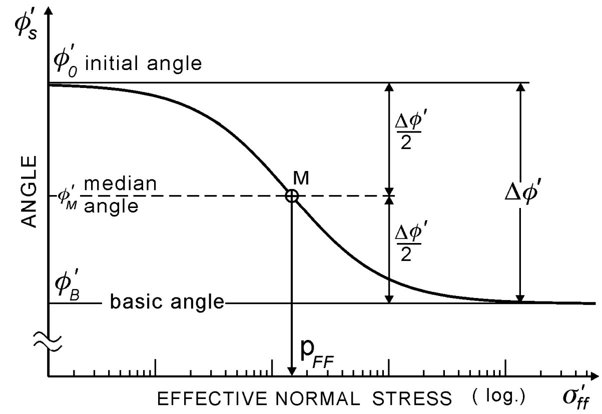

Figure 2. Secant angle and parameters of the non-linear failure envelope, semi-log plot.

Parameters have the following physical meaning:

ϕ_B' is the "basic angle of friction" that reflects friction close to the critical state, or at the constant volume. This angle is mobilized at high normal stress levels at which all dilatancy effects are suppressed. ϕ_B' lies in the range: $\phi_\mu' < \phi_B' \le \phi_{CV}'$ and $\phi_B' \approx \phi_{CV}'$ could be a reasonable assumption if testing is carried out in the conventional stress range. The range of ϕ_B' for coarse grained materials, like sand and rockfill, is 34^0 (+/-) 4^0, nearly a constant for a particular soil, largely independent of the stress path.

$\Delta\phi'$ "the angle of maximum dilatancy contribution" reflects density, angularity and the associated dilatancy effects close to the zero stress level. The maximum value is in the range from about 13^0 up to 30^0 for very dense coarse grained angular materials like crushed rock.

p_{FF} "the median angle pressure" reflects the compressibility of the soil skeleton related to soil density, the strength or the resistance of grains to crushing, which depends on the type of mineral, grading and grain roundness (Maksimović 1996).

It is widely accepted that the shearing strength of a particular sand with a void ratio e depends on relative density Dr which is here used as an index

$$I_D = \frac{e_{\max} - e}{e_{\max} - e_{\min}} = \frac{Dr(\%)}{100} \qquad (5)$$

where $e_{\max}$ and $e_{\min}$ are void ratios at loosest and densest state defined by the appropriate practical standards.

3 VALIDATION OF THE MODEL "M"

The described properties of the model offer the possibility to introduce the variable relative density I_D for description of new strength-density-stress level relationship and to test it's validity on a number of published test results as described by Maksimović 1993. It was suggested that the max. angle difference is practically proportional to relative density, and the median angle pressure is approximately linear function of relative density between the zero value at the critical relative density in the range Dr=20-25% and the max. value at of Dr=100%. This finding is further investigated here in order to examine the possibility to derive a reasonable, simple and practical correlation for the strength - stress level - density relationship for sands.

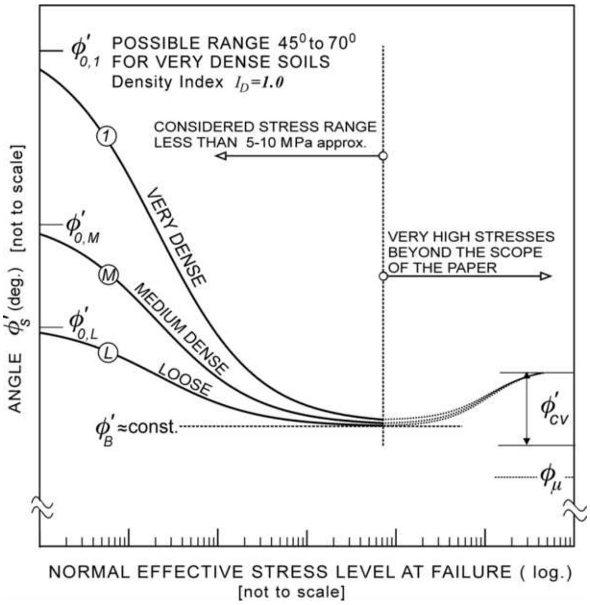

Figure 3. The angle of the shearing resistance depending on density and stress level. Qualitative semi-log plot, stress range of applicability of the model and the derived correlations.

