Earth Pressures Acting on a Vertical Shaft in Multi-layered $c-\varphi$ Soils
Les Pressions de Terre Agissant sur un Puits Vertical dans les Sols de $c-\varphi$ Multi-coupé en dégradé

I.M. Lee & D.H. Kim
Korea university, Seoul, Korea

K.Y. Kim & Y.H. Lee
Korea Electric Power Research Institute, Daejeon, Korea

H.P. Moon
Daelim Industrial Company Limited, Seoul, Korea

ABSTRACT

It is one of the important tasks to evaluate the earth pressure acting on a vertical shaft when designing underground structures. The earth pressure acting on a vertical circular shaft is less than that of a retaining wall in plain strain condition due to three dimensional arching effect. Several researchers have suggested equations to estimate the earth pressure on the vertical circular shaft and verified them by conducting model tests. However, each of estimations shows different results and is not applicable to the vertical shaft in multi-layered soils and/or $c-\varphi$ soils. In this study, a new equation for estimating the earth pressure acting on the vertical shaft in $c-\varphi$ soils is proposed by modifying the equations suggested by other researchers. A parametric study is performed to investigate the significance of the cohesion when estimating earth pressures in vertical shafts. A method which can estimate the earth pressure on vertical shafts in layered soils is also proposed by assuming a failure surface in layered soils and using the modified equation. To verify the proposed methods, measured data were collected from three in-situ vertical shafts constructed in multi-layered $c-\varphi$ soils and compared with analytical solutions.

RESUME

C'est une des tâches importantes d'évaluer la pression de terre agissant sur un puits vertical en concevant des structures souterraines. La pression de terre agissant sur un puits circulaire vertical est moins que ce d'un mur de soutènement dans la condition d'effort simple en raison de l'effet formant une voûte en trois dimensions. Plusieurs chercheurs ont suggéré des equations d'estimer la pression de terre sur le puits circulaire vertical et les ont verifies en accomplissant des épreuves modèles. Pourtant, chaque d'estimations montre de différents resultants et n'est pas applicable au puits vertical dans les sols de multi-coupé en dégradé et/ou les sols de $c-\varphi$. Dans cette etude, une nouvelle equation pour estimer la pression de terre agissant sur le puits vertical dans les sols $c-\varphi$ est propose en modifiant les equations suggérées par d'autres chercheurs. Une etude paramétrique est exécutée pour enquêter sur la signification de la cohésion en estimant des pressions de terre dans les puits verticaux. Une method qui peut estimer la pression de terre sur les puits verticaux dans les sols coupé en degrade est aussi propose en supposant une surface d'échec dans les sols coupés en degrade et en utilisant l'équation modifiée. Pour vérifier les méthodes proposées, les données mesurées ont été recueillies de trois dans la situation les puits verticaux construits dans les sols de $c-\varphi$ multi-coupé en degrade et ont été comparables avec les solutions analytiques.

Keywords: earth pressure, vertical shaft, multi-layered soil, layered $c-\varphi$ soil, in-situ measurement data

1 INTRODUCTION

Design and construction of vertical shaft increase with the increase of underground space usage. Proper estimation of earth pressures is a key factor in design of vertical shafts. Because of three dimensional arching effects, i.e. convex arching and/or inverted arching, the earth pressure acting on a circular type of vertical shaft is less than other types. Thus, the circular cross section of vertical shaft is generally used. Several researchers have suggested several methods how to evaluate the earth pressure acting on circular vertical shafts considering arching effects. Handy (1985) and Paik & Salgado (2003) proposed equations to obtain the lateral coefficient of active earth pressures considering arching effect. Prater (1985) assumed conical failure shape and proposed angle of failure surface dependent on shape ratio (H/R) and on internal friction angle to estimate the earth pressure in shafts. Britto & Kusakabe (1983) studied the stability of vertical shafts in cohesive soils by using upper bound method. Wong & Kaiser (1988) suggested a convergence confinement method in vertical shafts considering horizontal and vertical arching effects, separately. Shin (2004) proposed an equation of earth pressure action on a circular vertical shaft by applying tangential and radial stress to differential soil element. These previous researches for obtaining earth pressures are based on either cohesionless or cohesive soils, thus are difficult to apply directly to $c-\varphi$ soils.

