Analytical solutions for vertical drains considering soil disturbance

Les solutions analytiques pour les canalisations verticales considérant des troubles de sol

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ABSTRACT
Soil disturbance caused by the installation of prefabricated vertical drains (PVDs) in soft soil deposits has a detrimental effect on the rate of consolidation. Design methods available for PVDs capture the effect of soil disturbance typically by reducing the in situ hydraulic conductivity in the disturbed zone (also called the smear zone) with the assumption that the hydraulic conductivity is spatially constant over the entire disturbed (smear) zone. However, it has been shown recently through laboratory and field studies that the hydraulic conductivity has a spatial variation within the disturbed zone. Based on data available in the literature, four possible profiles of hydraulic conductivity versus radial distance from the vertical drain were identified. Analytical solutions were developed for the rate of consolidation considering these hydraulic conductivity profiles. The study shows that the consolidation rate depends on the hydraulic conductivity profile in the disturbed zone.

RÉSUMÉ
Les troubles de sol provoqués par l'installation de canalisations verticales préfabriquées (PVDs) dans les dépôts de sol mous ont un effet préjudiciable sur le taux de consolidation. Les méthodes de design disponibles pour PVDs capturent l'effet de troubles de sol d'une manière caractéristique en réduisant le dans la conductivité hydraulique situ dans la zone dérangée (a aussi appelé la zone de tache) en considérant que la conductivité hydraulique est spatialement constante sur le dérangé entier (la tache) la zone. Pourtant, il a été montré récemment par le laboratoire et les études sur le terrain que la conductivité hydraulique a une variation spatiale dans la zone dérangée. Basé sur les données disponibles dans la littérature, quatre profils possibles de conductivité hydraulique contre la distance radiale de la canalisation verticale ont été identifiés. Les solutions analytiques ont été développées pour le taux de consolidation considérant ces profils de conductivité hydrauliques. L'étude montre que le taux de consolidation dépend du profil de conductivité hydraulique dans la zone derange.

Keywords : prefabricated vertical drain, analytical solution, ground improvement, smear, soil disturbance

1 INTRODUTION

Prefabricated vertical drains (PVDs) have been successfuly used in conjunction with preloading for improvement of soft soils since the early 1970s (Holtz 1987, Holtz et al. 1991, Bergado et al. 1993, Lo and Mesri 1994). The installation of PVDs reduces the water-drainage path because of which consolidation occurs faster than it would if there were no drains, resulting in rapid increase in strength and stiffness of soft clayey soils.

Several theoretical and experimental research studies have been performed on PVDs for estimating the discharge rate of PVDs (i.e., the rate of consolidation) and for determining and mitigating the operational problems experienced at sites with PVD's installed in the ground (Barron 1948, Hansbo 1981, 1997, Bergado et al. 1991, 1993, Chai et al. 1997, Chai and Miura 1999, Basu and Madhav 2000, Indraratna and Redana 1997, Bo et al. 2003, Basu et al. 2006, Basu and Prezzi 2007, Walker and Indraratna 2006, Sathananthan and Indraratna 2006). Soil disturbance caused during the installation of PVDs by closed-ended mandrels slows down the soil consolidation rate. Traditionally, the effect of soil disturbance is taken into account in calculations by assuming a reduced, spatially-invariant soil hydraulic conductivity in the disturbed zone (also called the smear zone) surrounding the PVD. In fact, Hansbo (1981) presented a paper in the 10th ICSMFE that has become part of standard practice today on the estimation of the degree of soil consolidation assisted by PVDs considering the presence of a smear zone.

Recent experimental investigations have, however, shown that the hydraulic conductivity within the disturbed zone is not spatially constant (Onoue et al. 1991, Madhav et al. 1993, Indraratna and Redana 1998, Sharma and Xiao 2000). Figure 1 shows the profiles of hydraulic conductivity k (normalized with respect to the *in situ*, undisturbed hydraulic conductivity k_c) in the vicinity of vertical drains, observed in laboratory and field studies, as functions of normalized radial distances from the drains. The radial distance r is normalized with respect to the equivalent mandrel radius $r_{m,eq}$, obtained by equating the cross sectional area of of the actual mandrels with equivalent circles.

