Dilatancy and shear strength behavior of sand at low confining pressures

Dilatance comportement et la résistance au cisaillement de sable à faible pression de confinement

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ABSTRACT

Sand dilates with shearing at a rate that increases with increasing relative density (D_R) and decreases with increasing effective confining stress (σ'_c). The peak friction angle of a sand depends on its critical-state friction angle and on dilatancy. In this paper, we develop a simple correlation between peak friction angle, critical-state friction angle, and dilatancy based on triaxial compression and plane-strain compression test data for sand for a range of confining pressures from very low levels to approximately 196 kPa.

RÉSUMÉ

Le sable se dilate avec la tonte à un taux qui augmente avec la densité relative (D_R) et diminue avec l'augmentation de l'efficacité de confinement (σ'_c). Le pic d'un angle de frottement du sable dépend de son état critique-angle de frottement et de dilatance. Dans ce papier, nous développons une simple corrélation entre l'angle de frottement maximal, l'angle de frottement dans l'état critique et l'angle de dilatance basé sur la compression triaxiale et plane-strain compression de données de test pour le sable d'une gamme de pressions de confinement de niveaux très bas d'environ 196 kPa

Keywords : sand; low confining stress; dilatancy; shear strength; friction angle.

1 INTRODUCTION

Sand exhibits dilatancy, which reduces with decreasing relative density (D_R) and increasing confining pressure (σ'_c). Bolton (1986), based on an examination of a large number of plane-strain and triaxial tests, proposed an equation to express this relationship. His paper did not contain data for very low confining stresses, so it is not known whether the correlation proposed in it would be applicable to situations of low confinement. An obvious example of cases in which that low-confining-stress correlation would be needed is the simulation of 1g model tests. The interpretation of in-situ tests at very shallow depths or prediction of local soil response at shallow depths would also require good understanding of soil response at low confinement. Another application in which it may become important in the first half of this century is construction on the Moon.

In this paper, we analyze data for drained triaxial compression (TX) and plane-strain compression (PSC) tests on sand under low confining pressures with the aim of quantifying the dependence of dilatancy and friction angle on relative density and confining stress. The effect of the intermediate principal stress has been explicitly incorporated in the equation for the analysis of plane-strain test data.

2 STRESS-DILATANCY RELATION AND FRICTION ANGLE CORRELATION

Rowe (1962) developed stress-dilatancy theory based on the analogy between the irregular packing of soil particles and regular assemblies of spheres or cylinders and on the hypothesis that a minimum energy ratio at failure is achieved. De Josselin de Jong (1976) put the theory on more solid basis by considering the laws of friction. Since then, many researchers have worked on linking dilatancy with soil state (e.g., Been and Jefferies, 1985, Manzari and Dafalias, 1997, Yu, 1998, Li et al., 1999, Li and Dafalias, 2000, Salgado et al., 2000, Li, 2002).

The dilatancy angle (ψ) is defined as:

$$\sin\psi = \frac{d\varepsilon_1 + k d\varepsilon_3}{d\varepsilon_1 - k d\varepsilon_3} \tag{1}$$

where $d\varepsilon_1$ and $d\varepsilon_3$ = principal strain increments; k = 1 for plane-strain; and k = 2 for triaxial test conditions. Bolton (1986), examining a large volume of data for various sands, concluded that, for both triaxial and plane-strain loading, the following relationship was valid:

$$-\left(\frac{d\varepsilon_v}{d\varepsilon_1}\right)_p = 0.3 I_R \tag{2}$$

where $d\varepsilon_v$ is the volumetric strain increment. The subscript p is used to indicate that the quantity is measured or calculated at the point at which peak strength is reached. The quantity I_R is referred to as the relative dilatancy index for both plane-strain and triaxial tests. It is expressed in terms of soil state as:

$$I_R = I_D\left(Q - \ln\frac{100\sigma'_{mp}}{p_A}\right) - R \tag{3}$$

where $I_D = D_R/100$ = relative density, ranging from 0 to 1; σ'_{mp} is the mean effective stress at peak shear strength; Q, R = fitting parameters that depend on the intrinsic sand characteristics; p_A = reference stress = 100kPa. The peak friction angle (ϕ_p) is written in terms of I_R as:

$$\phi_p = \phi_c + A_\psi I_R \tag{4}$$

where ϕ_c is the critical state friction angle; A_ψ = 3 (for triaxial conditions) and A_ψ = 5 (for plane-strain conditions) produced acceptable matches to the experimental data considered by Bolton (1986).