This paper tries to propose a new equation to evaluate the earth pressures acting on circular vertical shafts in $c-\varphi$ soils and/or multi-layered soils. The formula is derived by assuming conical failure shape and by adopting Mohr-Coulomb failure criterion. In-situ measurement data are collected and converted into earth pressures in three in-situ sites to verify the appropriateness of the proposed equation.

2 DERIVATION OF EARTH PRESSURES ACTING ON VERTICAL SHAFTS

2.1 *The coefficient of tangential earth pressure*

The coefficient of tangential earth pressure (λ) is defined to be the ratio of tangential stress to vertical stress. The earth pressure on vertical shafts decreases with the increase of tangential stress acting on the failure surface, thus with increase of λ value. Wong & Kaiser (1988) suggested that λ value is not 1.0 at elastic state; but will reach 1.0 at plastic state. Prater (1977) stated that the earth pressure will be overestimated when λ is assumed to be zero and underestimated when λ is assumed to be 1.0. Thus, he maintained λ value between K_0 (coefficient of lateral earth pressure at rest) and K_a (coefficient of Rankine's active earth pressure) and finally proposed '$\lambda = 1-\sin\varphi$' being appropriate to estimate the earth pressure. Lee et al. (2007) conducted physical model tests and suggested that λ value of '$1-\sin\varphi$' is conservative enough to estimate the earth pressure

Proceedings of the 17th International Conference on Soil Mechanics and Geotechnical Engineering
M. Hamza et al. (Eds.)

doi:10.3233/978-1-60750-031-5-1409

acting on the vertical shaft. In this study, λ is assumed to be '$1-\sin\varphi$'.

2.2 *The coefficient of radial earth pressure*

The coefficient of radial earth pressure is defined as the ratio of radial stress to vertical stress. Rankine (1857) suggested the coefficient of active earth pressure (K_a) without considering wall friction angle (δ). Paik & Salgado (2003) assumed miner principle stress direction is changing in concave shape, such as elliptic, catenary and parabolic due to fictional resistances of wall, and proposed the coefficient of earth pressure (K_w) as follows:

$$K_w = \frac{3(N\cos^2\theta + \sin^2\theta)}{3N-(N-1)\cos^2\theta} \tag{1}$$

where,

$$\theta = \tan^{-1}\left[\left((N-1)\pm\sqrt{(N-1)^2-4N\tan^2\delta}\right)/2\tan\delta\right]$$

and

$$N = \tan^2(45+\phi/2)$$

Equation 1 proposed by Paik & Salgado(2003) is used in this study as the coefficient of radial earth pressure.

2.3 *Equation for earth pressure on circular vertical shaft*

The assumptions adopted in this study for the derivation of earth pressures are as follow: [1] conical failure shape is assumed with β angle of '$45+\varphi/2$' (shown in Figure 1); [2] λ (coefficient of tangential earth pressure) is assumed to be '$1-\sin\varphi$'; and [3] Equation 1 is used for K_w (coefficient of radial earth pressure).

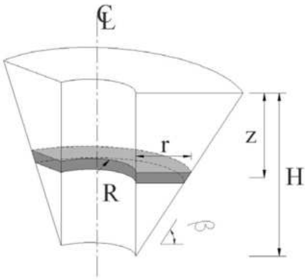
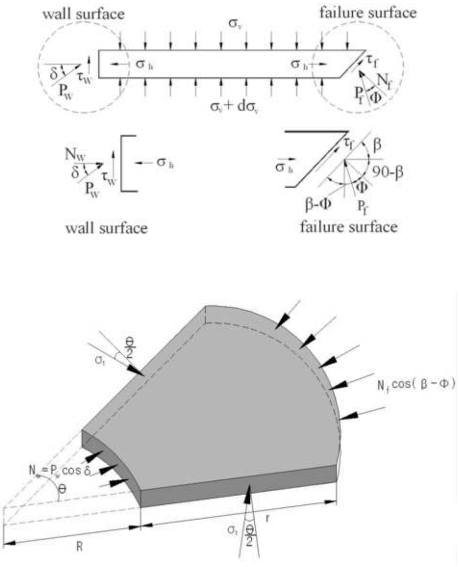


Figure 1. Conical failure shape of vertical shaft

As shown in Figure 1, R is radius of circular vertical shaft and r is radius to failure surface, where, H is total depth of shaft, z is depth below ground surface and β is angle of failure surface. Figure 2(a) shows stresses acting on the differential soil element behind a shaft wall. Derivation of the equation is summarized as follows.