It is clear from Figure 1 that the analysis of Hansbo (1981) with a spatially constant hydraulic conductivity profile in the smear zone (case a of Figure 2) is not strictly valid in real problems. Consequently, we developed analysis with different possible hydraulic conductivity profiles in the disturbed zone (cases b, c, d and e in Figure 2) that may be encountered in practice.

2 ANALYSIS

2.1 *Problem definition*

We considered a drain installed in a saturated, soft clay deposit with a circular cross section of radius r_d and a length equal to the entire thickness of the soil deposit. An annular cylinder of soil with inner and outer radii r_d and r_c (measured from the center of the drain) is considered as the unit cell (Figure 2); r_d and r_c are the drain radius and unit cell radius, respectively. Vertical flow in the unit cell is neglected (Leo 2004). The only pervious boundary of the unit cell considered in the analysis is

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the interface between the drain and the unit cell. This results in radially convergent horizontal flow of water towards the drain. Assuming a homogeneous deposit with no horizontal strain in the soil cylinder, flow patterns are identical along any horizontal plane. Hence, analysis considering only one such horizontal plane with axisymmetric flow is sufficient to solve the problem. We further assume that the vertical strain within the unit cell (due to consolidation) is spatially uniform. This represents the case of "equal-strain" consolidation (Richart 1959). In addition, flow of water is assumed to follow Darcy's law.

For cases b and c, two distinct zones within the disturbed zone, namely the smear and transition zones, are assumed with annular cross sections such that their outer radii (as measured from the center of the drain) are r_{sm} and r_{tr}, respectively, with $r_d < r_{sm} < r_{tr} < r_c$ (Figure 2). r_{sm} and r_{tr} are referred to as the smear zone and transition zone radii, respectively. For case d, there is a single disturbed zone with outer radius r_{tr}. Case e is a combination of cases b, c and d. For all the cases, the undisturbed zone lies between $r_{tr} \le r \le r_c$ where r is the radial coordinate measured from the center of the drain.

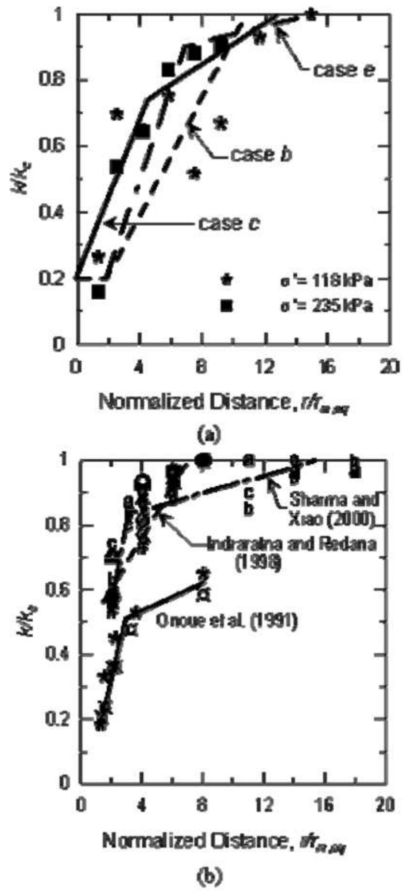

Figure 1. Hydraulic conductivity profiles from (a) field samples (Madhav et al. 1993), (b) laboratory model studies.

