ABSTRACT

In Newtonian mechanics, shear resistance of material is assumed to be linear proportion to shear rate, in which the proportional constant is defined as a coefficient of viscosity. However, the rate-dependent or viscous behaviour of soil can not simply assume to be linear, but it can be explained using Rate Process Theory (Mitchell and Soga, 2005). The rate effect on shear strength of soil and creep settlement are two well-known examples of the viscous behaviour of soil. In addition, Vaid and Campanella (1977) have shown an experimental evidence of the rate effect on the shear strength of Haney Clay. This paper aims to present the non-linear viscous behaviour of Bangkok Clay using an experiment programme and modelling. The experiments are performed on normally consolidated clay which taken from the Centre of Bangkok. All specimens are isotropically consolidated and undrained compression sheared varying with five different shear rates. The rate-dependent hyperplasticity model (Puzrin and Houlsby, 2003 and Likitlersuang and Houlbsy, 2006) based on Rate Process Theory are employed in this study. The model is calibrated against the undrained triaxial sheared tests of Bangkok Clay.

RÉSUMÉ


Keywords : Undrained shear test, Coefficient of viscosity, Rate Process Theory, Bangkok Clay
discussions on the rate-effects on the stress-strain-strength characteristic of undrained Bangkok clay are addressed.

2 HYPERPLASTICITY THEORY

The rate-dependent hyperplasticity model is described by Puzrin & Houlsby (2003). The model is to describe the shear behaviour of a rate-dependent material, and is expressed in terms of a single pair of stress/strain variables. It can, however, be readily extended to a full tensorial form (see for instance Puzrin & Houlsby (2003) and Houlsby & Puzrin (2006)). This model makes use of the continuous hyperplasticity framework described in detail by Houlsby & Puzrin (2006). Such models are based on a thermodynamic approach to plasticity theory, in which the entire constitutive response of the model (including all coupling effects) is defined once two potential functionals are specified. The chosen potential functionals for the Puzrin & Houlsby (2003) model are the Gibbs Free Energy ($G$) and the Flow Potential ($\nu$) (which is related to the dissipation function).

The model employs a continuous field of viscoplastic yield surfaces (effectively an infinite number of surfaces). As formulated by Puzrin & Houlsby (2003) it incorporated rate effects, using Rate Process Theory, but did not take into account temperature variations.

Likitlersuang & Houlsby (2006) presented a series of developments of constitutive models for soil mechanics, in which all models explained in term of triaxial stress-strain parameters:

\[ p = \frac{1}{3}(\sigma_1 + 2\sigma_3), \quad q = \sigma_1 - \sigma_3 \] (1)

\[ \varepsilon_p = \varepsilon_1 + 2\varepsilon_3, \quad \varepsilon_q = \varepsilon_1 - \varepsilon_3 \] (2)

where subscripts 1 and 3 represent major and minor principal stress (or strain), respectively. The model employs the Modified Cam-Clay (MCC) yield surface shape which is success to model the stress-strain-strength characteristic of normally consolidated clay. As we known that a major criticism of the MCC model is the fact that it describes the behaviour of soils inadequately at small strains and it is poor performance with respect to cyclic loading and no modelling of the effects of immediate past history. To remedy this situation, the continuous hyperplasticity with kinematic hardening yield surface is combined with the MCC surfaces. The following expressions are Gibbs Free Energy potential and Flow potential for describing the continuous kinematic hardening Modified Cam-Clay (KHMCC) (Likitlersuang & Houlsby, 2006).

The Gibbs Free Energy functional is defined as:

\[ g = -\frac{p^2}{2K} - \frac{q^2}{6G} - \left( p \int_0^1 \dot{\alpha}_p d\eta + q \int_0^1 \dot{\alpha}_q d\eta \right) \]

\[ + \int_0^1 \left( \frac{1}{2} \dot{H}_p \dot{\alpha}_p^2 + \frac{1}{2} \dot{H}_q \dot{\alpha}_q^2 \right) d\eta \] (3)

where $K$ and $G$ are elastic bulk and shear modulus respectively, $\eta$ is an “internal coordinate” which effectively specifies size of the yield surface, $\alpha_p = \frac{1}{2} \dot{\alpha}_p (\eta) d\eta$ and $\alpha_q = \frac{1}{2} \dot{\alpha}_q (\eta) d\eta$ are kinetic internal variable functions in which play a role as volumetric and deviator plastic strains respectively. The “kernel functions” $\dot{H}_p(\eta)$ and $\dot{H}_q(\eta)$ define the basic shape of the stress-strain curves for consolidation ($p$-variable) and shear ($q$-variable) respectively.