For PSC and TX conditions, Eqs. (2) and (4) imply, respectively, the following:

$$\phi_p = \phi_c + 0.8\psi_p \tag{5}$$

$$\phi_p = \phi_c + 0.5\psi_p \tag{6}$$

where ψ_p is the peak dilatancy angle.

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3 FRICTION ANGLE AND DILATANCY RATE CORRELATION FOR TOYOURA SAND

The literature contains extensive TX and PSC test data. For Toyoura sand, TX data can be found in Fukushima and Tatsuoka (1984) and Tatsuoka (1987). PSC data for Toyoura sand are available in Tatsuoka et al. (1986) and Tatsuoka (1987). For these data, we define three ranges of confining stress: $\sigma'_{mp} < 50$ kPa , $50 \leq \sigma'_{mp} \leq 100$ kPa and $\sigma'_{mp} > 100$ kPa respectively. According to their relative densities, specimens are classified as "very loose" (0-15%), "loose" (15-35%), "medium" (35-65%), "dense" (65-85%) or "very dense" (85-100%). The data available for Toyoura sand are very complete, including tests performed at very low confining pressures, and will be used in this paper to further explore the concepts and friction angle correlation advanced by Bolton (1986).

Plane-strain compression can best be visualized in terms of a test on a sample such that two of the sides of the sample are fixed, while the other two are free to move. On the sides where movement is possible, the stress σ'_3 is constant throughout the test. With the increase of σ'_1, the sample contracts vertically and, for soil densities of practical interest, i.e., from loose to very dense sands, tries to expand horizontally. But it can expand only in the direction of application of σ'_3. In the direction normal to the boundaries (in which no movement is allowed in a traditional plane-strain testing device), the sample pushes against the walls, causing an increase in the normal stress there, so that $\sigma'_2 > \sigma'_3$. The presence of the boundary also interferes with the rolling of sand particles over one another even at critical state, thus introducing a kinematic constraint on the sand sample, making the rolling of particles over each other more difficult than under triaxial conditions, where the particles have an extra degree of freedom. In plane-strain compression, sand particles require extra energy to move, with the consequence that ϕ_c becomes larger than in triaxial compression.

The effect of stress path on critical-state friction angle (ϕ_c) has been studied by several researchers, including Wang and Lade (2001), Alshibli and Williams (2005), Lam and Tatsuoka (1988), Matsuoka and Nakai (1982), Sutherland and Mesdary (1969), and Yoshimine (2005). For our analyses, the value of ϕ_c is chosen based on the observations that, for a particular type of sand, it would not change with a change of initial relative density or initial confining stress, but that it will vary with the loading path. Experimental observations have shown that ϕ_c is higher for PSC conditions than for TX conditions (Pradhan et al., 1988 and Yoshimine, 2005). Several researchers have proposed ϕ_c values of Toyoura sand for TX conditions. Examples of proposed values are: 31.6 ^{o} (Verdugo & Ishihara, 1996), 31.1^o (Wang et al., 2002), and 31.2 ^{o} -34.4^o, with an average ϕ_c value of 32.8 ^{o} (Fukushima and Tatsuoka, 1984 and Tatsuoka, 1987). Tatsuoka et al. (1986), Pradhan et al. (1988) and Yoshimine (2005) contended that ϕ_c can be 3-5^o higher in PSC tests than in TX. According to the literature, ϕ_c values for Toyoura sand in PSC conditions are: 34.5^o – 38^o (Tatsuoka et al., 1986 and Pradhan et al., 1988) and 36^o (Tatsuoka, 1987, for loading perpendicular to the predominant orientation of the particles). For our analyses, we consider $\phi_c = 32.8^o$ for TX and 36^o for PSC conditions.

