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Simplified phase change model for artificially frozen ground subject to water seepage

Modèle de changement de phase simplifiée pour la congélation artificielle du sol compte tenue du courant de l'eau souterraine

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ABSTRACT

The behaviour of artificially frozen ground is largely dependent on its temperature-dependent thermal and hydraulic properties. The existing ratio of ice to unfrozen water has to be correctly represented for a realistic ascertainment of this behaviour and thus a forecast of the freezing time during icing work. The phase change model presented here can be used to describe the unfrozen water content and thus the freezing behaviour as a function of the temperature. Optimisation possibilities can then be investigated on this basis in the run-up to icing measures.

RÉSUMÉ

Le comportement du sol congelé artificiel est essentiellement dépendant des paramètres caractéristiques thermiques et hydrauliques, qui sont dépendant de la température. Pour un mieux comprendre le comportement du sol et une meilleure prévision de la durée de la congélation, il est nécessaire de déterminer les proportions entre glace et sol non congelé correctement. À l'aide du modèle de changement de phase, quelle se présente ici, il est possible de décrire la teneur de l'eau non congelé et le comportement du sol congelé dépendant de la température. À cause de cette modèle il est possible d'analyser différentes possibilités d'optimisation en avance.

Keywords : ground freezing, frozen soil, phase change model, unfrozen water content

1 INTRODUCTION

The freezing behaviour of artificially frozen ground has recently gained in popularity in a number of construction projects. An artificial withdrawal of heat temporarily freezes the in-situ soil and changes its properties so that it provides the primary stabilisation itself as a static supporting element. What's more, closed frost bodies are watertight. Frost bodies can also be produced in very heterogeneous soils and have no lasting negative effects on the soil and groundwater on account of their reversibility. The technical advantages of the process, however, are opposed by the high costs of the refrigerating capacity that has to be provided over a longer period of time to produce and maintain the frost body.

Apart from the strengths that can be achieved for the frost body, the reliable forecast of frozen times and the frost propagation are an essential planning parameter and basis of the calculation. Up to now, these forecasts have been based on very simple models and often do not take sufficient account of the existing thermal influences on the frost body. Unknown thermal influences or those that are ignored during planning have to be compensated during construction by additional installations and a higher energy input with high consequential costs. This is particularly true of a groundwater flow which can represent a decisive thermal input for an icing measure. In a subsoil with a strong flow the frost propagation may even come to a complete standstill so that a reliable exploration and calculational consideration of any groundwater flows is existentially important in the run-up phase for the safety and profitability of an icing measure.

The effect of the flow results in a coupled thermal transport-groundwater flow problem that cannot be solved with a self-contained analysis. Furthermore, the temperature-dependence of the thermal soil properties and the phase change of the pore water give the problem a very non-linear character, so that a numerical method is required. This can then be used to draw up a flow-modified freezing plan with a considerable savings potential.

2 PROPERTIES OF FROZEN SOIL

The thermal and hydraulic properties of soils display a distinctive temperature dependency in the temperature range that is relevant for subsoil freezing. This is due on the one hand to the temperature-dependent properties of the individual soil constituents themselves, though also and in particular on the changing quantity ratio of water and ice during the freezing process. This process does not have an isothermal course at a certain freezing temperature but within a freezing interval. Unfrozen water can still be found in soil at temperatures far below the freezing point of pure water. This is why it is important to define the volumetric content of ice and water as a function of the temperature.

