Mathematical characterization of pumice and quartz sands

Caractérisation mathématique de sables ponce et quartzeux

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ABSTRACT
The typical triaxial stress behaviour of pumice and quartz sands are mathematically characterized by the theoretical equations derived from the Principle of Natural Proportionality. Reference is made to triaxial compression tests results under confining stress of up to $200kPa$ performed in loose and dense specimens of both types of sands.

RÉSUMÉ
Le comportement sous contraintes triaxiales typiques des sables ponce et quartzeux est caractérisé mathématiquement au moyen des équations dérivées du principe de Proportionnalité Naturelle. On fait référence aux résultats d'essais de compression triaxiale, sous contrainte de confinement supérieure à 200kPa, réalisés sur des sables lâches et denses des deux types.

Keywords : constitutive relations; sands; pumice and quartz sands; triaxial compression tests

1 INTRODUCTION

During the Practitioner/Academic Forum, held on occasion of the 16th. International Conference on Soil Mechanics and Geotechnical Engineering (ICSMGE), September 2005, in Osaka, Japan, the third Practitioner: Stephen Crawford, (Tonkin & Taylor Ltd., New Zealand), in his reply for discussion on issue 1: "Give an example of a problem that you have encountered where research is perceived to be lacking and would have been of benefit in developing a solution", proposed by Harry Poulos, Chairman, (Coffey Geosciences Pty Ltd., Australia), commented that quartz and pumice sand specimens subjected to controlled CPT tests in the laboratory show significantly different behaviour as evidenced in Figure 1 (Crawford, 2005).

When the author read such a presentation he immediately was tempted to apply the general theoretical equations given by the Principle of Natural Proportionality (PNP) to mathematically describe such behaviour. The results are presented in this paper.

2 THEORETICAL EQUATIONS

The stress-strain behaviour corresponds to the pre-peak normal function Y_N with $\nu=1$, ν being the shear exponent (Juárez-Badillo 1994, 1997, 1999), as represented in Figure 2, and it is expressed by:

$$e_a = \frac{1}{3}\mu\, x_f \ln\left(1-\frac{x}{x_f}\right) \tag{1}$$

where $x=(\sigma_1-\sigma_3)/\sigma_{co}$, $e_a=$ axial natural strain, $x_f=x$ for $e_a=\infty$. The initial confining pressure is equal to σ_{co} and μ is the shear coefficient. On the other hand, the natural axial strain e_a in term of common strain is given by:

$$e_a = \ln(1+\varepsilon) - \frac{\varepsilon_v}{3} \tag{2}$$

where ε is the common strain (Cauchy) and ε_v the natural volumetric strain.

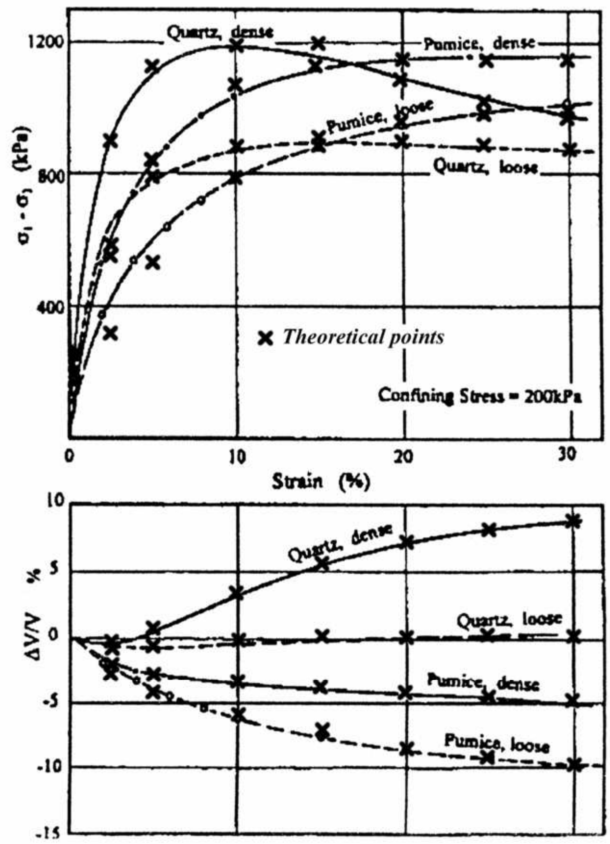

Figure 1. Typical Triaxial Behaviour of Pumice & Quartz Sands

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Equation 1 may be written

$$x = x_f \left[1 - \exp\left(\frac{3e_a}{\mu x_f} \right) \right] \tag{3}$$

In our case, since $\sigma_{co} = 200 kPa$ and disregarding ε_v in Equation 2, this expression can be written as:

$$\sigma_1 - \sigma_3 = (\sigma_1 - \sigma_3)_f \left\{ 1 - \exp\left[\frac{600 \ln(1+\varepsilon)}{\mu(\sigma_1 - \sigma_3)} \right] \right\} \tag{4}$$

where $(\sigma_1 - \sigma_3)_f = \sigma_1 - \sigma_3$ for $e_a = \infty$.

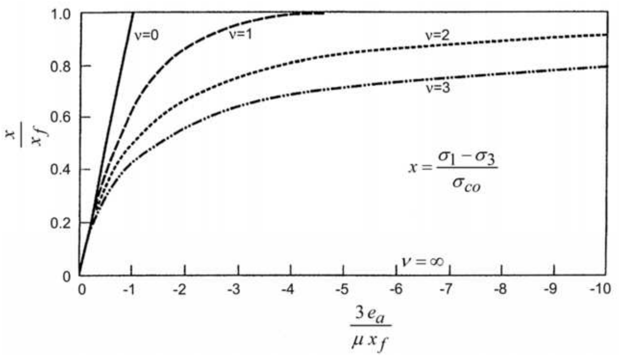

Figure 2. Prepeak normal function Y_N

The post-peak ductility function Y_D , Figure 3, reads as follows:

$$\sigma_1 - \sigma_3 = (\sigma_1 - \sigma_3)_\infty + [(\sigma_1 - \sigma_3)_1 - (\sigma_1 - \sigma_3)_\infty] \left[\frac{\ln(1+\varepsilon)}{\ln(1+\varepsilon_1)} \right]^{-1/\nu} \tag{5}$$

where $(\sigma_1 - \sigma_3)_\infty = (\sigma_1 - \sigma_3)$ at $e_a = \infty$; $[(\sigma_1 - \sigma_3)_1, \varepsilon_1]$ are the coordinates of a known point and ν is now defined as the ductility coefficient. It should be observed that in triaxial compression tests e_a and ε have negative values.

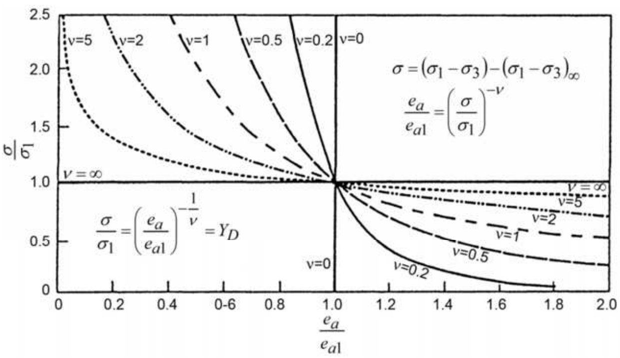

Figure 3. Postpeak ductility function Y_D

The author formerly found that the general equation for volume changes given by the PNP in the case of clays (Juárez-Badillo 1969, 1975), (Juárez-Badillo & Rico-Rodríguez, 1975) reads as follows:

$$\frac{V}{V_o} = \left\{ \frac{\sigma_e + \left[\alpha \sigma_c \frac{\sigma_c}{\sigma_e} - \alpha(\sigma_e - \sigma_c) \right] y}{\sigma_{eo}} \right\}^{-\gamma} \tag{6}$$

where σ_{co} and σ_{eo} are the initial consolidation and equivalent consolidation pressures, respectively, whereas σ_c and σ_e are the ongoing consolidation and equivalent consolidation pressures (in the virgin branch of the compression curve), respectively; γ is the natural compressibility coefficient and *y* is the sensitivity function, Figure 4, given by:

$$y = \left[1 + \left(\frac{e_a}{e_a^*} \right)^{-\beta} \right]^{-1} \tag{7}$$

where α and β are pore pressure parameters $(\alpha \le 1)$ and $e_a^* = e_a$ at $y = 0.5$. All volume changes may be considered as induced by actual or virtual pore pressures dissipation. Observe in Equation 6 that the term σ_e is due to the existing isotropic pressure σ_c ; the next two terms in brackets result from the disturbance of the clay structure, the former being positive term due to the applied σ_c whereas the latter is a negative term due to the release of the ongoing stored pressure $\sigma_s = \sigma_e - \sigma_c$. The sensitivity function y represents the disturbance mechanism of the structure and it ranges from 0 to 1 when e_a varies from 0 to ∞ . For the case of granular soils like sands the interlocking mechanism between particles resembles the preconsolidation procedure of clays. The overconsolidation factor in clays, $OCF = \sigma_{eo} / \sigma_{co}$ is equivalent to the initial amount of interlocking. When $OCF = 1, \sigma_{eo} = \sigma_{co}$ and there is no interlocking effects. The OCF increases with the relative density of the sand and with the hardness of its particles.

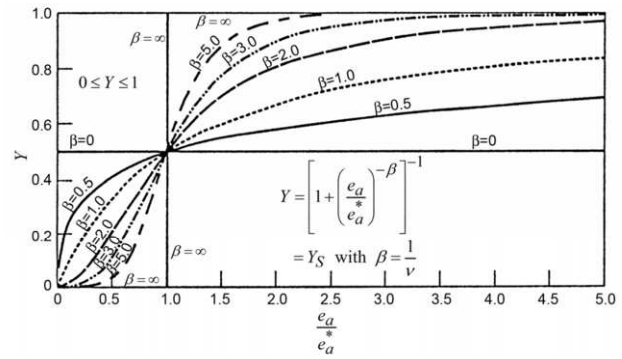

Figure 4. Sensitivity function Y

The relationship between the equivalent consolidation pressures and the consolidation pressures becomes equal to:

$$\frac{\sigma_e}{\sigma_{eo}} = \left(\frac{\sigma_c}{\sigma_{co}} \right)^\rho = \left(1 + \frac{\Delta\sigma_c}{\sigma_{co}} \right)^\rho \tag{8}$$

where ρ is the relationship between the expansibility and compressibility coefficients γ_p and γ namely:

$$\rho = \frac{\gamma_p}{\gamma} \tag{9}$$

Substituting Equation 8 into Equation 6 we obtain:

$$\frac{V}{V_o} = \left\{ \left(1 + \frac{\Delta\sigma_c}{\sigma_{co}} \right)^\rho + \alpha \left[\left(\frac{\sigma_{co}}{\sigma_{eo}} \right)^2 \left(1 + \frac{\Delta\sigma_c}{\sigma_{co}} \right)^{2-\rho} \ldots\ldots \right.\right.$$
$$\left.\left. \ldots\ldots - \left(1 + \frac{\Delta\sigma_c}{\sigma_{co}} \right)^\rho + \frac{\sigma_{co}}{\sigma_{eo}} \left(1 + \frac{\Delta\sigma_c}{\sigma_{co}} \right) \right] y \right\}^{-\gamma} \tag{10}$$

For triaxial compression tests we may write:

$$\frac{\Delta V}{V_o}=\left\{\left(1+\frac{\sigma_1-\sigma_3}{3\sigma_{co}}\right)^{\rho}+\alpha\left[\left(\frac{\sigma_{co}}{\sigma_{eo}}\right)^{2}\left(1+\frac{\sigma_1-\sigma_3}{3\sigma_{co}}\right)^{2-\rho}\ldots\ldots\right.\right.$$
$$\left.\left.\ldots\ldots\ldots\ldots-\left(1+\frac{\sigma_1-\sigma_3}{3\sigma_{co}}\right)^{\rho}+\frac{\sigma_{co}}{\sigma_{eo}}\left(1+\frac{\sigma_1-\sigma_3}{3\sigma_{co}}\right)\right]y\right\}^{-\gamma}-1 \qquad (11)$$

where $(\sigma_1-\sigma_3)$ is given by Equation 4 for the pre-peak region and Equation 5 for the post-peak region whereas y is determined from Equation 7.