We plot (ϕ_p-ϕ_c) vs I_D and I_R in Figs. 1(a) and (b), respectively, for both TX and PSC conditions. Bolton (1986) showed that, for Q =10 and R=1, I_R can be obtained from strains using Eq. (2) for both TX and PSC conditions. Eq. (2) also fits well the values of $(d\varepsilon_v/d\varepsilon_1)_p$ and I_R for the data examined in this paper. Both I_R and $(d\varepsilon_v/d\varepsilon_1)_p$ are taken at peak strength. With the ϕ_c values we selected for the analysis, Fig.1(a) shows that the triaxial data points lie approximately at the same locus as the plane-strain data points, which would imply a similar contribution of dilatancy to shear strength in both cases. Fig.1(b) shows that the value of A_ψ to be used in equation (4) would be 3.8 for both TX and PSC cases. For $A_\psi = 3.8$ for both TX and PSC conditions, we can rewrite equations (5) and (6) as a single equation:

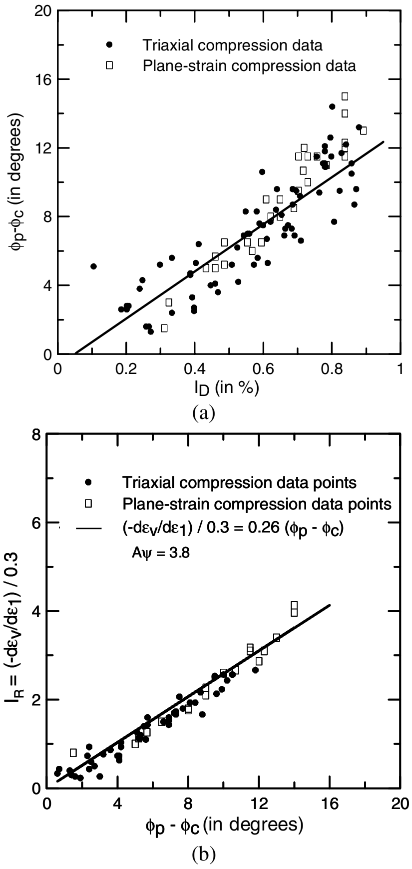

Fig 1: Triaxial and plane-strain compression data points for Toyoura sand for determining A_ψ : (a) (ϕ_p-ϕ_c) vs I_D and (b) (ϕ_p-ϕ_c) vs I_R

$$\phi_p = \phi_c + 0.62\psi_p \approx \phi_c + 0.6\psi_p \tag{7}$$

Eq. (4) can be rewritten as:

$$\phi_p = \phi_c + A_\psi I_R \tag{8}$$

with A_ψ = 3.8 for both triaxial and plane-strain conditions. Rearranging Eqs. (3) and (8), we obtain the following equation

$$\frac{\phi_p - \phi_c}{3.8} + I_D \ln \frac{100\sigma'_{mp}}{p_A} = I_D Q - R \tag{9}$$

for triaxial and plane-strain conditions, where

$$\sigma'_{mp} = \frac{\sigma'_{1p} + 2\sigma'_{2p}}{3} \tag{10}$$

for triaxial compression and

$$\sigma'_{mp} = \frac{\sigma'_{1p} + \sigma'_{2p} + \sigma'_{3p}}{3} \tag{11}$$

for plane-strain compression.

In plane-strain compression, the relationship between σ'_1, σ'_2, and σ'_3 can be expressed by the following equation

$$b = \frac{\sigma'_2 - \sigma'_3}{\sigma'_1 - \sigma'_3} \tag{12}$$

Tatsuoka et al. (1986) showed that the value of b in equation (12) lies between 0.2 and 0.3 at peak for Toyoura sand. Drained simple shear tests on Toyoura sand by Pradhan et al. (1988) produced b in the 0.22-0.33 range at critical state. Lam and Tatsuoka (1988) observed b values in 0.2-0.5 range, with b varying with the height-to-width ratio of the samples. We have assumed b = 0.25. The effect of b is only considered in the calculation of the mean stress (by calculating b using Eq.(12) and substituting it into Eq. (11)). Note that the value of ϕ_c in PSC is influenced by the intermediate principal stress that develops as a result of loading all the way to critical state, but that is already reflected in the values of ϕ_c used in our calculations, which are different for TX and PSC conditions. Mean stress values were calculated at peak shear strength. Equation (9) gives a simple correlation of dilatancy and peak friction angle with D_R and confining stress. Bolton (1986) found that R=1 and Q=10 provided a good fit for clean silica sands when the same ϕ_c value is assumed for both PSC and TX. Bolton (1986) specified a maximum value for I_R of 4, given the absence of experimental data at low confining stresses. In this paper, we obtain Q and R values for Toyoura sand for both plane-strain and triaxial conditions considering also data at low confining stresses. Analysis results are summarized in Tables 1 and 2.