Irrespective of the temperature, the total porosity n can be calculated with the familiar equation:

$$n = 1 - \frac{\rho_d}{\rho_s} \qquad (1)$$

The resulting volume fraction of the solid n_m in the overall volume is a complementary parameter:

$$n_m = 1 - n \qquad (2)$$

Only fully saturated soils are considered in this example so that the following still has to apply according to Figure 1 for unfrozen and frozen soil:

unfrozen: $n = n_w$ (3)

frozen: $n = n_w + n_i$ (4)

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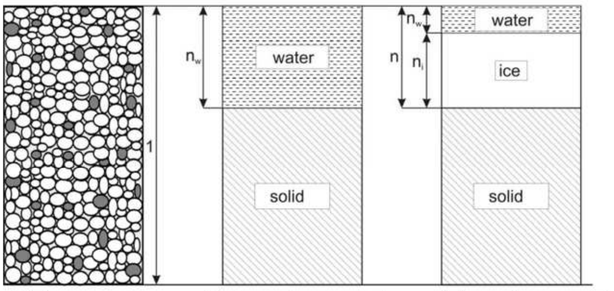

a) heterogeneous composition
b) phase change model for the unfrozen soil
c) phase change model for the frozen soil

Figure 1. Homogenized soil composition of a saturated soil in the case of unfrozen and frozen soil.

If the course of the volumetric contents as a function of the temperature is known, the temperature-dependent properties of the overall system, consisting of soil, water and ice, can be taken into account by a suitable averaging of the corresponding thermal parameters of the individual constituents. These are essentially the thermal conductivity λ [W/mK] and the thermal capacity or the volumetric thermal capacity relative to the volume c_v [J/m³K]. In a fully saturated state, the thermal conductivity of the soil is determined by averaging according to Johansen and Frivik (1980) over the weighted geometric mean of the thermal conductivities of the individual constituents. Taking the ice phase in a frozen state into account, the thermal conductivity can be calculated from the following averaging:

$$\lambda = (\lambda_m)^{1-n} \cdot (\lambda_w)^{n_w} \cdot (\lambda_i)^{n_i} \tag{5}$$

This approach is also used in the calculation model presented below.

The thermal capacity is determined from the weighted arithmetic mean of the individual constituents:

$$c_v = c_{v,m} \cdot n_m + c_{v,w} \cdot n_w + c_{v,i} \cdot n_i \tag{6}$$

The latent crystallisation heat that is released during the phase change of the pore water due to a reorganisation of the atoms during the formation of ice and which delays the freezing process also has a decisive effect on the propagation of a frost body. The latent heat of pure water is L = 333600 J/kg. During thawing the latent heat ensures the inertia of frozen soil compared to a fast defrosting. This offers an important safety aspect in the event of short-term malfunctions in the freezing system.

Figure 2 shows the idealised temperature-dependent courses of the thermal conductivity and thermal capacity of a soil taking into account the latent heat.

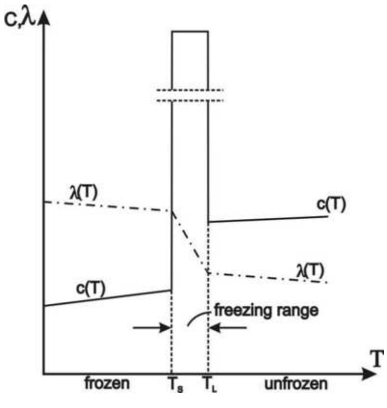

Figure 2. Idealised courses of the thermal conductivity λ and thermal capacity c of a soil taking into account crystallisation heat

According to the method of equivalent thermal capacity, the latent heat is often taken into account through a corresponding increase in the thermal capacity in the freezing interval. The overall volumetric thermal capacity of a soil can hence be expressed as follows, taking the latent heat into account:

$$c_v(T) = c_{v,m} \cdot n_m + c_{v,w} \cdot n_w + c_{v,i} \cdot n_i + \rho_w \cdot L \cdot \frac{\partial n_w}{\partial T} \tag{7}$$