Equilibrium condition of horizontal force acting on the differential soil element (shown in Figure 2) gives

$$\int_0^{2\pi} N_w R\,d\theta dz + \int_0^{2\pi} 2\sigma_t \sin\left(\frac{d\theta}{2}\right) r dz + \int_0^{2\pi} \tau_f \frac{\cos\beta}{\sin\beta}\left(r+R\right) d\theta dz = \int_0^{2\pi} N_f \frac{\cos(90-\beta)}{\sin\beta}\left(r+R\right) d\theta dz \tag{2}$$



(a)

(b)

Figure 2. Equilibrium of stress at soil element

In Equation 2, $N_w\ (=P_w\cos\delta)$ is lateral stress acting on wall surface of element and is same as σ_h, and τ_f is defined as shear strength at failure surface. N_w and σ_t will be expressed as

$$N_w = \sigma_h = K_w\sigma_v - 2c\sqrt{K_w} \tag{3}$$

$$\sigma_t = \lambda\sigma_v \tag{4}$$

where, σ_v is vertical stress at certain depth z. If $d\theta$ is very small then Equation 5 can be obtained by substituting Equation 3 and 4 into Equation 2:

$$N_f = \frac{RK_w\sigma_v + r\lambda\sigma_v + c\dfrac{\cos\beta}{\sin\beta}\left(r+R\right) - 2Rc\sqrt{K_w}}{r+R}\,\frac{\tan\beta}{\tan\beta-\tan\phi} \tag{5}$$

From Figure 2(a), equilibrium of vertical stress gives

$$\sigma_v A + A d_z\gamma = \left(\sigma_v + d\sigma_v\right)A + 2\pi R\tau_w dz + 2\pi\left(r+R\right)\left\{\tau_f\sin\beta + N_f\sin\left(90-\beta\right)\right\}\frac{dz}{\sin\beta} \tag{6}$$

where, A is area of differential soil element, τ_f is shear strength at failure surface, and τ_w is shear strength at wall surface.

The first order differential Equation 7 can be obtained by substituting Equation 5 into Equation 6.

$$\frac{d\sigma_v}{dz} + S\sigma_v = T \tag{7}$$

where, γ is soil density,

$$T = \gamma - \frac{2\pi}{A}c\left\{\left(r+R\right) + \left(\frac{r+R}{\tan\beta} - 2R\sqrt{K_w}\right)\frac{1+\tan\beta\tan\phi}{\tan\beta-\tan\phi}\right\}$$

and

$$S=\frac{2\pi}{A}\left\{\left(K_w R+\lambda r\right)\frac{1+\tan\beta\tan\phi}{\tan\beta-\tan\phi}\right\}$$

Equation 7 can be solved with the boundary condition, $\sigma_v = 0$ when z=0. Then, σ_v is

$$\sigma_v=-\frac{T}{S}e^{-Sz}+\frac{T}{S} \tag{8}$$

where, K_w is coefficient of radial earth pressure (shown in Equation 1) and λ is coefficient of tangential earth pressure. Finally, the equation of earth pressures acting on vertical shafts can be obtained as

$$p_i=K_w\sigma_v-2c\sqrt{K_w} \tag{9}$$

2.4 *Parametric study*

A parametric study is performed to verify appropriateness of the derived equation and to investigate the effect of cohesion on earth pressures. Table 1 shows each case of parametric studies. To observe change of earth pressures according to cohesion variaton, internal friction angle (φ), density of soil (γ) and wall friction angle (δ) are assumed to be constant. Another parametric study is also done to compare the sensitivity of either cohesion or friction angle on the earth pressure (see Table 2). In Table 2, assuming Case 5 as the reference case, the cohesion is increased by 1.5 times in Case 6; the tan φ is again increased by 1.5 times in Case 7. Radius and height of the hypothetical vertical shaft are 2 m and 20 m, respectively.

Table 1. Properties of soils for parametric study 1

	γ (kN/m^3)	c (kPa)	φ (°)	δ (°)	Method
Case 1	18	0.0	30.0	0.0	Rankine
Case 2	18	0.0	30.0	0.0	Proposed
Case 3	18	5.0	30.0	0.0	Proposed
Case 4	18	10.0	30.0	0.0	Proposed

