Non-linear viscous behaviour of undrained Bangkok clay

Comportement visqueux non linéaire de l'argile non drainée de Bangkok

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ABSTRACT

In Newtonian mechanics, shear resistance of material is assumed to be linear proportion to shear rate, in which the proportional constant is defined as a coefficient of viscosity. However, the rate-dependent or viscous behaviour of soil can not simply assume to be linear, but it can be explained using Rate Process Theory (Mitchell and Soga, 2005). The rate effect on shear strength of soil and creep settlement are two well-known examples of the viscous behaviour of soil. In addition, Vaid and Campanella (1977) have shown an experimental evidence of the rate effect on the shear strength of Haney Clay. This paper aims to present the non-linear viscous behaviour of Bangkok Clay using an experiment programme and modelling. The experiments are performed on normally consolidated clay which taken from the Centre of Bangkok. All specimens are isotropically consolidated and undrained compression sheared varying with five different shear rates. The rate-dependent hyperplasticity model (Puzrin and Houlsby, 2003 and Likitlersuang and Houlsby, 2006) based on Rate Process Theory are employed in this study. The model is calibrated against the undrained triaxial sheared tests of Bangkok Clay.

RÉSUMÉ

Dans la mécanique newtonienne, la résistance au cisaillement de la matière est supposée d'être linéaire proportionnellement aux taux de cisaillement, dans laquelle la proportion constante est définie comme un coefficient de viscosité. Toutefois, le comportement visqueux ou dépendant du taux des sols ne peut tout simplement pas assumer d'être linéaire, mais elle peut être expliquée en utilisant la théorie des « Rate Process » (Mitchell et Soga, 2005). Le taux d'effet sur la résistance au cisaillement des sols et celui du tassement du fluage sont deux exemples bien connus de la viscosité du comportement des sols. En outre, Vaid et Campanella (1977) montre une preuve expérimentale de l'effet sur les taux de la résistance au cisaillement de l'argile. Cet article vise à présenter le comportement visqueux non linéaire de l'agile de Bangkok à l'aide d'un programme d'expérimentation et de modélisation. Les expériences sont faites sur l'argile normalement consolidé qui est prise du centre de Bangkok. Tous les échantillons sont consolidés isotrope et cisaillés non drainés en variant avec cinq taux différents de cisaillement. Le modèle hypo-plasticité dépendant du taux (Puzrin et Houlsby, 2003 et Likitlersuang et Houlsby, 2006) basé sur la théorie des « Rate Process » est utilisé dans cette étude. Le modèle est calibré par rapport à l'essais triaxial cisaillé non drainé de l'agile de Bangkok.

Keywords : Undrained shear test, Coefficient of viscosity, Rate Process Theory, Bangkok Clay

1 INTRODUCTION

Rheology is a science for the study of materials whose deformation characteristics vary not only with load but also with time rate of load application. It is well-known that clays are rheological materials; that is, their behaviour depends on rate of loading or rate of prescribed deformation. The rate-dependent (i.e., time-dependent) behaviour of engineering materials could be simply explained as Newtonian material, which assumes that shear resistance is linear proportion to shear rate. However, there are many evidences showing that the viscous behaviour of clays could not simply assume as linear (ex. Vaid & Campanella, 1977).

Rate-dependent models for soil need to address the viscoplastic behaviour. This is often achieved by expressing the strain ε as a function of stress σ and time *t* (i.e. $\varepsilon = f(\sigma, t)$). This potentially leads to two theoretical problems. Firstly, such relationships may violate the Laws of Thermodynamics (or at least it may not be possible to demonstrate that they do not) unless each feature of the model is carefully checked for consistency with thermodynamic laws (see Houlsby & Puzrin (2006). Secondly, the using of time explicitly as a variable in the constitutive law requires the specification of a reference (starting) time. This can in practice be difficult to define in certain problems, and violates certain requirements of objectivity of constitutive models. The material behaviour itself does not depend on time as such, but only in relation to changes of loading. Viscous behaviour under applied stress or strain increments can be more consistently expressed by expressing the constitutive equation in terms of either stress rate or strain rate instead of time. In this paper we effectively define the strain as a function of stress and strain rate: $\varepsilon = f(\sigma, \dot{\varepsilon})$.