Only 4 data sets on the shearing strength of sand obtained by drained triaxial compression test taken from the literature are shown due to space limitations. The parameters or functions of the suggested model are obtained by minimizing the norm of error by the method outlined earlier (Maksimovic, 1992), where the norm of error is a conventional "the least square fit". The iterative procedure used to generate results shown in 4 figures (Fig.4 to Fig. 7) was as follows. In the first step the constant basic angle ϕ'_B is estimated either as the reported value of ϕ'_{CV} or as the best fit to available data, particularly when the reported ϕ'_{CV} has not provided the best fit. In the second step for each reported constant density the parameters $\Delta\phi'$ and p_{FF} are computed by fitting to the eq. (2). It is found that both parameters depend on density index. The values are plotted in the diagram on the top right area of each of 4 figures and the linear regression applied to derive the linear relationship between these two parameters and the density index. Linear relationships are shown as two lines which show that the relations between these two variables are practically linear functions of the density index I_D. In the third step the derived linear relationships are used to compute curves which correspond to the densities and data for which the results are reported (shown as full lines), and for representative values of density indices of 0.25, 0.50, 0.75 and 1.0 (shown as dashed lines).

Results of triaxial compression test on Mol sand along the stress path with constant octahedral stress for three stress levels and four different relative densities are shown in Fig. 4. The basic angle obtained by curve fitting in this case was significantly lower than the ϕ'_{CV} value, and the same is found for Monterey #0 sand shown in Fig. 5. For Ticino sand in Fig.6 and Fulung sand in Fig.7 $\phi'_B = \phi'_{CV}$ provide a reasonably good fit.

Computed and the test values are compared in Fig. 8. It can be seen that the scatter is very small indicating that the derived correlations for each considered sand is very good though for each sand the different set of linear relationships are derived as shown in Fig. 4 to Fig.7. Author believes that the proposed approach is practical and valid for any sand tested in triaxial