3 PRACTICAL APPLICATION

Application of Equations 4, 5, 7 and 11 to the experimental data of Figure 1, made it possible to obtain the theoretical values of the different parameters involved which are indicated in Table 1.

To obtain the parameters shown in the Table 1, imagination, sensibility, experience and a complete knowledge of the different non linear equations is a prerequisite as well as full understanding of the relative importance of all the terms in Equation 6.

Table 1. Theoretical values of the parameters

Sand	State	σ_{co} kPa	Pre-peak Y_N			Post-peak Y_D				Volume change					
			ν	$(\sigma_1-\sigma_3)_f$ kPa	μ	ν	$(\sigma_1-\sigma_3)_\infty$ kPa	ε_1	$(\sigma_1-\sigma_3)_1$ kPa	γ	ρ	σ_{eo}/σ_{co}	α	β	ε^*
Pumice	loose	200	1	1000	0.040	-	-	-	-	0.065	(0.5)	1	1	3	-0.20
	dense	200	1	1150	0.020	-	(1000)	-	-	0.065	0.5	2.3	1	3	-0.20
Quartz	loose	200	1	900	0.016	2	800	-0.30	880	0.050	0.2	4	1	3	-0.10
	dense	200	1	1200	0.009	1	800	-0.30	980	0.050	0.2	20	1	2	-0.07

Notes: Values in parenthesis are assumed without experimental data.

In Equation 6 the term σ_e is due to the isotropic stress increment. The term $-\alpha(\sigma_e-\sigma_c)$ increases with overconsolidation in clays and with interlocking in sands, it is equal to zero in normally consolidated clays and different from zero in many loose sands due to partial interlocking in them. The term $(\alpha\sigma_c)(\sigma_c/\sigma_e)$ decreases with overconsolidation in clays and with interlocking in sands.

A brief description of how the different parameters were found is pertinent. For the pre-peak region, a value of $(\sigma_1-\sigma_3)_f$ in Equation 4 is found from the last part of the experimental curve and subsequently the value of μ is determined from the initial part of it; both values are adjusted until a very good fit is obtained. For the post-peak region, after a good experimental point is fixed a value of $(\sigma_1-\sigma_3)_\infty$ is assumed and the value of the ductility parameter ν is obtained from the complete experimental curve. Observe that $(\sigma_1-\sigma_3)_\infty$ is supposed to be the same for either loose or dense conditions.

In what refers to volume change, parameters γ and ρ for both types of sands are independent of their relative density (loose or dense). Assuming $\alpha=1$ and considering that usually $\beta>1(2\,or\,3)$ the value of γ is determined in terms of an assumed value of ρ from the initial part of the experimental curve $(\varepsilon\leq5\%)$. Then the value of σ_{eo}/σ_{co} is found from the final part of the experimental curve. Later on ε^* is determined at the middle point of the volume change, say at $\varepsilon=30\%$, as given by Equation 11 with $\alpha=1$, and $\alpha=0$ and finally the value of β is found from the complete experimental curve. Repeat the whole process adjusting the different parameters until a very good fit is obtained.

It would be great benefit if other triaxial compression and extension tests are performed at different values of σ_{co} to improve the values of γ and ρ , to find out the law of variation of σ_{eo}/σ_{co} with respect to σ_{co} and the relative density as well as the variation of β and ε^* in terms of those factors.

4 CONCLUSIONS

A very simple and important conclusion can be derived as follows: Sands subjected to drained triaxial shear tests behave just the same way as clays, with particle interlocking representing the overconsolidation of clays.

The author sincerely hopes that these results will help narrow somewhat the gap between practitioners and academics in what refers to practical and theoretical applications witch was the main objective of the forum held on the occasion of the 16 ICSMGE in Japan in September 2005.

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