A kinematic hardening modified Cam-clay model (Likitlersuang & Houlsby, 2006) is developed for describing the clay behaviour including the rate-effect. The model is developed within a theoretical framework ("continuous hyperplasticity") that is rooted firmly in thermodynamic principles. Thus the resulting model is guaranteed to be consistent with the Laws of Thermodynamics. A detailed exposition of continuous hyperplasticity is not possible here, but is given by Houlsby & Puzrin (2006). The development of this approach may also be traced through Likitlersuang & Houlsby (2006).

This paper aims to present the non-linear viscous behaviour using the rate-dependent hyperplasticity model from Puzrin & Houlsby (2003) and extended by Likitlersuang & Houlsby (2006). The model is calibrated against the undrained triaxial sheared tests of Bangkok clay (Saksuphan, 2006). Some

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discussions on the rate-effects on the stress-strain-strength characteristic of undrained Bangkok clay are addressed.

2 HYPERPLASTICITY THEORY

The rate-dependent hyperplasticity model is described by Puzrin & Houlsby (2003). The model is to describe the shear behaviour of a rate-dependent material, and is expressed in terms of a single pair of stress/strain variables. It can, however be readily extended to a full tensorial form (see for instance Puzrin & Houlsby (2003) and Houlsby & Puzrin (2006)). This model makes use of the continuous hyperplasticity framework described in detail by Houlsby & Puzrin (2006). Such models are based on a thermodynamic approach to plasticity theory, in which the entire constitutive response of the model (including all coupling effects) is defined once two potential functionals are specified. The chosen potential functionals for the Puzrin & Houlsby (2003) model are the Gibbs Free Energy (g) and the Flow Potential (w) (which is related to the dissipation function). The model employs a continuous field of viscoplastic yield surfaces (effectively an infinite number of surfaces). As formulated by Puzrin & Houlsby (2003) it incorporated rate effects, using Rate Process Theory, but did not take into account temperature variations.

Likitlersuang & Houlsby (2006) presented a series of development of constitutive models for soil mechanics, in which all models explained in term of triaxial stress-strain parameters:

$$p = \frac{1}{3}\left(\sigma_1 + 2\sigma_3\right),\; q = \sigma_1 - \sigma_3 \tag{1}$$

$$\varepsilon_p = \varepsilon_1 + 2\varepsilon_3,\; \varepsilon_q = \varepsilon_1 - \varepsilon_3 \tag{2}$$

where subscript 1 and 3 represent major and minor principal stress (or strain), respectively. The model employs the Modified Cam-Clay (MCC) yield surface shape which is success to model the stress-strain-strength characteristic of normally consolidated clay. As we known that a major criticism of the MCC model is the fact that it describes the behaviour of soils inadequately at small strains and it is poor performance with respect to modelling cyclic loading and no modelling of the effects of immediate past history. To remedy this situation, the continuous hyperplasticity with kinematic hardening yield surface is combined with the MCC surfaces. The following expressions are Gibbs Free Energy potential and Flow potential for describing the continuous kinematic hardening Modified Cam-Clay (KHMCC) (Likitlersuang & Houlsby, 2006).

The Gibbs Free Energy functional is defined as:

$$\begin{aligned} g = &-\frac{p^2}{2K} - \frac{q^2}{6G} - \left(p\int_0^1 \hat{\alpha}_p d\eta + q\int_0^1 \hat{\alpha}_q d\eta \right) \\ &+ \int_0^1 \left(\frac{1}{2}\hat{H}_p \hat{\alpha}_p{}^2 + \frac{1}{2}\hat{H}_q \hat{\alpha}_q{}^2 \right) d\eta \end{aligned} \tag{3}$$

where K and G are elastic bulk and shear modulus respectively, η is an "internal coordinate" which effectively specifies size of the yield surface, $\alpha_p = \int_0^1 \hat{\alpha}_p(\eta) d\eta$ and $\alpha_q = \int_0^1 \hat{\alpha}_q(\eta) d\eta$ are kinetic internal variable functions in which play a role as volumetric and deviator plastic strains respectively. The "kernel functions" $\hat{H}_p(\eta)$ and $\hat{H}_q(\eta)$ define the basic shape of the stress-strain curves for consolidation (p-variable) and shear (q-variable) respectively.