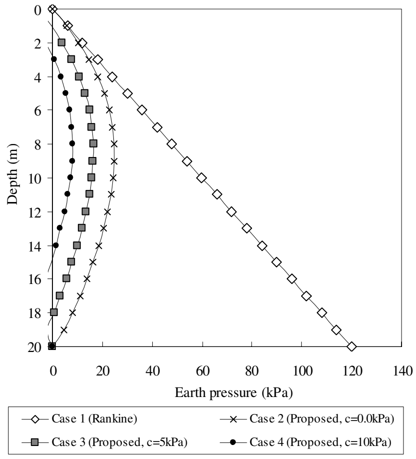


Figure 3. Variation of earth pressures with the variation of cohesion

Table 2. Properties of soils for parametric study 2

	γ (kN/m^3)	c (kPa)	φ (°)	tan φ	δ (°)
Case 5	18	5.0	30.0	0.57	0.0
Case 6	18	7.5	30.0	0.57	0.0
Case 7	18	5.0	40.9	0.87	0.0

Figure 3 clearly shows that reduction of earth pressures is significant with cohesion increase. Moreover, Figure 4 shows that the cohesion is more sensitive to the variation of earth pressures than the friction angle.

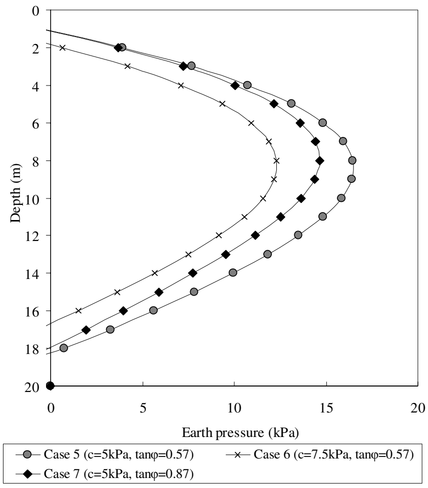


Figure 4. Variation of earth pressures due to increment of either c or tan φ

2.5 *Estimation of earth pressures in layered soils*

The earth pressure distribution for layered soils can also be estimated by using the modified equation assuming the failure surface is continuous as shown in Figure 5.

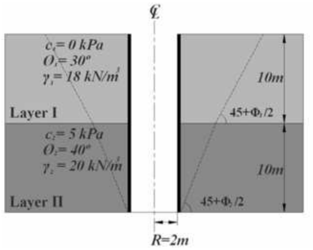


Figure 5. Failure surfaces and material properties of layered soils

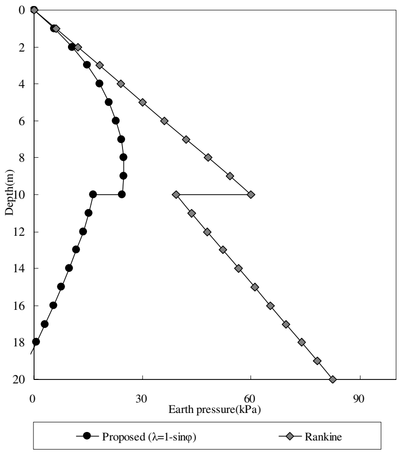


Figure 6. Earth pressure distributions in layered soils

Figure 6 shows results of earth pressures estimated by the proposed method along with those values calculated using Rankine's theory. We can see bump in the boundary of two layers.

3 FIELD MEASUREMENT

Three vertical shafts were constructed in Busan with circular cross sections, which have diameters ranging from 14 to 17m. The depth of excavation ranged from 57.5 to 64.7m. H-piles, concrete walls and H-ring beams were used in soil layers and rockbolts and shotcrete were used in rock layers as support systems. The ground is composed of 5 soil and/or rock layers, and properties of each layer are shown in Table 3. As shown in Table 3, the strength parameters of rock mass which were used in shaft design, were somewhat conservative. In-situ measurements were made and the measured data were collected from strain gauges attached to ring beams, piezometers, water level gauges, transducers for measuring the axial force of rockbolts and shotcrete stress cells at vertical sections of each shaft.

Table 3. Properties of soils and rocks

	Alluvial			Weathered soil			Weathered rock	Soft rock	Hard rock
	#1	#2	#3	#1	#2	#3			
Unit weight (kN/m^3)	17	18	18	19	19	23	21	23	25
Cohesion (kPa)	0.0	0.0	0.0	20	15	20	30	400	1000
Internal friction angle(°)	30	35	35	30	30	30	30	32	38

Figure 7 shows the converted earth pressures from measured data and earth pressure distributiona calculated from the proposed method. Comparisons are made only up to depth of weathered soil layers excluding rock layers. Pre-loading was applied during shaft excavation in order to reduce the horizontal displacement. Therefore, the measured in-situ data were converted into earth pressures either by taking the pre-loading into consideration or by not taking into consideration. Comparison results show that the converted earth pressures are somewhat smaller than estimated values from the proposed equation. However, if the pre-loading is taken into account, those two values match resonally well.

