Stability of circular tunnels in soft ground

Stabilité de tunnels circulaires en terrain meuble

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ABSTRACT

This paper considers the undrained stability of a circular tunnel in soft ground where the strength increases linearly with depth. Solutions are obtained which enable designers to compute the tunnel pressure that is needed to maintain stability of an unsupported heading during the construction process. These solutions take account of the ground surcharge, the tunnel geometry, the soil unit weight, a non-homogeneous undrained shear strength, and are expressed in dimensionless form.

The numerical results have been obtained using finite element formulations of the upper and lower bound theorems of limit analysis. By obtaining upper and lower bound estimates on the tunnel pressure, the actual tunnel pressure required to maintain stability can be bracketed from above and below. This provides an inbuilt error indicator for the accuracy of the solutions. Some upper bounds are also presented from the use of simple rigid block mechanisms. These mechanisms are shown to give accurate upper bounds in some cases. Where possible, the theoretical predictions are compared against experimental data obtained from centrifuge and laboratory tests.

RÉSUMÉ

Ce papier traite de la stabilité non drainée de tunnels circulaires en terrain meuble dont la résistance mécanique augmente de manière linéaire avec la profondeur. Les solutions obtenues permettent aux concepteurs de calculer les pressions nécessaires pour maintenir la stabilité du front non supporté pendant le processus de construction. Ces solutions qui prennent en compte la surcharge due au terrain, la géométrie du tunnel, le poids du sol et la résistance au cisaillement non drainée, sont exprimées de façon adimensionnelle.

Les résultats numériques ont été obtenus en utilisant une formulation aux éléments finis des théorèmes d'analyse limite cinématique et statique. L'obtention des limites supérieure et inférieure des pressions permet l'encadrement des pressions requises pour la stabilité. Cette méthode fournit également un indicateur de l'erreur permettant d'estimer l'exactitude des solutions. Certaines limites supérieures sont également obtenues par l'application de simples mécanismes de blocs rigides, ce qui, dans certains cas, fournit des résultats précis. Lorsque c'est possible, les prédictions théoriques sont comparées aux données expérimentales obtenues à partir de tests en centrifuge et en laboratoire.

Keywords : tunnel stability, limit analysis, plasticity

1 INTRODUCTION

This paper investigates the undrained stability of a circular tunnel in a soil medium where the shear strength increases linearly with depth. The stability of the tunnel is found using numerical formulations of the limit analysis theorems as well as semi-analytical rigid-block mechanisms. The geometry of the tunnel, modelled under plane strain conditions, is shown in Figure 1. The soil medium around the tunnel is modelled as a non-uniform Tresca material with an undrained cohesion at the ground surface (c_{u0}), a unit weight (γ) and a strength factor (ρ). The soil unit weight and the cohesion are usually known and are necessary to determine the stability of the tunnels. The strength factor determines the rate at which the shear strength increases linearly with depth and a factor of zero represents a homogeneous medium of uniform strength. The strength of the soil at any given depth can be described as:

$$c_u(z) = c_{u0} + \rho z \tag{1}$$

For the undrained analysis of tunnels, where the constant volume condition applies, it is useful to describe the stability in terms of the dimensionless parameter $(\sigma_s - \sigma_t)/c_{u0}$ which is a function of H/D, $\gamma D/c_{u0}$ and $\rho D/ c_{u0}$. Using this set of dimensionless parameters allows a compact set of stability charts to be obtained which are useful for design purposes.

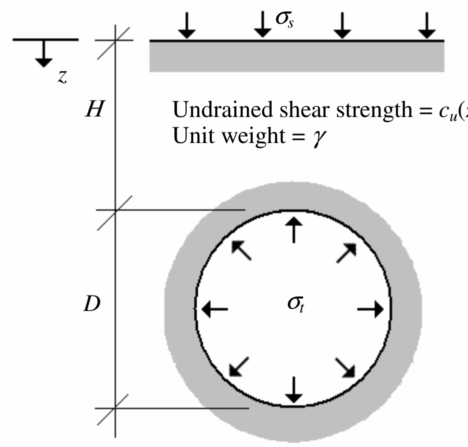

Figure 1. Plane strain circular tunnel in a non-uniform Tresca material.