The $\hat{H}_p(\eta)$ expression used for the consolidation behaviour is:

$$\hat{H}_p(\eta) = \frac{K}{2\left(a_p - 1\right)}\left(1 - r_p \eta\right)^3 \tag{4}$$

where a_p is the non-linearity parameter of the consolidation curve and r_p controlling the asymptote of the consolidation curve, which is related to the slope of the normal consolidation line (λ). The volumetric stress-strain relationship known as consolidation curve can be represented (different from conventional plot in soil mechanics) as in Figure 1(a).

The $\hat{H}_q(\eta)$ function is simply expressed as hyperbolic stress-strain curve:

$$\hat{H}_q(\eta) = \frac{3G}{2\left(a_q - 1\right)}\left(1 - \eta\right)^3 \tag{5}$$

where a_q acting as a shaping parameter controlling the rate of change of tangent stiffness. The typical shape of the deviator stress-strain curve can be illustrated as shown in Figure 1(b).

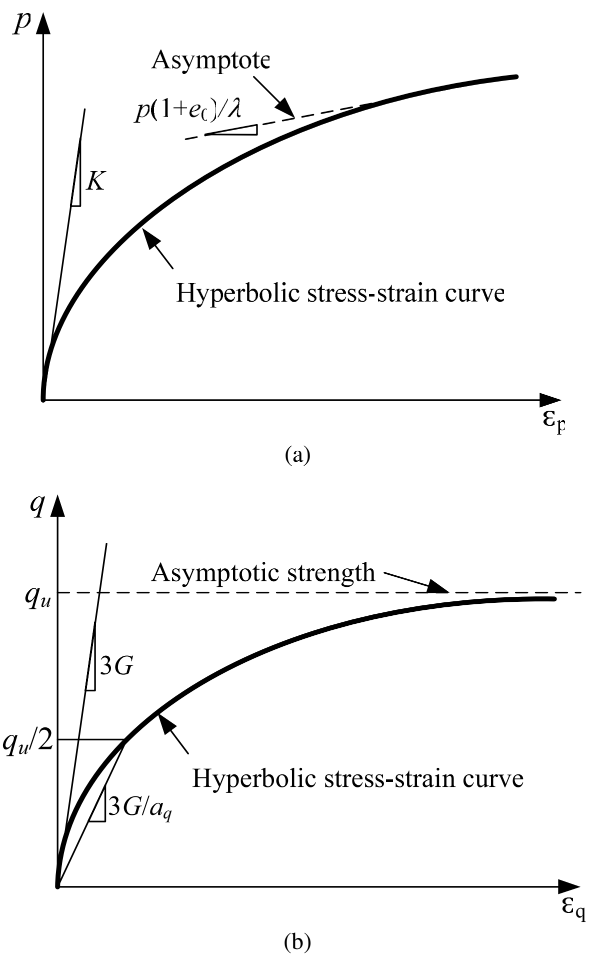

Figure 1. (a) Typical volumetric stress-strain relationship (consolidation curve) from model (b) Typical deviator stress-strain relationship from model

The flow potential is specified as:

$$w = \mu r^2 \int_0^1 \left(\cosh\left(\frac{\left\langle \sqrt{\hat{\chi}_p^2 + \hat{\chi}_q^2 / M^2} - \hat{H}_p \hat{\alpha}_p \eta \right\rangle}{\mu r} \right) - 1 \right) d\eta \quad (6)$$

The "generalised stresses" $\hat{\chi}_p(\eta)$ and $\hat{\chi}_q(\eta)$ are the quantities those are conjugate to the internal variables $\hat{\alpha}_p$ and $\hat{\alpha}_q$ respectively, which is expressed as:

$$\hat{\chi}_p = -\frac{\partial g}{\partial \hat{\alpha}_p} = p - \hat{H}_p \hat{\alpha}_p \quad (7a)$$

$$\hat{\chi}_q = -\frac{\partial g}{\partial \hat{\alpha}_q} = q - \hat{H}_q \hat{\alpha}_q \quad (7b)$$