Apart from the thermal properties, the hydraulic conductivity k_f also displays a temperature-dependent behaviour. The permeability k [m²] of a soil, on the other hand, depends solely on the physical characteristics of the crystalline structure. This is not subject to any significant temperature effect and is thus much more suitable than the k_f-value as a constant input value for the simulation of icing measures. The following relationship exists between permeability and hydraulic conductivity via the temperature-dependent fluid properties, density ρ_f and viscosity η:

$$k_f = \frac{k \cdot \rho_f \cdot g}{\eta} \tag{8}$$

Apart from the temperature-dependent change in hydraulic conductivity due to the change in density and viscosity, the remaining unfrozen water content also affects the hydraulic conductivity. The hydraulic conductivity of the frozen soil $k_{f,g}$ can be derived from the hydraulic conductivity of the unfrozen soil $k_{f,u}$ through coupling to the volumetric increase in the ice content according to Jame and Norum (1980):

$$k_{f,g} = 10^{-E \cdot n_i} \cdot k_{f,u} \tag{9}$$

In equation (9), E stands for an impedance factor for which there is no quantitative relationship to a soil property in today's literature. The reference values quoted are approx. E = 2.5 for silty soils, 5 to 15 for sandy soils and approx. 20 for gravely soils (Lundin 1990).

3 UNFROZEN WATER CONTENT

The unfrozen water content w_u is crucial to describe the state of a frozen soil since all volumetric soil contents and thus the thermal and hydraulic properties can be derived directly from this parameter. The knowledge of the course of the unfrozen water content for temperatures below freezing point is consequently essential for any reliable freezing time forecasts. The following figure shows the unfrozen water content for various soils.

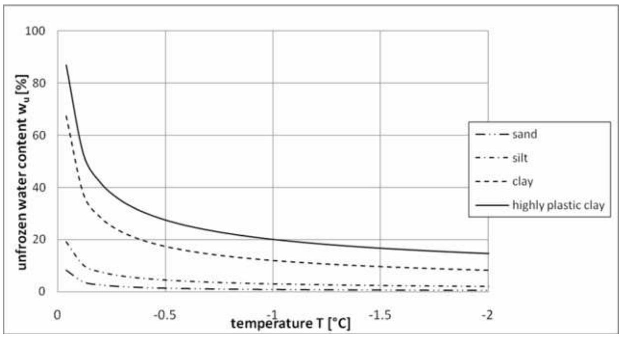

Figure 3. Course of the unfrozen water content for various soil types (after Jessberger 1990)

The course of the unfrozen water content can be determined in both experiments and theoretical models. Anderson and Tice (1972) built on the results of Nerseova and Tsytovich (1963) and investigated the determining factors for the unfrozen water content in soil and came to the result that only the temperature and specific surface are relevant for the course. They formulated the following empirical relationship to describe the w_u-course:

$$\ln(w_u) = 0{,}2618 + 0{,}5519 \cdot \ln(S_s) - 1{,}449 \cdot S_s^{-0{,}264} \cdot \ln(|T'|) \quad (10)$$

This equation can be used to determine the temperature-dependent course of the unfrozen water content solely from the specific surface S_s. The temperature difference between the liquid temperature T_L and existing temperature T is the decisive temperature T´ since it represents the temperature below freezing point per definition (Civan 2000).

If one plots the unfrozen water content for a known specific surface against the temperature, the resulting curve is typical for an exponential function (see Figure 3.), and can be approximated very well by means of regression through two parameters a and b according to the following equation:

$$w_u(T) = a \cdot |T'|^b \quad (11)$$

The prefactor a is positive, the exponent b is always negative.

4 PHASE CHANGE MODEL

The main parameter to determine the soil-specific input parameter for a freezing simulation with our own phase change model is the specific surface. This is simply calculated via a spherical model from the grain distribution and then allows the determination of the w_u-curve by means of equation (10), from which the thermal and hydraulic soil properties described above can be derived.

The specific surface S_s of a body is defined as the ratio of its surface to its mass. With a decreasing diameter, the ratio of surface to volume and thus the specific surface increases for a ball. This is why coarse-grain soils always have a smaller surface than fine-grain soils from geometric considerations.

