# Uncertainty in predicted hydraulic conductivity functions of unsaturated soils

# Incertitude dans les fonctions de perméabilité prédites de sols non saturés

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## ABSTRACT

Hydraulic conductivity function data from the technical literature for different soil types, defined using different experimental approaches, are used to assess the uncertainty involved in empirical prediction models. Empirical predictions were found to lead to an error in hydraulic conductivity of 1 to 4 orders of magnitude, with the greatest discrepancies at low moisture contents. This finding emphasizes the importance of soil-specific characterization of hydraulic properties for use in design, using either flow pump permeameter testing or column infiltration testing. An infiltration example is used to highlight the impact of uncertainty in predicted hydraulic conductivity functions on engineering analyses. Uncertainty in the hydraulic conductivity function was found to have the greatest impact on suction profiles observed during wetting under infiltration rates close to the saturated hydraulic conductivity.

## RÉSUMÉ

Les données de fonction de perméabilité de la littérature technique pour de différents types de sol définis en utilisant de différentes approches expérimentales sont utilisées pour évaluer l'incertitude impliquée dans les modèles de prédiction empiriques. Les prédictions empiriques peuvent causer une erreur dans la perméabilité de 1 à 4 ordres de grandeur, avec les plus grandes contradictions aux teneurs en humidité basses. Cette conclusion accentue l'importance de caractérisation spécifique de sol de propriétés hydrauliques pour l'utilisation dans le design, en utilisant couler la pompe permeameter la mise à l'essai ou la mise à l'essai d'infiltration de colonne accélérée. Un exemple d'infiltration est utilisé pour accentuer l'impact d'incertitude dans les fonctions de conductivité hydrauliques prédites en ingénierie des analyses. L'incertitude dans la fonction de conductivité hydraulique a le plus grand impact sur les profils de succion sous les taux d'infiltration près de la conductivité hydraulique saturée.

Keywords : Hydraulic conductivity, unsaturated soils, statistical models for hydraulic conductivity prediction

## 1 INTRODUCTION

The long-term sustainability of engineering designs has become a critical concern in practice. In geotechnical engineering, such designs will consider soil hydraulic and mechanical behavior under physical and climatic loading in order to reach a quantified level of risk, promote efficient use or re-use of materials, and minimize environmental impacts of a system over its expected lifecycle. Unsaturated soils mechanics provides an important set of tools that can be used to address sustainability concerns in many geotechnical systems. Specifically, prediction of the changes in stiffness, strength, or volume of an unsaturated soil associated with water flow due to climatic interaction can be used to better quantify the long-term response of geotechnical systems above the water table (pavements, retaining walls, landfill covers *etc.*). There has been reluctance to consider the beneficial aspects of partial saturation in geotechnical engineering practice, as saturation often presents a worst-case performance scenario for a geotechnical structure. However, engineers are realizing that this assumption may lead to over-conservative designs that result in wasted materials.

In the case that geotechnical engineers do consider water flow through unsaturated soils, they often rely on empirical predictions of the hydraulic properties, albeit with potentially high uncertainty. Although a multitude of experimental approaches have been proposed to measure the hydraulic properties over the past centuyry, their use is still limited to academia due to long test durations and complex experimental setups and procedures. Nonetheless, if the magnitude of uncertainty is prohibitively high when using predicted properties, use of experimental techniques in geotechnical practice should be re-considered.

Along these lines, the goals of this paper are to (a) present data from the literature on the hydraulic conductivity of unsaturated soils to highlight natural variability for different soil types, (b) assess the error involved in using empirical predictions of the hydraulic conductivity function, and (c) assess the impact of this error on the prediction of matric suction profiles during infiltration through an unsaturated soil layer.

## 2 BACKGROUND

Flow of water in unsaturated soils can be described using three non-linearly related variables, namely the volumetric moisture content θ (or degree of saturation), the matric suction ψ (or capillary pressure if the air pressure is non-zero), and the hydraulic conductivity *k*. The soil water retention curve (SWRC) is defined as the relationship between volumetric moisture content and suction, and represents the energy needed (*i.e.*, the suction) to de-saturate the soil to a given moisture content. The hydraulic conductivity function (or *k* -function) is defined as the relationship between hydraulic conductivity and suction (or volumetric moisture content), and reflects the decrease in available pathways for water flow as a soil desaturates. The governing equation for water flow analyses in unsaturated soils is Richards' equation, as follows:

$$\frac{d\theta}{d\psi}\frac{\partial\psi}{\partial t}=\frac{\partial}{\partial z}\left[k(\psi)\left[1-\frac{1}{\rho_w g}\frac{\partial\psi}{\partial z}\right]\right] \qquad (1)$$

Inspection of this equation indicates that the slopes of the SWRC θ(ψ) and *k*-function *k* (ψ) must be know to determine the distribution in matric suction with time and space.