The stability of the tunnel is obtained by application of finite element limit analysis to calculate upper and lower bounds on the factor $(\sigma_s - \sigma_t)/c_{u0}$ for the problem shown in Figure 1. These techniques utilize linear finite elements to formulate an optimization problem that is solved using second order conic programming. Safe estimates for the true solution of the stability $(\sigma_s - \sigma_t)/c_{u0}$ were obtained using the lower bound method based on the principle that any set of loads supported by

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a statically admissible stress field will underestimate the true collapse load. The upper bound method, which uses the fact that any kinematically admissible velocity field will provide an unsafe solution on the true collapse load, provides an estimate of the collapse load that is above the true solution. Using both methods in tandem enables the true collapse load to be bracketed from above and below and, as the accuracy of each of the bounds is increased, the true solution is known with more certainty. A semi-analytical method using a series of rigid block mechanisms, as described by Chen (1975), was also used to find upper bounds on the stability of the tunnel as a check of the validity of the finite element analyses.

2 FINITE ELEMENT LIMIT ANALYSIS

Finite element formulations of the upper and lower bound theorems provide a means by which they can be applied to the analysis of complex engineering problems. The upper bound theorem is based on the notion that the imposed loads cannot be carried by the soil mass if, for any kinematically admissible failure mechanism (or velocity field), the rate of work done by the external forces exceeds the internal rate of dissipation (Chen 1975). A kinematically admissible velocity field is one which satisfies both the flow rule and the velocity boundary conditions.

The lower bound theorem is based upon the notion that if an equilibrium state of stress can be found that satisfies the stress boundary conditions and the yield criterion, then the imposed loads can be safely carried by a soil mass. If such a state of stress can be found it gives a lower bound (or safe) solution and underestimates the true collapse load (Chen 1975). The stress field that meets all of the above criteria is known as a "statically admissible stress field". Limit analysis is the most useful when both the upper and lower bounds are both computed, this allows us to know that the true solution lies between the two bounds while also giving an estimate of the error. This error is simply the difference between the two bounds.

The finite element implementations used throughout this study are based upon the original formulations of Sloan (1988, 1989) who used linear finite elements and linear programming to solve the resulting optimization problem. Since this initial work, finite element limit analysis has evolved significantly and the techniques used in this paper are based upon the procedures described in Lyamin & Sloan (2002a,b) and Krabbenhoft *et al* (2005, 2007).

In this study, upper and lower bounds on the stability of a circular tunnel are found for a range of values of H/D, $\gamma D/c_{u0}$ and $\rho D/c_{u0}$. The main focus is to examine the effects of the soil strength parameter ($\rho D/c_{u0}$) while also considering the effects of the tunnel depth (H/D) and the soil unit weight ($\gamma D/c_{u0}$). A finite element mesh which is representative of those used for the upper and lower bound analyses of a tunnel with H/D = 1 is shown in Figure 2. This mesh, which has been chosen for clarity, is in fact coarser than the actual meshes used in the finite element analyses. The mesh is essentially the same for both the upper and lower bounds, however, the boundary conditions are set differently. The velocity boundary conditions are for the upper bounds and include both the horizontal, u, and the vertical, v, velocities. The stress boundary conditions are for the lower bound analyses and consist of the normal stress, σ_n, and the tangential stress, τ.

The lower bound analysis is performed by solving a optimization problem to find a statically admissible stress field which maximizes $(\sigma_s - \sigma_t)/c_{u0}$, while the upper bound analysis is performed by solving a similar optimization problem to find the kinematically admissible velocity field that minimizes the quantity $(\sigma_s - \sigma_t)/c_{u0}$. Note that the elements furthest from the tunnel and shown as dashed lines are extension elements which are used to extend the statically admissible lower bound stress field beyond the defined mesh and over the semi-infinite domain. These elements are unnecessary for the upper bound analyses, but efforts were made to ensure that the plastic zone was contained entirely inside the mesh boundaries. If the upper bound mesh is made too small, the solution is still a valid upper bound but may be too high.