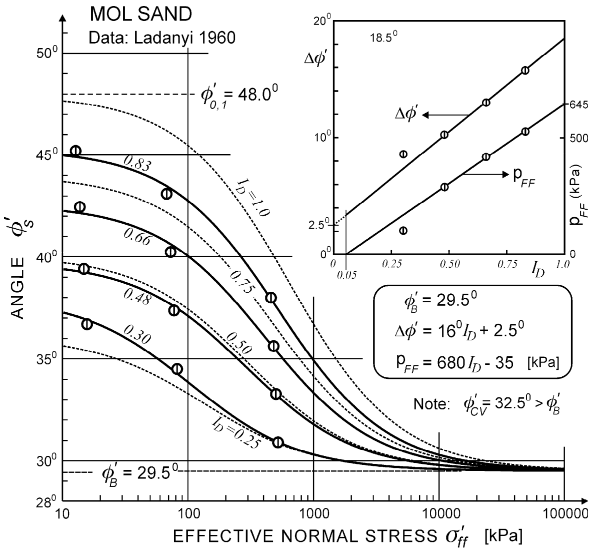

Figure 4. Strength-stress level-density relationship for Mol sand

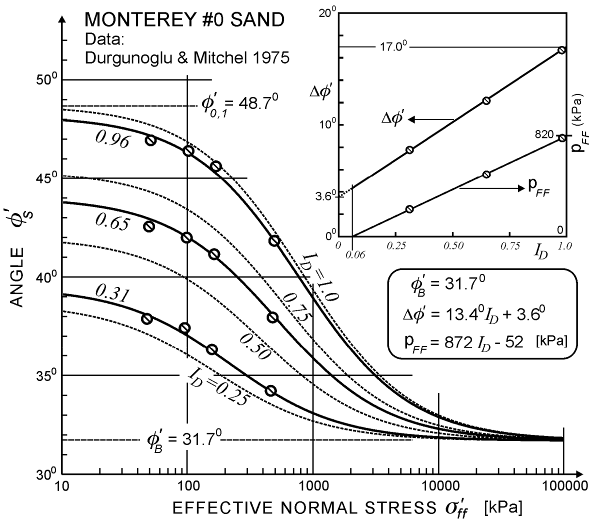

Figure 5. Strength-stress level-density relationship for Monterey #0 sand

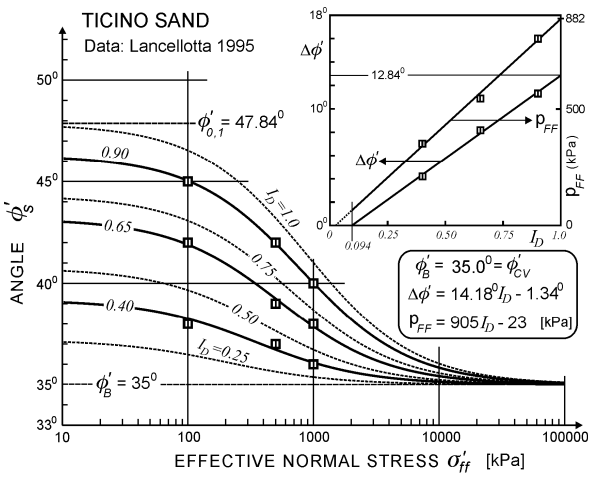

Figure 6. Strength-stress level-density relationship for Ticino sand

compression. On the other hand, one can apply the proposed relationship for alluvial and beech quartz sands encountered in practice with similar grain shape, grain size distribution with coefficient of uniformity C_U<2.5 approximately considered

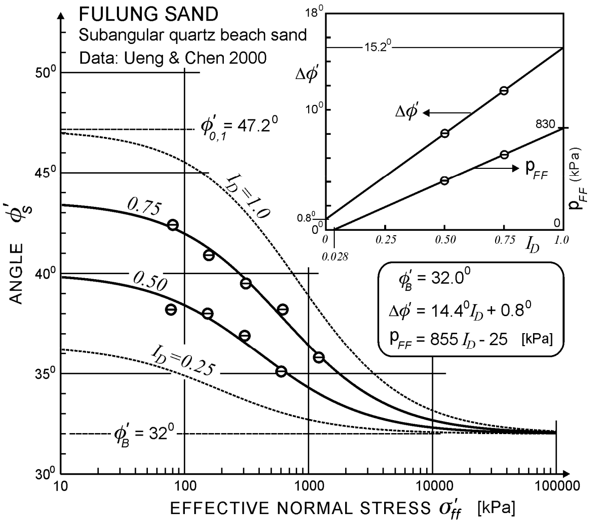

Figure 7. Strength-stress level-density relationship for Fulung sand

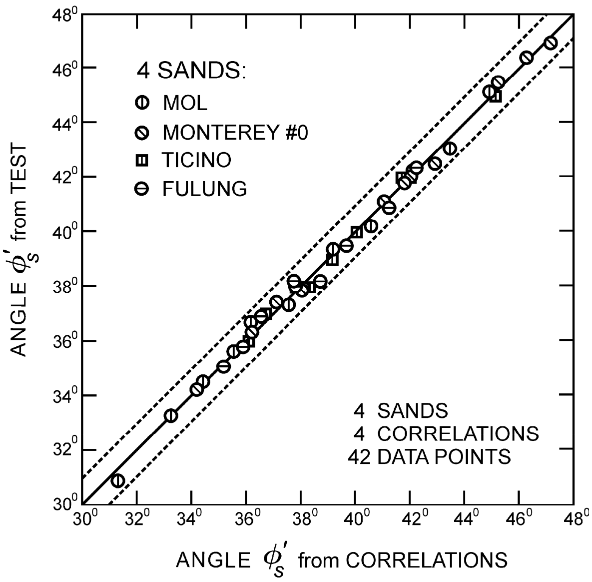

Figure 8. Computed and test values for 4 sands.