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Although some researchers have developed approaches to predict the shape of the SWRC from the pore size distribution of soils (Arya *et al.* 1999; Simms and Yanful 2002) or through empirical correlations (*e.g.*, Zapata et al. 1999), poor results may be obtained because the hydrualic properties of unsaturated soils are a function of many variables. Specifically, the SWRC and *k* -function are sensitive to soil structure variables such as pore size distribution (Burger and Shackelford 2001; Simms and Yanful 2002), proportions of sand and clay fractions (Chiu and Shackelford 1998), clay mineralogy (Miller *et al.* 2002), compaction conditions (Meerdink *et al.* 1996), volume changes (Huang *et al.* 1984; Parent *et al.* 2007), and stress state (Ng and Pang 2005). Further, the hydraulic characteristics are affected by hysteresis upon wetting and drying (Topp and Miller 1966) so they are not unique soil properties as inferred by empirical and theoretical predictions. The wide range of variables that may impact the SWRC has led to support in engineering practice for its experimental determination. ASTM test method D6836 describes procedures and setup for five common experimental approaches, the hanging column test, gravimetric and volumetric pressure chamber tests (axis translation), chilled mirror hygrometer test, and centrifuge permeameter test.

In order to use experimental SWRC data to solve Richards' equation for water flow in unsaturated soils, the data must be converted to a continuous function by fitting a power law, hyperbolic, or polynomial function. There are several models available, although the most commonly used in practice is that proposed by van Genuchten (1980):

$$\theta = \theta_r + (n - \theta_r)\left[1 + (\alpha_{vG}\psi)^{N_{vG}}\right]^{-\left(1-\frac{1}{N_{vG}}\right)} \qquad (2)$$

where $\alpha_{vG}$ and $N_{vG}$ are fitting parameters, *n* is the porosity, and $\theta_r$ is the volumetric moisture content at residual conditions.

In contrast to the relatively well-established state-of-the-practice for determination of the SWRC, the *k*-function has not received the same attention in practice. The *k*-function is sensitive to many of the same variables as is the SWRC, and can potentially can vary more significantly. Although many tests have been proposed to determine the hydraulic conductivity of unsaturated soils (column flow test, pressure plate outflow test, centrifuge permeameter), their use is typically restricted to academia with only a single standardized test method (ASTM D6527). Instead of experimental determination, statistical models based on pore size distributions are commonly used in practice to predict the *k*-function from the shape of the SWRC, many of which are summarized well by Leong and Rahardjo (1997). These approaches assume that the soil is an interconnected series of pores having a size distribution characterized by the shape of the SWRC. Statistical models neglect physicochemical and hydrophobic effects on moisture flow (attraction of water to soil or geosynthetic), and neglect film flow on particle surfaces.

The most commonly used predictive *k*-function in practice is obtained by substituting the van Genuchten (1980) model into the Mualem (1976) model, as follows:

$$k(\theta) = k_s\sqrt{\frac{\theta-\theta_r}{\theta_s-\theta_r}}\left[1-\left(1-\left(\frac{\theta-\theta_r}{\theta_s-\theta_r}\right)^{1/\left(1-\frac{1}{N_{vG}}\right)}\right)^{\left(1-\frac{1}{N_{vG}}\right)}\right]^2 \qquad (3)$$

where $k_s$ is the saturated hydraulic conductivity. $k(\psi)$ can be defined by substituting Eq. (2) for θ into Eq. (3). There has only been limited experimental evaluation of the validity of the predicted *k*-function for use in practice.