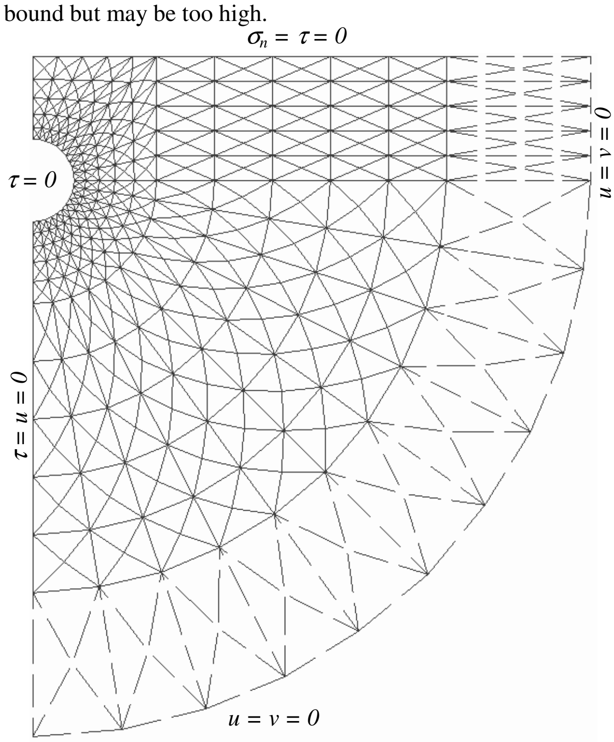

Figure 2. Example finite element mesh showing boundary conditions.

3 RIGID BLOCK ANALYSIS

Semi-analytical rigid block methods were also used to find upper bound estimates for the tunnel stability. These provided an additional check on the limit analysis solutions and gave upper bounds that are close to the finite element limit analysis solutions. Two of the rigid block mechanisms that were considered are shown in Figure 3. The geometry of the blocks is allowed to vary while being suitably constrained such that the discontinuity lengths and areas cannot become negative. The minimum upper bound for each mechanism was obtained by optimizing its geometry a Hooke-Jeeves algorithm with discrete steps. This method works by performing two different types of searches, an exploratory search and a pattern search. The rigid block analyses are extremely quick taking of the order of just one second. Provided an appropriate mechanism is chosen, this technique gives a fairly accurate upper bound to compare the finite element solutions against and does not need large amounts of computational power.

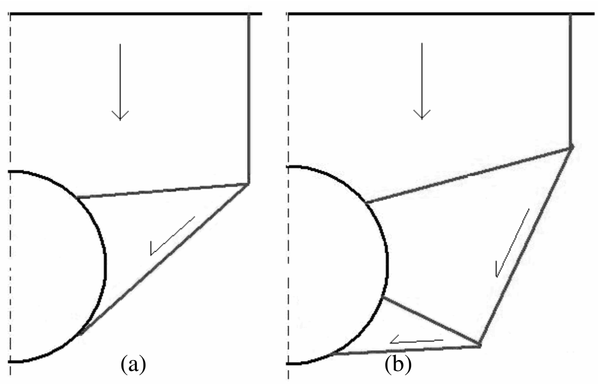

Figure 3. Rigid block mechanisms showing velocity discontinuities.

4 RESULTS AND DISCUSSION

As the stability of the plane strain tunnel geometry shown in Figure 1 is defined by the dimensionless stability number $(\sigma_s - \sigma_t)/c_{u0}$, it is possible to simplify the analysis of the problem by setting the surcharge pressure to zero. For the purpose of analysis, further simplification is achieved by setting the tunnel diameter (D) and the undrained shear strength at the ground surface (c_{u0}) to unity. Using the dimensionless parameters described above (H/D, $\gamma D/c_{u0}$ and $\rho D/ c_{u0}$), this reduces the variables in the parametric study to the tunnel depth (H), the soil unit weight (γ) and the strength factor of the soil (ρ). It is important to note that this paper investigates active collapse only and does not consider a possible passive tunnel "blow out" failure. Active collapse is driven by a combination of the action of gravity (γ) and the surcharge pressure (σ_s), while the resistance is provided by a combination of the shear strength of the soil ($c_u(z)$) and the internal tunnel pressure (σ_t).

Results obtained for the stability of the case with $H/B = 5$ are summarized in Figure 4 as a plot of stability $(\sigma_s - \sigma_t)/c_u$ versus $\gamma D/c_{u0}$. This chart shows that the finite element upper and lower bounds lie, for the most part, within a few percent of each other. The rigid block upper bounds also give a good approximation to the finite element solutions. The mechanisms that provided the best upper bound from the rigid block analysis are shown in Figure 3; for shallow tunnels with low $\gamma B/c_{u0}$ values Figure 3(a) gave the best solutions, while deep tunnels were modelled better by 3(b). This is because the failure mode for shallow tunnels with low $\gamma B/c_{u0}$ values has a plastic zone that lies entirely above the centreline of the tunnel. Deep tunnels tend to have a more extensive plastic zone that develops beyond the tunnel invert. The mechanism featured in Figure 3(b) captures this much better than that shown in Figure 3(a).