However, for a particular case of undrained behaviour of clay, the expression of flow potential in Equation (6) can be simplified to one-dimensional case as:

$$w = \mu r^2 \int_0^1 \left(\cosh\left(\frac{\left\langle \left|\hat{\chi}_q\right| - k_0 \exp\left(\hat{\alpha}_q / \alpha_0\right)\eta \right\rangle}{\mu r} \right) - 1 \right) d\eta \quad (8)$$

where k_0 and α_0 are material parameters calibrated to fit experimental data. This is because it is simple for numerical implementation in spreadsheet programme.

The above form of flow potential employs Rate Process Theory, which is required two parameters: μ is the viscosity at small strain rate and r is a constant with the dimensions of strain rate. $\langle \ \rangle$ denote Macaulay brackets: $\langle x \rangle = 0, \ x < 0; \ \langle x \rangle = x, \ x \geq 0$. For strain rates much smaller than $r/2$ the additional stress due to viscous effects is approximately $\mu\dot{\varepsilon}$ and therefore corresponds to linear viscosity, whilst for strain rates much larger than $r/2$ the additional stress is approximately $\mu r \log(2\dot{\varepsilon}/r)$. In between there is a transition, which is shown in Figure 2.

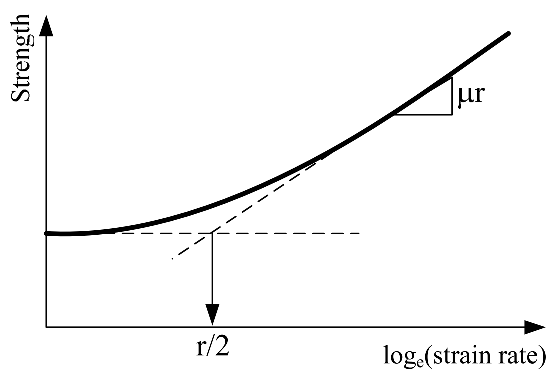

Figure 2. Viscous effect on the strength implied by Rate Process Theory

Application of the standard procedures described by Houlsby & Puzrin (2006) gives the equation for the increment of the internal variable:

$$w = \mu r^2 \int_0^1 \left(\cosh\left(\frac{\left\langle \left|\hat{\chi}_q\right| - k_0 \exp\left(\hat{\alpha}_q / \alpha_0\right)\eta \right\rangle}{\mu r} \right) - 1 \right) d\eta \quad (8)$$

where $S(x)$ is the extended signum function in which $S(x) = -1, x < 0; \ -1 \leq S(x) \leq 1, x = 0;$ and $S(x) = 1, x > 0$

The incremental stress-strain response is then given by:

$$d\varepsilon_q = \frac{dq}{3G} + \left[\int_0^1 r \sinh\left(\frac{\left\langle \left|\hat{\chi}_q\right| - k_0 \exp\left(\hat{\alpha}_q / \alpha_0\right)\eta \right\rangle}{\mu r} \right) S\left(\hat{\chi}_q\right) d\eta \right] dt \quad (9)$$

Above equations are all that is required to compute the complete stress-strain response in undrained compression shear. The model requires five material parameters: G, a_q, k_0, α_0, μ and r, is capable of modelling an elasto-visco-plastic response including the effect of strain rate.

3 EXPERIMENTAL PROGRAMME AND MODEL CARIBRATION

An experimental study on the rate-effects on the undrained stress-strain-strength behaviour of Bangkok has been carried out at Chulalongkorn University (Saksuphan, 2006). The specimens have been taken from 7.5 – 9.5 m depth in the Centre of Bangkok. The average index properties are plastic limit (*PL*) of 44%, liquid limit (*LL*) of 95%, natural moisture content (w_n) of 84%, and specific gravity (G_s) of 2.67.