## 3 EXPERIMENTAL K-FUNCTIONS IN THE LITERATURE

Experimental SWRC data for different clay soils is shown in Figure 1(a). Due to the wide range of matric and total suctions in most soils, it is conventional to plot suction on a logarithmic scale. When plotted on a semi-logarithmic scale, the SWRC for most soils follows an "S" shaped curve that mimics the pattern of the soils' pore size distribution (Lu and Likos 2005). The SWRC data for the clays and sand-clay in Figure 1(a) show a relatively gradual decrease in moisture content with changes in suction over several orders of magnitude. The experimental *k*-functions for these same soils shown on a log-log scale in Figure 1(b) indicates that the hydraulic conductivity also gradually decreases with suction. In general, the *k*-functions follow an inverted "L" shaped curve, where the hydraulic conductivity is close to $k_s$ for suctions below the point at which air enters the specimen.

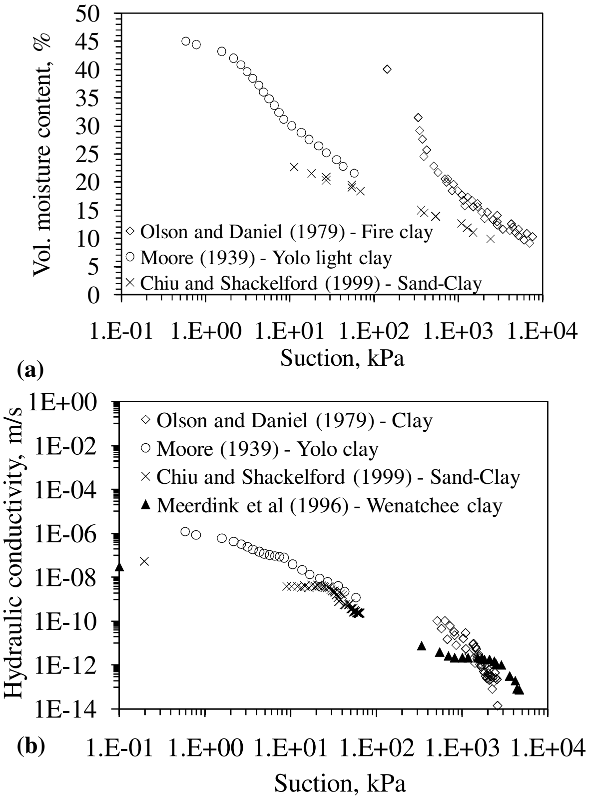

Figure 1 : Data for clays: (a) SWRC ; and (b) *k*-functions

Experimental SWRC and *k*-function data for silts are shown in Figures 2(a) and 2(b), respectively. Due to the different approaches used to characterize silts and clays, the silt *k*-functions are shown as a function of moisture content. The SWRCs for the silts still shows a gradual decrease in moisture with suction, although the rate of decrease is greater for the silts and a more abrupt air-entry suction is noted in some of the silts as well. Experimental SWRC and *k*-function data for sands are shown in Figures 3(a) and 3(b), respectively. An interesting observation from this data is that the shape of most SWRCs are similar, but the shapes of the *k*-functions are much different. In fact the *k*-functions for sands with similar SWRC often differ by orders of magnitude [*e.g.*, compare the data from Youngs et al. (2001) with Sigda and Wilson (2005)]. Near saturation, both the silts and the sands tend to have higher hydraulic conductivity values than the clay soils in Figure 1. However, as silts and sands desaturate, they reach lower values of permeability than the clay for similar suctions (or degree of saturation). Fine-grained materials can retain more water in the pores as suction increases, so there are available pathways for water flow even when highly unsaturated. An important feature of the *k*-functions of all of the soil types is the shape in dry conditions (near residual saturation). The *k*-functions all tend to show a decreasing trend downward with . On a logarithmic scale, this indicates a sharp decrease in *k*.