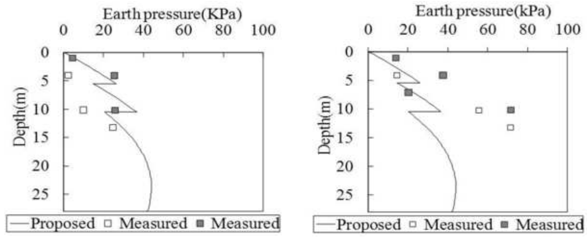
4 CONCLUSION

A new equation for estimating earth pressures acting on vertical shafts in $c-\varphi$ soils is proposed in this paper. The method which can estimate earth pressures on vertical shafts in multi-layered soils is also suggested. Parametric study is performed to assess the sensitivity of cohesion on the earth pressure and it was found that the decrease of the earth pressure is significant with the increase of cohesion.

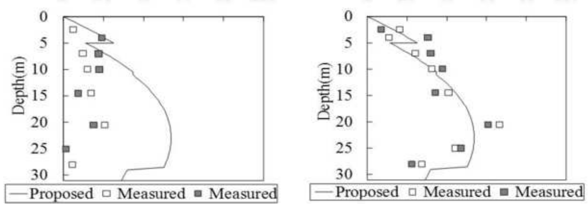
In-situ measured data are collected from three in-situ vertical shafts sites installed at multi-layered soils. The estimated values obtained using the proposed method are in good agreement with most of the earth pressures converted from the measured data although converted pressures are pretty much dependent on pre-loading conditions.

ACKNOWLEDGMENTS

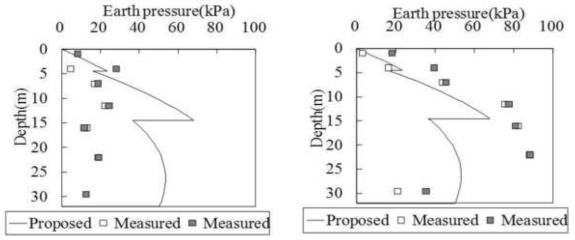
This work was supported by Korea Institute of Construction and Transportation Technology Evaluation and Planning under the Ministry of Land, Transport and Maritime Affairs in Korea (Grant No. 04-C01) and Korea Electric Power Research Institute (KEPRI). The support is gratefully acknowledged.



(a) Shaft #1 without pre-loading (b) Shaft #1 with pre-loading



(c) Shaft #2 without pre-loading (d) Shaft #2 with pre-loading



(e) Shaft #3 without pre-loading (f) Shaft #3 with pre-loading

Figure 7. Comparison between measured data and estimated results

REFERENCE

Britto, A. M. & Kusakabe, O. 1983. Stability of unsupported axisymmetric excavation in soft clay. Journal of Geotechnical Engineering, ASCE, Vol.109, No.5, pp.666-681.

Handy, R. L. 1985. The arch in soil arching. Journal of Geotechnical Engineering, ASCE, Vol. 111, No.3, pp.302-318.

Lee, I. M., Moon, H. P., Lee, D. S., Kim, K. R. & Choi, M. S. 2007. Earth pressure of vertical shaft considering arching effect in layered soils, Tunnelling Technology, Vol.9, No.1, pp49-62.

Paik, K. H. & Salgado, R. 2003. Estimation of active earth pressure against rigid retaining walls considering arching effects. Geotechnique, Vol.53, No.7, pp.643-653.

Prater, E. G. 1977. An examination of some theories of earth pressure on shaft linings. Canadian Geotechnical Journal, Vol.14, No.1, pp. 91-106.

Rankine, W. J. M. 1857. On the stability of loose earth. Phil. Trans. Royal. Society, London. Vol.147, No.1, pp. 9-27.

Shin, Y. W. 2004. Earth pressure acting on the cylindrical retaining wall of a shaft in cohesionless soils. Ph.D. Thesis, Hanyang University, Korea.

Wong, R. C. K. & Kaiser, P. K. 1988. Design and performance evaluation of vertical shafts; rational shaft design method and verification of design method. Canadian Geotechnical Journal, Vol.25, No.2, pp. 320-337.