2.2 *Average excess pore pressure*

The hydraulic conductivity $k_{sm}(r)$ in the smear zone ($r_d \le r \le r_{sm}$) is either constant at k_s (for cases b and e) or linearly varying (for cases c and d). As an example, for case c:

$$k_{sm}(r) = k_s + (r - r_d)(k_t - k_s)/(r_{sm} - r_d) \tag{1}$$

In the transition zone, a linear variation of the hydraulic conductivity, of the form $k_{tr}(r) = A + Br$, is assumed for all the cases. As an example, for case b, k_{tr} varies from k_s at $r = r_{sm}$ to k_c at $r = r_{tr}$ which can be mathematically described as:

$$k_{tr}(r) = (k_s r_{tr} - k_c r_{sm})/(r_{tr} - r_{sm}) + (k_c - k_s)r/(r_{tr} - r_{sm}) = A + Br \tag{2}$$

for $r_{sm} \le r \le r_{tr}$. In the undisturbed zone, the hydraulic conductivity is a constant at k_c.

Applying Darcy's law and equating the compressive volume strain rate $\partial\varepsilon_v/\partial t$ of the outer hollow soil cylinder, within the unit cell, of arbitrary thickness ($r_c - r$) to the rate of water flow out of the cylinder into the inner soil cylinder of radius r, we get for case b:

$$\partial u_c/\partial r = (\gamma_w/2k_c)(r_c^2/r - r)\partial\varepsilon_v/\partial t;\ r_{tr} \le r \le r_c \tag{3a}$$

$$\partial u_{tr}/\partial r = (\gamma_w/2k_{tr})(r_c^2/r - r)\partial\varepsilon_v/\partial t;\ r_{sm} \le r \le r_{tr} \tag{3b}$$

$$\partial u_{sm}/\partial r = (\gamma_w/2k_s)(r_c^2/r - r)\partial\varepsilon_v/\partial t;\ r_d \le r \le r_{sm} \tag{3c}$$

where u_c, u_{tr} and u_{sm} are pore pressures in the undisturbed, transition and smear zones, respectively, and γ_w is the unit weight of water. Integrating the above equations with respect to r, substituting k_{tr} from equation (2) and using the boundary and continuity conditions $u_{sm} = 0$ at $r = r_d$, $u_{tr} = u_{sm}$ at $r = r_{sm}$ and $u_c = u_{tr}$ at $r = r_{tr}$,we get:

$$u_{sm} = (\gamma_w/2k_s)\{r_c^2\ln(r/r_d) - (r^2 - r_d^2)/2\}\partial\varepsilon_v/\partial t \tag{4a}$$

$$\begin{aligned} u_{tr} = {} & (\gamma_w/2)[(r_c^2/A)\ln\{k_s r/(A+Br)r_{sm}\} - (1/B^2)\{A + Br - k_s \\ & - A\ln[(A+Br)/k_s]\} + (1/k_s)\{r_c^2\ln(r_{sm}/r_d) \\ & - (r_{sm}^2 - r_d^2)/2\}]\partial\varepsilon_v/\partial t \end{aligned} \tag{4b}$$

$$\begin{aligned} u_c = {} & (\gamma_w/2)[(1/k_c)\{r_c^2\ln(r/r_{tr}) - (r^2 - r_{tr}^2)/2\} \\ & + (1/k_s)\{r_c^2\ln(r_{sm}/r_d) - (r_{sm}^2 - r_d^2)/2\} + (r_c^2/A)\ln(r_{tr}k_s/r_{sm}k_c) \\ & - (1/B^2)\{k_c - k_s - A\ln(k_c/k_s)\}]\partial\varepsilon_v/\partial t \end{aligned} \tag{4c}$$