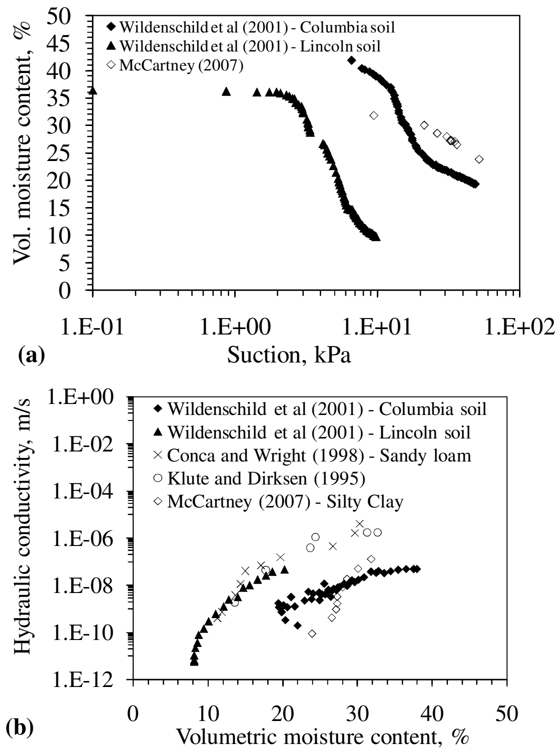

Figure 2 : Data for silts: (a) SWRC ; and (b) *k*-functions

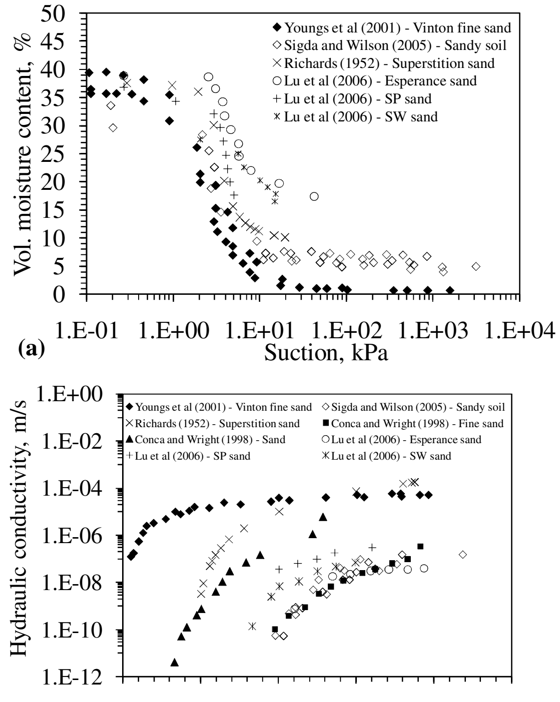

Figure 3 : Data for sands: (a) SWRC ; and (b) *k*-functions

The *k*-function is intended to capture the proportionality between hydraulic gradient and water flow rate, and is thus only relevant for conditions in which the water phase in the soil is continuous. In other words, when the water phase becomes discontinuous, the hydraulic conductivity is negligible or zero. However, when the water phase in the soil becomes discontinuous, water vapor phase transport by diffusion dominates the migration of moisture in the soil. Because the theoretical prediction of the *k*-function provides no lower bound on the hydraulic conductivity of a soil, the boundary between liquid and vapor phase transport is difficult to assess when using predictive models like that shown in Eq. (3). Evaluation of the experimental SWRCs and *k*-functions in Figures 1 through 3 indicates that this boundary likely occurs in the vicinity of the suction range in which the SWRC begins to flatten out with increasing suction.

## 4 COMPARISON BETWEEN FITTED, PREDICTED, AND EXPERIMENTAL K-FUNCTIONS

The van Genuchten (1980) model in Eq. (2) was fit using least squares optimization to the different experimental SWRC shown in Figure 1(a), 2(a), and 3(a). The parameters from this fit were then used in Eq. (3) to predict the *k*-function, as shown by the solid line in Figure 4. In addition, Eq. (3) was fit directly to the *k*-function data, shown by the dashed line in Figure 4. For the k-function data for the soil shown in Figure 4 reported by Moore (1939), the predicted *k*-function differs from the experimental results by two orders of magnitude at low degrees of saturation, and also by a substantial difference at higher degrees of saturation.

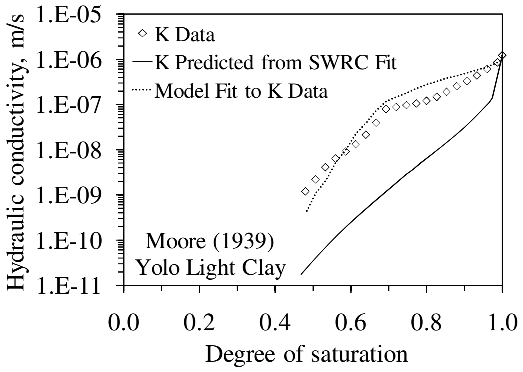

Figure 4 : Example of a comparison between the *k*-function predicted from the best-fit parameters for the SWRC with that fitted directly to experimental data

The difference between the fitted and predicted *k*-function can be assessed through the magnitude of the α parameter in the van Genuchten (1980) model. The smaller this value, the steeper the slope of the *k*-function with respect to either suction or moisture content. A summary of the fitted α values is shown in Figure 5. The data in this figure indicate that the α value obtained from the fit to the SWRC (*i.e.*, predicted *k*) generally overestimates the α value corresponding to a direct fit to the *k* data.