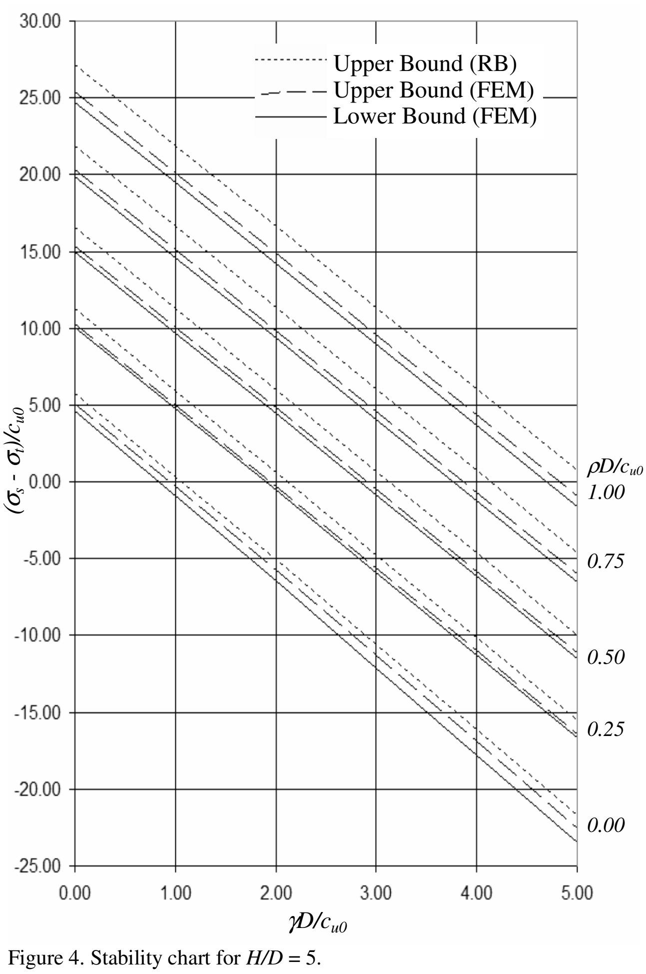

Figure 4. Stability chart for $H/D = 5$.

Rigorous undrained stability results for a circular tunnel in clay have been given by Mair (1979), Davis *et al.* (1980) and Sloan and Assadi (1992). Figure 5 shows a comparison between the new results, the experimental results of Mair (1979), and the limit analysis results obtained by Sloan and Assadi (1992) for the uniform strength case ($\rho D/ c_{u0} = 0$). The $\gamma B/c_{u0}$ parameter is approximately equal to 2.6 and the values for the new results and those of Sloan and Assadi (1992) were found by interpolation between $\gamma B/c_{u0} = 2$ and 3. The assumption of a linear variation between $\gamma B/c_{u0}$ values was justified by inspecting the various stability charts such as those shown in Figure 4. It is, of course, important to compare any theoretical predictions with experimental results whenever possible to assess the reliability of the approach. The results of Mair (1979) are the most comprehensive set of experimental results available and were performed in the Cambridge centrifuge using a Kaolin clay that was relatively uniform in strength.