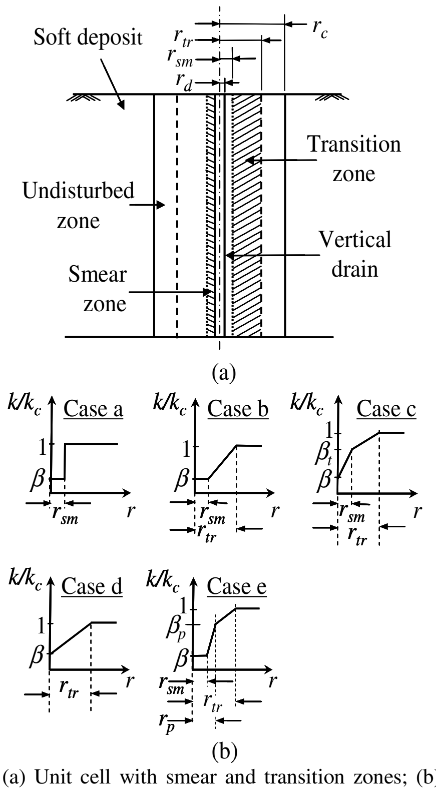

Figure 2. (a) Unit cell with smear and transition zones; (b) Hydraulic conductivity profiles in the disturbed zone surrounding the PVD.

If $\bar{u}$ is the average excess pore pressure throughout the unit cell, then the following equation can be written:

$$\pi\left(r_c^2 - r_d^2\right)\bar{u} = \int_{r_d}^{r_{sm}} 2\pi r u_{sm}dr + \int_{r_{sm}}^{r_{tr}} 2\pi r u_{tr}dr + \int_{r_{tr}}^{r_c} 2\pi r u_c dr \tag{5}$$

Substituting u_{sm}, u_{tr} and u_c from equations (4a), (4b) and (4c), respectively, in equation (5) and rearranging the terms we obtain:

$$\bar{u} = (\gamma_w r_c^2/2k_c)\mu\partial\varepsilon_v/\partial t \tag{6}$$

where μ is given by (after truncating the terms with negligible contributions):

$$\mu = \ln(n/q) + (1/\beta)\ln(m) + (q-m)\ln(\beta q/m)/(\beta q - m) - 3/4 \quad (7)$$

in which $n = r_c / r_d$, $m = r_{sm} / r_d$, $q = r_{tr} / r_d$ and $\beta = k_s / k_c$.

The above equation of μ is obtained for case b. Similar equations of μ can be obtained for cases c, d and e (the equation of μ for case a was obtained by Hansbo (1981)). Note that equation (6) for $\bar{u}$ is valid for all the cases. For case c, μ is given by:

$$\mu = \ln(n/q) + (m-1)\ln(\beta m/\beta_t)/(\beta m - \beta_t) + (q-m)\ln(\beta_t q/m)/(\beta_t q - m) - 3/4 \quad (8)$$

where $\beta_t = k_t/k_c$. For this case, the hydraulic conductivity $k_{sm}(r)$ in the smear zone varies linearly from k_s at $r = r_d$ to k_t at $r = r_{sm}$. In the transition zone, the hydraulic conductivity $k_{tr}(r)$ varies linearly from k_t at $r = r_{sm}$ to k_c at $r = r_{tr}$.

For case d, the hydraulic conductivity increases monotonically over the entire disturbed zone from k_s at $r = r_d$ to k_c at $r = r_{tr}$. Consequently, the equation of μ is obtained as:

$$\mu = \ln(n/q) + (q-1)\ln(\beta q)/(\beta q - 1) - 3/4 \quad (9)$$

For case e, the hydraulic conductivity remains constant at k_s within the smear zone ($r_d \le r \le r_{sm}$) and increases in the transition zone following a bilinear curve with one slope from k_s (at $r = r_{sm}$) to k_p (at $r = r_p$) and with another slope from k_p (at $r = r_p$) to k_c (at $r = r_{tr}$). With this variation, the following equation is obtained for μ:

$$\mu = \ln(n/q) + (1/\beta)\ln(m) + (p-m)\ln(\beta p/\beta_p m)/(\beta p - \beta_p m) + (q-p)\ln(\beta_p q/p)/(\beta_p q - p) - 3/4 \quad (10)$$

where $p = r_p/r_d$ and $\beta_p = k_p/k_c$.