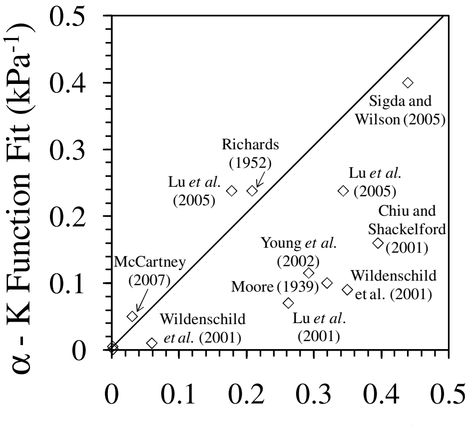

Figure 5: Comparison between α parameters for different soils obtained from fitting the van Genuchten (1980) model to the *k*-function data and to the SWRC data

## 5 IMPLICATIONS AND RECOMMENDATIONS

Numerical solutions to Richards' equation [Eq. (1)] for the case of steady-state, downward infiltration at a discharge velocity q (m/s) through a soil layer with a water table depth of 10 m are shown in Figure 6(a). This analysis was chosen as it is sensitive primarily to the shape of the *k*-function. The parameters used in this analysis were obtained using the van Genuchten-Mualem (1980) model fit to the SWRC and *k*-function for the soil tested by Moore (1939). The difference in the suction profiles for the two parameter sets are shown in Figure 6(b). The results in these figures indicates that as the infiltration rate q decreases, the error in prediction of the suction profile increases (i.e., assuming that the fitted *k*-function is the correct fit). Plate load tests reported in the literature indicate that a difference in suction of 17 kPa as shown may have significant implications on soil stiffness and foundation settlement (Costa *et al.* 2003).

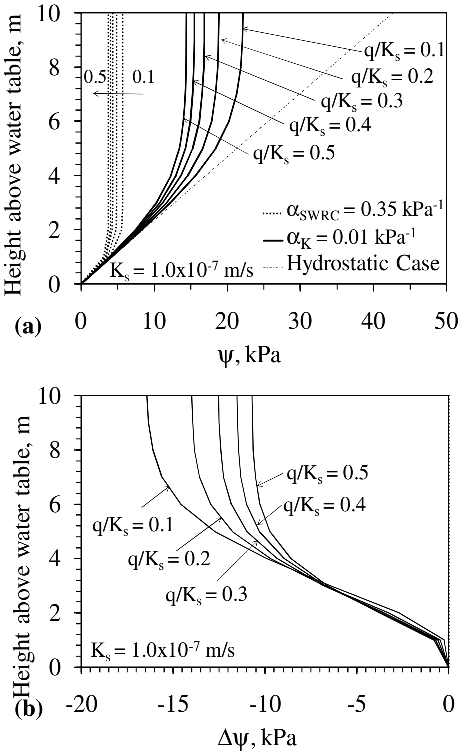

Figure 6: (a) Predicted suction profiles with parameters for Moore (1939); (b) Difference in suction profiles due to error

The selection of an appropriate testing approach to determine the K-function depends on the ability to control of the variables mentioned in Section 2 as well as on the testing time. For sands and silts, a column infiltration test is most appropriate, which can be performed in a an expedited manner in a centrifuge permeameter (McCartney 2007). For clays and deformable soils, the flexible-wall flow pump permeameter is recommended as it provides the best control of the stress-state, volume change, and water outflow (Znidarcic *et al.* 1990).

## 6 CONCLUSIONS

An important source of reluctance to implement lessons learned in academic research on unsaturated soil mechanics in geotechnical practice was discussed. Empirical predictions of the hydraulic conductivity function can lead to errors of 1 to 4 orders of magnitude, with the greatest discrepancies at low moisture contents. Analyses in this paper emphasize the need for experimental characterization of the SWRC and *k*-function, or in the least, soil-specific validation of predictions, for use in the design of geotechnical systems in unsaturated soils.

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