The $\dot{H}_p(\eta)$ expression used for the consolidation behaviour is:

\[ \dot{H}_p(\eta) = \frac{K}{2(a_p - 1)} (1 - r_p \eta)^3 \] (4)

where $a_p$ is the non-linearity parameter of the consolidation curve and $r_p$ controlling the asymptote of the consolidation curve, which is related to the slope of the normal consolidation line ($\lambda$). The volumetric stress-strain relationship known as consolidation curve can be represented (different from conventional plot in soil mechanics) as in Figure 1(a).

The $\dot{H}_q(\eta)$ function is simply expressed as hyperbolic stress-strain curve:

\[ \dot{H}_q(\eta) = \frac{3G}{2(a_q - 1)} (1 - \eta)^3 \] (5)

where $a_q$ acting as a shaping parameter controlling the rate of change of tangent stiffness. The typical shape of the deviator stress-strain curve can be illustrated as shown in Figure 1(b).
The flow potential is specified as:

\[ w = \mu r^2 \int_0^1 \left( \cosh \left( \frac{\left[ \kappa^2 + \frac{\hat{\alpha}^2}{\mu r} - \dot{H}_p \hat{\alpha}_p \eta \right]}{\mu r} \right) - 1 \right) d\eta \]  (6)

The “generalised stresses” \( \dot{\hat{\alpha}}_p(\eta) \) and \( \dot{\hat{\alpha}}_q(\eta) \) are the quantities those are conjugate to the internal variables \( \hat{\alpha}_p \) and \( \hat{\alpha}_q \) respectively, which is expressed as:

\[
\begin{align*}
\dot{\hat{\alpha}}_p &= -\frac{\partial \hat{\alpha}_p}{\partial \hat{\alpha}_p} = p - \dot{H}_p \hat{\alpha}_p & (7a) \\
\dot{\hat{\alpha}}_q &= -\frac{\partial \hat{\alpha}_q}{\partial \hat{\alpha}_q} = q - \dot{H}_q \hat{\alpha}_q & (7b)
\end{align*}
\]

However, for a particular case of undrained behaviour of clay, the expression of flow potential in Equation (6) can be simplified to one-dimensional case as:

\[ w = \mu r^2 \int_0^1 \left( \cosh \left( \frac{\left[ \hat{\eta} q - k_0 \exp(\hat{\alpha}_q/\alpha_0) \eta \right]}{\mu r} \right) - 1 \right) d\eta \]  (8)

where \( k_0 \) and \( \alpha_0 \) are material parameters calibrated to fit experimental data. This is because it is simple for numerical implementation in spreadsheet programme.

The above form of flow potential employs Rate Process Theory, which is required two parameters: \( \mu \) is the viscosity at small strain rate and \( r \) is a constant with the dimensions of strain rate. ( ) denote Macaulay brackets: \( \{ x \} = 0, \ x < 0; \ \{ x \} = x, \ x \geq 0 \). For strain rates much smaller than \( r/2 \) the additional stress due to viscous effects is approximately \( \mu \dot{\varepsilon} \) and therefore corresponds to linear viscosity, whilst for strain rates much larger than \( r/2 \) the additional stress is approximately \( \mu r \log(2 \dot{\varepsilon}/r) \). In between there is a transition, which is shown in Figure 2.

Figure 2. Viscous effect on the strength implied by Rate Process Theory

Application of the standard procedures described by Houlsby & Puzrin (2006) gives the equation for the increment of the internal variable:

\[ w = \mu r^2 \int_0^1 \left( \cosh \left( \frac{\left[ \hat{\eta} q - k_0 \exp(\hat{\alpha}_q/\alpha_0) \eta \right]}{\mu r} \right) - 1 \right) d\eta \]  (8)

where \( S(x) \) is the extended signum function in which \( S(x) = -1, x < 0; \ -1 \leq S(x) \leq 1, x = 0; \ \text{and} \ S(x) = 1, x > 0 \). The incremental stress-strain response is then given by:

\[
\begin{align*}
d\epsilon &= \frac{dq}{3G} + \\
&+ \int_r \sinh \left( \frac{\hat{\eta} q - k_0 \exp(\hat{\alpha}_q/\alpha_0) \eta}{\mu r} \right) S(\hat{\eta} q) d\eta 
\end{align*}
\]  (9)

Above equations are all that is required to compute the complete stress-strain response in undrained compression shear. The model requires five material parameters: \( G, \ a_q, \ k_q, \ \alpha_0, \mu \) and \( r \), is capable of modelling an elasto-visco-plastic response including the effect of strain rate.