4 DISCUSSION

4.1 *Relative dilatancy index and peak friction angle correlation*

The peak friction angle (ϕ_p) in sands is a result of the combined effects of relative density, mean effective stress, loading path, and basic frictional shear strength (as reflected in the value of the critical-state friction angle). The correlation for peak friction angle proposed by Bolton (1986) and reexamined in this paper can capture, in a simple manner, the effects of all of these factors. An additional factor, not accounted for here, is fabric anisotropy.

The loading path affects both the value of ϕ_c and the value of the mean effective stress (through the effect of b on the mean effective stress calculated for plane-strain conditions). The relative density appears in the expression for I_R as seen in Eq.(3) The confining stress also appears in the same expression but also affects Q. Finally, the parameter A_ψ in Eq. (8) would be able to capture any difference in the contribution of dilatancy to ϕ_p that there might be between PSC and TX, although, for Toyoura sand, for the data considered for the present paper, it has resulted approximately the same.

In the procedure we followed to determine the parameters in the friction angle and dilatancy index correlations, we first found the relationship between $(d\varepsilon_v/d\varepsilon_1)_p$ and I_R that optimized Eq. (3) and then determined A_ψ for the totality of triaxial and plane-strain compression data, with I_R calculated from the strains using Eq. (2), ϕ_p measured in each test, and the appropriate ϕ_c depending on whether the test was a TX or PSC test. The correlation between I_R, relative density and mean stress (and the values of Q and R) is then found by fitting Eq. (9) to data separated based on confining stress level.

4.2 *Q and R*

Table 1 summarizes the Q and R values for TX tests on Toyoura sand obtained by fitting Eq. (9) to Toyoura sand datasets corresponding to different confinement levels. The relative densities of the sand samples for each confining stress level varied from 30% to 90%. The Q values fall in the 7.7-10.0 range, with regression coefficients 0.839 to 0.997 when R is set to 1.

Table 1: Values of Q and R for TX tests on Toyoura sand

σ'_{3p} (kPa)	σ'_{mp} (kPa)	Data points	Best Fit Q	R	r^2	Trend line set with R=1 Q	R	r^2
4	9.3	11	6.9	0.47	0.928	7.7	1	0.914
6.2	14.3	10	6.2	-0.23	0.943	8.1	1	0.839
11.2	25.8	12	7.4	0.13	0.99	8.7	1	0.954
20.8	47.2	11	7.5	0.03	0.987	9	1	0.945
50.3	108.4	7	8.9	0.79	0.999	9.3	1	0.997
99.3	207.5	13	9.3	0.8	0.997	9.7	1	0.996
197.2	412.4	8	9.6	0.73	0.999	10	1	0.997

Table 2: Values of Q and R for PSC tests on Toyoura Sand

σ'_{3p} (kPa)	σ'_{mp} (kPa)	Data points	b	Best Fit Q	R	r^2	Trend line set with R=1 Q	R	r^2
4.9	15.7	10	0.25	9.2	1.57	0.971	8.4	1	0.963
9.8	29.7	5	0.25	10.4	2.1	0.985	8.8	1	0.961
49	151.2	4	0.25	10.2	0.88	0.998	10.3	1	0.998
68.6	196.7	6	0.25	10.1	0.9	0.993	10.2	1	0.993
98	297.1	6	0.25	11	1.2	0.992	10.7	1	0.99