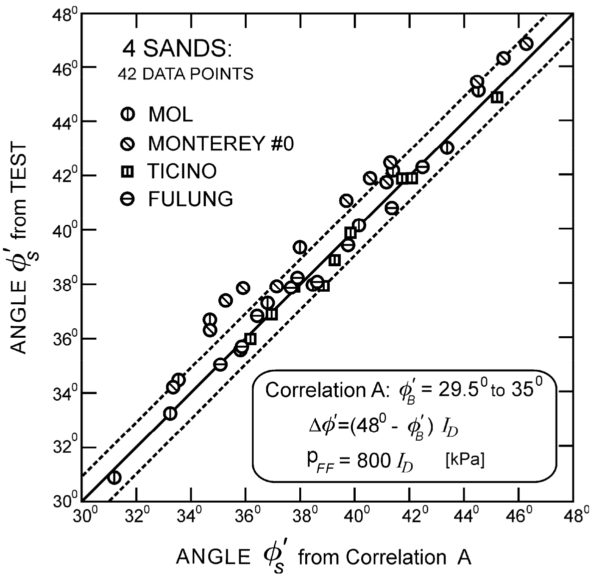

Figure 9. Computed and test values based on tentative correlation.

here, though these data are not shown here due to space limitations, and can be traced in enclosed references. Parameter p_{FF} is significantly larger for well graded and rounded sand grains, but this is outside the scope of this paper.

An attempt is made to derive a practical single set of relationships which would, reasonably and simply as possible, represent the relationship for all four examples. Tentative correlation with the scatter is shown in Fig.9. Within the error of (+/-) 1^0 are 35 data points, while for the remaining 7 data points the difference does not exceed 2^0 only and on the safe side. The common feature of all four examples is that the initial angle of the failure envelope drawn from the origin is about 48^0 for I_d=1.0 being in the range 47.2^0 to 48.7^0.

For most practical applications in limit equilibrium methods, with an error not exceeding 0.2^0 on the safe side, σ'_{ff} can be replaced by σ'_n, p_{FF} by p_N in eq. (2) and the angle of the shearing resistance is simply $\phi'_S = \tan^{-1}(\tau / \sigma'_n)$.

4 CONCLUSIONS

Analyses of 4 published sets of results presented here show, as a practical approximation, that the max. angle difference $\Delta\phi'$ is proportional to relative density and the median angle pressure p_{FF} is linear function of the density index I_D. The proposed approach and relationships could be useful in practical interpretation of shearing strength tests on sands and application in practice.

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REFERENCES:

Bishop, A.W. (1966). The strength of soils as engineering materials, Sixth Rankine Lecture, Geotechnique, Vol.16, No.2, 89-130.

Bolton, M.D. (1986) The strength and dilatancy of sands, Geotechnique 36, No.1, 65-78, and (1987) Author's reply, Geotechnique 37, No.2, 225-226.

Durgunoglu H.T. and Mitchel J.K. (1975). Static penetration resistance of soils. Proc. ASCE Spec.Conf. In Situ Measurements of Soil Properties, Raleigh, NC. 151-189.

Golder, H.Q. (1941). The ultimate bearing pressures of rectangular footings, Journ. Inst. of Civ. Engrs., 17, 161-174.

Lancellotta, R. (1995) Geotechnical Engineering, Balkema, Rotterdam.

Ladanyi, B., (1960). Etude des relations entre les contraintes et des deformations lors du cisaillement des sols pulverulents. Annls Trav. Publ. Belg. No.3, 1-30.

Maksimović, M. (1989).Nonlinear failure envelope for coarse grained soils, XII ICSMFE, Rio de Janeiro, Vol.1, 731-734

Maksimović, M. (1993) Nonlinear failure envelope for the limit state design, Limit state design in geotechnical engineering, International symposium, Danish Geot. Society, Bul. 10, Copenhagen, 131-140.

Maksimović, M. (1996) A Family of Non-linear failure envelopes for Non-Cemented Soils and Rock Joints, WWW Electric Journal for Geotechnical Engineering, Jubilee Volume, Oct., Ppr9607.

Taylor, D.W., (1948). Fundamentals of soil mechanics. John Wiley and Sons, New York.

Ueng, T.S. and Chen, T. J. (2000) Energy aspects of particle breakage in drained shear of sands. Geotechnique 50, No.1, 65-72.