3 EXPERIMENTAL PROGRAMME AND MODEL CARIBRATION

An experimental study on the rate-effects on the undrained stress-strain-strength behaviour of Bangkok has been carried out at Chulalongkorn University (Saksuphan, 2006). The specimens have been taken from 7.5 – 9.5 m depth in the Centre of Bangkok. The average index properties are plastic limit (PL) of 44%, liquid limit (LL) of 95%, natural moisture content \( (w_n) \) of 84%, and specific gravity \( (G_s) \) of 2.67.

The model has been calibrated against each of tests from Saksuphan (2006), in which it report tests on normally consolidated clays which were undrained compression sheared. All specimens were isotropically consolidated at 150 kPa and undrained compression sheared varying the shear rate of 0.010, 0.105, 0.536, 1.056 and 9.820 % per minute respectively. The appropriate parameter values were also determined as shown in Table 1. The process of parameter fitting involved some optimisation of parameter values, but the values do not represent rigorous least square optimal values. However, some values of the model parameters could be guided from a comprehensive study of the prediction of a continuous hyperplasticity model for Bangkok Clay by Likitlersuang & Houlsby (2007).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Unit</th>
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<tr>
<td>3G</td>
<td>15.5</td>
<td>MPa</td>
</tr>
<tr>
<td>( k_0 )</td>
<td>11.5</td>
<td>kPa</td>
</tr>
<tr>
<td>( \mu )</td>
<td>4.2</td>
<td>MPamin</td>
</tr>
<tr>
<td>( r )</td>
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<td>min(^{-1})</td>
</tr>
<tr>
<td>( a_q )</td>
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<td>(-)</td>
</tr>
<tr>
<td>( \alpha_0 )</td>
<td>0.40</td>
<td>(-)</td>
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The predictions of model with test results are presented in Figures 3 and 4. Figure 3 shows the prediction of the deviator stress-strain curves. Figure 4 presents the variation of undrained strength with strain rate. The model is obviously able to capture...
accurately effect of strain rate on stress-strain curve and undrained strength of Bangkok clay.

The particular form of the flow potential was chosen so that it results in viscous behaviour corresponding to Rate Process Theory (for a useful discussion of this theory see Mitchell & Soga, 2005). The viscous component of the stress $\sigma_{\text{visc}}$ in the model is related to the plastic strain rate $\dot{\alpha}$ through an expression of the form:

$$\dot{\alpha} = r \sinh \left( \frac{\sigma_{\text{visc}}}{\mu r} \right)$$

The values of $r = 0.0005 \text{ min}^{-1}$ and $\mu r = 2.1 \text{ kPa}$ can be observed from Figure 4, which agree with the physical meaning illustrating in Figure 2. The values of $r = 0.0005 \text{ min}^{-1}$ and $\mu / \sigma_\text{c}' = 28 \text{ min}$ (normalised by the pre-consolidation pressure, $\sigma_\text{c}'$) can be comparable with the analysis result of undrained test on Haney Clay from Puzrin & Houlsby (2003), in which they reported $r = 0.008 \text{ min}^{-1}$ and $\mu / \sigma_\text{c}' = 2.8 \text{ min}$.

**Figure 3.** Comparison between stress-strain curves for undrained tests on Bangkok clay and the model prediction

**Figure 4.** Variation of undrained strength with strain rate: comparison between model prediction and experimental data

4 CONCLUSION

It is remarkable that a model entirely encapsulated by the two potential functions: energy function and flow potential and using only six material parameters is able to capture the undrained behaviour caused by rate effects. It is also shown that each of model parameters can be given a clearly defined physical meaning interpretation. The model includes the non-linear viscous response from Rate Process Theory, which requires two viscous parameters: $\mu$ acting as the coefficient of viscosity at small strain rate and $r$ playing the turning point between small and large strain rate effects. The experimental result of undrained compression test varying shear strain rate on Bangkok clay can used to confirm the performance of the model.

REFERENCES