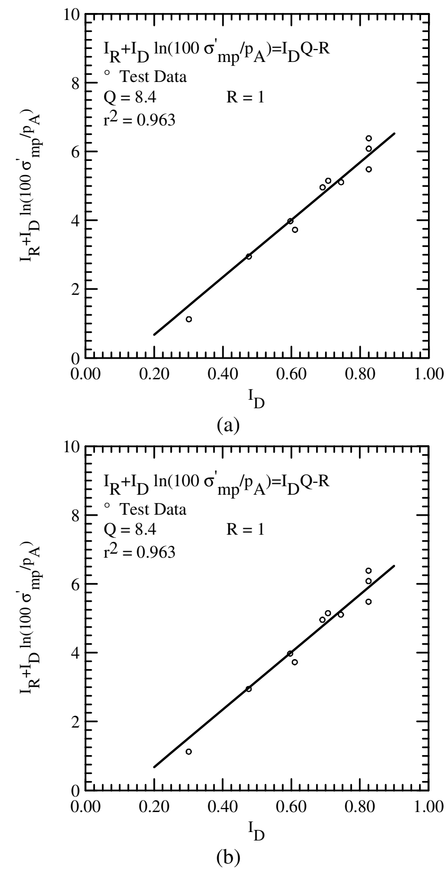

Fig 2: Best-fit Q and R values for plane-strain tests on Toyura sand at b=0.25 and (a) σ'_{3p}= 4.9 kPa, (b) σ'_{3p}= 9.8 kPa

Table 2 has the results of analyses for plane-strain tests on Toyoura sand. The stress parameter b is taken as 0.25 in the calculation of the mean stress that appears in Eq. (9). Q increases from 8.4 to 10.7 with increasing σ'_3 if R is set equal to 1. Fig. 2 shows that the proposed Q values fit the data well with a regression coefficient higher than 0.96. Q is higher for PSC than for TX conditions at comparable values of D_R and σ'_c.

4.3 Q values from fits using R=1

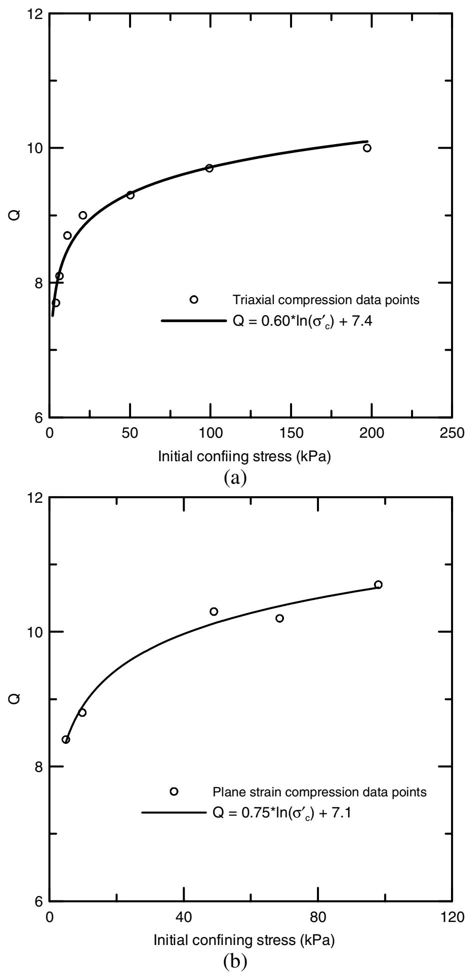

Fig. 3: Variation of Q values with increasing effective confining stress for (a) TX and (b) PSC tests

Figs. 3(a) and (b) illustrate the relationship between Q and initial confining stress σ'_c. The parameter Q increases logarithmically with increases of σ'_c in both triaxial compression and plane-strain compression. Based on these observations, we propose a simple relationship between Q and σ'_c:

$$Q=7.4+0.60\ln\sigma'_c \tag{13}$$

for triaxial compression, and

$$Q=7.1+0.75\ln\sigma'_c \tag{14}$$

for plane-strain compression.

These relations are expressed in terms of σ'_c for simplicity, and are strictly applicable to Toyoura sand within the range of stress levels and relative density considered in this study.

5 SUMMARY AND CONCLUSIONS

We analyzed the results of triaxial and plane-strain compression tests on Toyoura sand for initial consolidation stresses ranging from very low to 197 kPa to examine the dependence of dilatancy and friction angle on D_R and confining stress. The end-of-consolidation confining stress for collected test data was as low as 2 kPa in the case of triaxial compression and 4.9 kPa in the case of plane-strain compression. Appropriate values of the critical-state friction angle were used for triaxial compression and plane-strain compression.

A correlation of critical state, peak friction angle and soil state was established based on the Toyoura sand data for both triaxial and plane-strain compression. To obtain simple correlations of the peak dilatancy and peak friction angles with D_R and confining stress that could be directly comparable, R was set equal to 1. A set of Q values was found for plane-strain and triaxial compression. A logarithmic increase of Q with increasing confining stress was observed, and a simple relationship between Q and stress level was established.

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