The simplified calculational methods to determine S_s are based on the assumption that the soil consists of different-sized but ideal-round grains. The total surface of the soil results from the sum total of the single surfaces of all grains. An exact determination is not possible. The grain is thus classified approximately into size ranges in sections. These are characterised by an equivalent diameter d_i and their relevant mass content $\varphi_{m,i}$ of the overall grain. The mathematical surface of a grain is then determined as a weighted mean of the individual n size ranges according to equation (12):

$$S_s = \sum_{i=1}^{n} \frac{6}{d_i \cdot \rho_s} \cdot \varphi_{m,i} \quad (12)$$

The equivalent diameter of a range can be chosen in a number of different ways. Possible values, for example, are the diameter at the upper or lower size range boundary and the arithmetic or harmonic mean of the upper and lower value. Since the assumption of an ideal-round grain without roughness underestimates its surface compared to reality, the lower grain diameter of a size range should be chosen for the equivalent diameter. An experimental comparison for various sandy soils confirms that the resulting specific surface is relatively precise. The simple spherical model thus provides a method to determine the specific surface as the basis for the freezing behaviour of a non-cohesive soil with relatively little work.

Using the specific surface obtained from the spherical model it is now possible to map the course of the unfrozen water content according to Anderson and Tice. The evaluation of the exponential function leads to unrealistically high values for the unfrozen water content close to the freezing point. A ceiling function is thus introduced with the value of the water content prevalent in an unfrozen state so that the maximum water amount to be frozen is given a correct physical upper limit (see Figure 4.). If this ceiling is not set, unrealistically high freezing times arise on account of the clearly overestimated latent heat during the phase change.

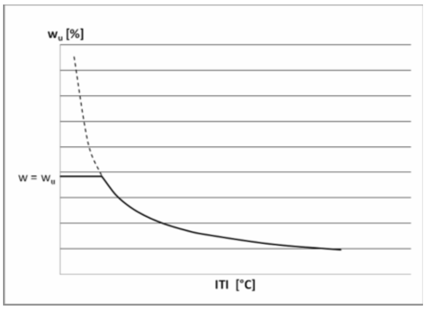

Figure 4. Capped course of the unfrozen water content for the phase change model

5 NUMERICAL IMPLEMENTATION AND VERIFICATION OF THE PHASE CHANGE MODEL

The finite-differences-program SHEMAT (Simulator for Heat and Mass Transport) was developed at the RWTH Aachen Chair for Applied Geophysics by Prof. Clauser's group and is a program system that can cope with the thermal-hydraulic coupling of 3D cases with a changing frost limit (Clauser 2003). The program was originally planned for geophysical problems in intrusive rocks and was recently modified at the Chair of Geotechnical Engineering for practical use in freezing measures. Using the aforementioned simplified phase change model, it is possible to obtain a sufficiently precise description of the freezing behaviour of a soil by specifying only a few standard geotechnical parameters.

The verification of the phase change model and its numerical implementation was carried out by a subsequent calculation of model tests both ignoring and taking into account the influence of a groundwater flow. The following figure shows the temperature curves measured in a model test with groundwater flow (Frivik & Comini 1982) and calculated with SHEMAT.

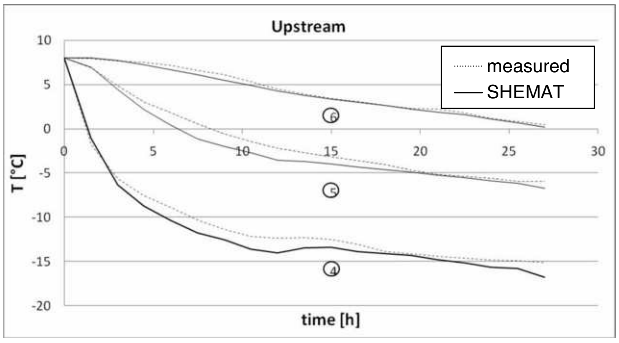

Figure 5. Measured temperature curves and curves calculated via SHEMAT for a model test with groundwater flow

The good concordance shows that it is possible to realistically map the freezing behaviour with the phase change model implemented in SHEMAT.