The model has been calibrated against each of tests from Saksuphan (2006), in which it report tests on normally consolidated clays which were undrained compression sheared. All specimens were isotropically consolidated at σ_c' = 150 kPa and undrained compression sheared varying the shear rate of 0.010, 0.105, 0.536, 1.056 and 9.820 % per minute respectively. The appropriate parameter values were also determined as shown in Table 1. The process of parameter fitting involved some optimisation of parameter values, but the values do not represent rigorous least square optimal values. However, some values of the model parameters could be guided from a comprehensive study of the prediction of a continuous hyperplasticity model for Bangkok Clay by Likitlersuag & Houlsby (2007).

Table 1. Model parameters for undrained compression test on Bangkok clay

Parameters	values	unit
$3G$	15.5	MPa
k_0	11.5	kPa
μ	4.2	MPamin
r	0.0005	min^{-1}
a_q	2.8	(-)
α_0	0.40	(-)

The predictions of model with test results are presented in Figures 3 and 4. Figure 3 shows the prediction of the deviator stress-strain curves. Figure 4 presents the variation of undrained strength with strain rate. The model is obviously able to capture accurately effect of strain rate on stress-strain curve and undrained strength of Bangkok clay.

The particular form of the flow potential was chosen so that it results in viscous behaviour corresponding to Rate Process Theory (for a useful discussion of this theory see Mitchell & Soga, 2005). The viscous component of the stress σ_{visc} in the model is related to the plastic strain rate $\dot{\alpha}$ through an expression of the form:

$$\dot{\alpha} = r \sinh\left(\frac{\sigma_{visc}}{\mu r}\right) \tag{10}$$

The values of r = 0.0005 min^{-1} and μr = 2.1 kPa can be observed from Figure 4, which agree with the physical meaning illustrating in Figure 2. The values of r = 0.0005 min^{-1} and μ/σ'_c = 28 min (normalised by the pre-consolidation pressure, σ'_c) can be comparable with the analysis result of undrained test on Haney Clay from Puzrin & Houlsby (2003), in which they reported r = 0.008 min^{-1} and μ/σ'_c = 2.8 min.

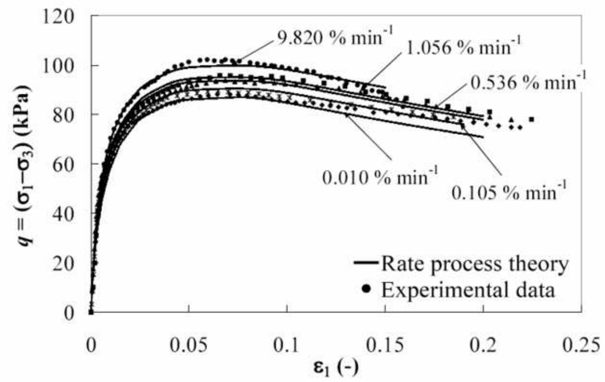

Figure 3. Comparison between stress-strain curves for undrained tests on Bangkok clay and the model prediction

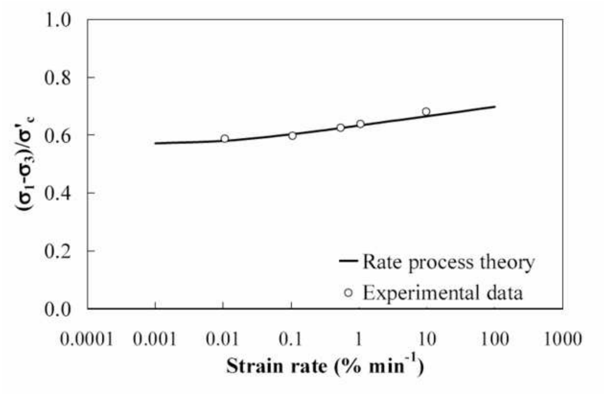

Figure 4. Variation of undrained strength with strain rate: comparison between model prediction and experimental data

4 CONCLUSION

It is remarkable that a model entirely encapsulated by the two potential functions: energy function and flow potential and using only six material parameters is able to capture the undrained behaviour caused by rate effects. It is also shown that each of model parameters can be given a clearly defined physical meaning interpretation. The model includes the non-linear viscous response from Rate Process Theory, which requires two viscous parameters: μ acting as the coefficient of viscosity at small strain rate and r playing the turning point between small and large strain rate effects. The experimental result of undrained compression test varying shear strain rate on Bangkok clay can used to confirm the performance of the model.

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