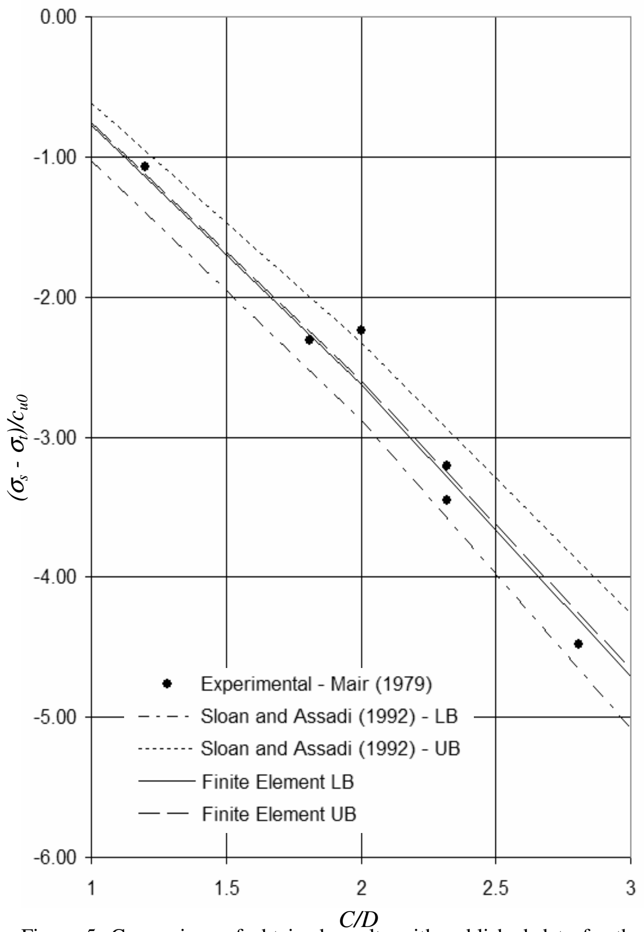

Figure 5. Comparison of obtained results with published data for the case where $\gamma B/c_{u0}$ is approximately 2.6.

Figure 5 shows that, for the uniform case, the new predictions are in a very close agreement with the experimental results of Mair (1979) as well as being an improvement over the theoretical results of Sloan and Assadi (1992).

A comparison of the new results with those of Sloan and Assadi (1992) is given in Figure 6 for a typical heterogeneous case with H/D equal to 4. It can be seen that, for cases where the soil is non-uniform with $\rho D/ c_{u0}$ being non-zero, the new bounds are a significant improvement. For the homogeneous case the improvements are less marked.

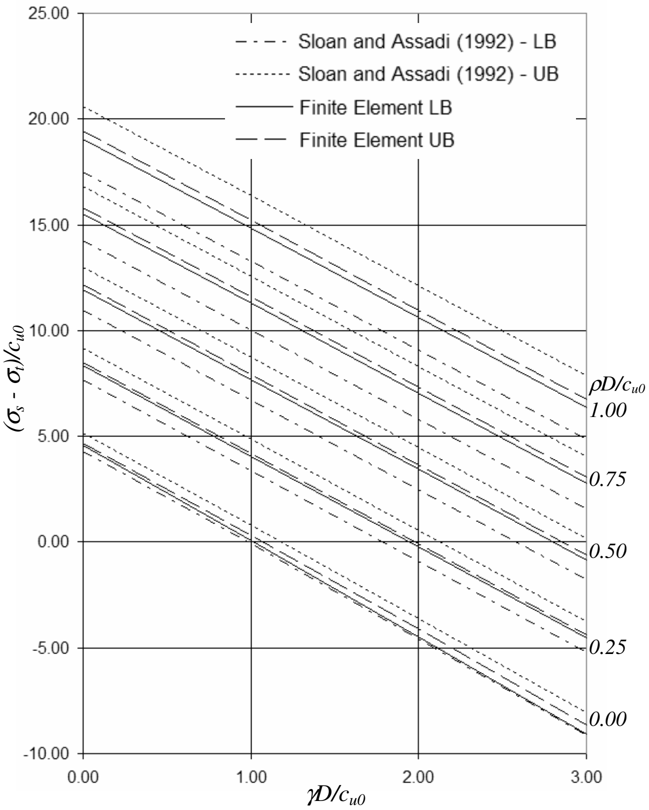

Figure 6. Comparison of obtained results with published data for a non-uniform case where H/D = 4.

Finally, it is important to understand the implications of the stability numbers shown in the stability charts. Since the stability number is $(\sigma_s - \sigma_t)/c_u$, this means that the special case where $\sigma_s = 0$ actually corresponds to the pressure on the face of the tunnel which just prevents collapse. A negative stability number implies that a compressive normal stress must be applied to the tunnel face to maintain stability, while a positive stability number means that no tunnel support is required to prevent collapse (in fact the tunnel face can actually support a tensile normal stress).

5 CONCLUSIONS

The stability of a circular tunnel in an undrained clay whose shear strength increases linearly with depth has been investigated under plain strain conditions. Upper and lower bound solutions for the stability of the tunnel for a variety geometries and soil conditions have been found using both semi-analytical upper bound limit analysis and numerical finite element limit analysis. Using these solutions, a stability chart in terms of dimensionless parameters has been generated for a typical case that would be useful for design purposes.

For situations where the soil strength is uniform, the bounds are a small improvement over the results of Sloan and Assadi (1992) and in very good agreement with the experimental results of Mair (1979). When the soil is non-uniform, the new results are a significant improvement over those of Sloan and Assadi (1992) while also covering a larger range of variables. The semi-analytical rigid block upper bound methods proved to give fairly accurate solutions with very little computational effort for shallower tunnels.

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