The derivation of the unfrozen water content from the grain distribution in the soil makes a complicated determination of thermal characteristics with a number of marginal conditions unnecessary. This is thus a practical instrument that can be used to optimise the freezing times under the effect of flows by means of a modified arrangement of the freezing pipes and their modes of operation.

6 OPTIMISATION APPROACHES FOR FREEZING MEASURES WITH A GROUNDWATER FLOW

A fictitious cross-cut with a clear inner diameter of 5.50 m, a necessary frost body thickness of 1.50 m and 18 evenly distributed freezing pipes was chosen to investigate possible optimisation approaches for freezing measures with a groundwater flow. Only half the cross-cut had to be simulated for reasons of symmetry. The influence of the groundwater flow on the frost progress was initially investigated. As the following figure shows, the freezing time rises disproportionately with an increase in the flow velocity.

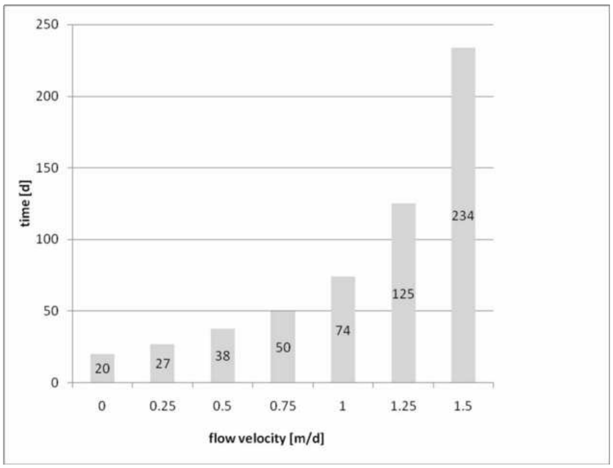

Figure 6. Rise in freezing time with an increasing flow velocity

Various optimisation approaches were investigated taking this as a basis. These show, for example, that by concentrating the freezing pipes on the upstream side of the flow, the freezing time can be reduced with the same number of pipes. Even better results are achieved by pre-cooling with additional freezing pipes in the upstream area.

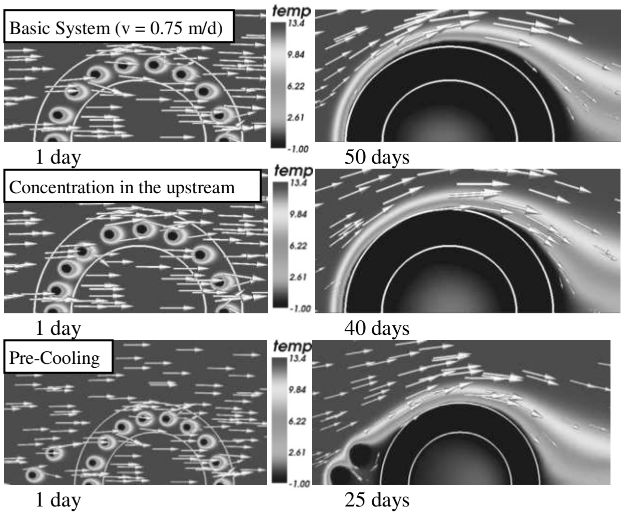

Figure 7. Frost body development based on temperature distribution with various pipe arrangements

As Fig. 7 shows, the freezing times can be greatly reduced compared to the basic system normally used with an equidistant arrangement of the pipes. Nevertheless, it must be remembered that altered pipe arrangements may have the opposite effect and prolong the freezing time if the groundwater situation is misinterpreted. If, however, the groundwater conditions are sufficiently well known and directionally stable, the phase change model presented here is an instrument that can be used to optimise freezing measures in the planning phase, thus leading to long-term cost savings.

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