Finally, we also reproduce the expression of μ for case a, as obtained by Hansbo (1981), for the sake of completeness:

$$\mu = \ln(n/q) + (1/\beta)\ln(m) - 3/4 \quad (11)$$

2.3 *Degree of consolidation*

Assuming that all the excess pore pressure due to preloading is developed instantly, the volumetric strain rate can be related to stress change rate as:

$$\partial\varepsilon_v/\partial t = m_v \partial\bar{\sigma}'/\partial t = -m_v \partial\bar{u}/\partial t \quad (12)$$

where $\bar{\sigma}'$ is the average effective stress in the unit cell due to preloading at the end of consolidation, $\bar{u}$ is the average excess pore pressure at the time of load application, and m_v is the coefficient of volume compressibility. Defining the coefficient of consolidation in the horizontal direction $c_h = k_c/m_v\gamma_w$ and the corresponding time factor $T = c_h t/4r_c^2$, respectively, and substituting equation (6) in equation (12), we get:

$$d\bar{u}/dt + (2k_c/m_v\gamma_w r_c^2\mu)\bar{u} = 0 \quad (13)$$

Solving equation (13) using the initial condition that $\bar{u} = \bar{u}_0$ at $t = 0$, where $\bar{u}_0$ is the initial average excess pore pressure, and using the definitions of c_h and T, we get the change in average excess pore pressure with time:

$$\bar{u} = \bar{u}_0 \exp(-8T/\mu) \quad (14)$$

The degree of consolidation U at a particular time t (or time factor T) is the ratio of the excess pore pressure dissipated to the initial excess pore pressure. U can be mathematically expressed as $U = 1 - \bar{u}/\bar{u}_0$. Substituting this in equation (14), the following expression for the degree of consolidation can be obtained:

$$U = 1 - \exp(-8T/\mu) \quad (15)$$

2.4 *Equivalent radius*

The analytical solutions are valid for drains with circular cross sections and for cylindrical disturbed zones. In order to use these solutions for PVDs, an equivalent circular radius $r_{d,eq}$ of the PVD has to be calculated (Hansbo 1981):

$$r_{d,eq} = (b_w + b_t)/\pi \quad (16)$$

where b_w and b_t are the width and thickness of the PVD, respectively. For PVDs installed in rectangular or triangular patterns, the unit cells are rectangular or hexagonal in shape (in plan). In order to use the analytical solutions, the rectangular or hexagonal unit cells need to be replaced by equivalent circles having the same area. For a rectangular installation pattern, the equivalent radius of the unit cell is given by:

$$r_{c,eq} = \sqrt{s_x s_y / \pi} \quad (17)$$

where s_x and s_y are the spacings of the PVDs in two mutually perpendicular directions. For a triangular installation pattern with a spacing s, the equivalent radius of the unit cell is given by:

$$r_{c,eq} = \left(\sqrt{3}/2\pi\right)^{0.5} s \quad (18)$$

3 RESULTS

We studied the influence of the different hydraulic conductivity profiles described above on the consolidation rate. Figure 3 shows plots of degree of consolidation U versus time factor T for PVDs installed in a rectangular arrangement with 1 m ($r_{c,eq}$ = 564.2 mm) center-to-center spacing. Four hydraulic conductivity profiles (cases a, b, c and d) are considered. The PVDs are assumed to have a cross section of 100 mm × 4 mm ($r_{d,eq}$ = 33.1 mm). A mandrel with rectangular cross section 125 mm × 50 mm ($r_{m,eq}$ = 44.6 mm) is considered. The extent of the disturbed zone is defined by $r_{sm} = 2r_{m,eq}$ (except for case d) and $r_{tr} = 12r_{m,eq}$ (data presented in Figure 1 was used to define the disturbed zone). The degree of disturbance β at the drain surface is taken as 0.2. For case c, $\beta_t = 0.6$ is assumed.

Figure 3 indicates that the hydraulic conductivity profile in the disturbed zone has a definite impact on the rate of consolidation. For U = 90%, the time factor T corresponding to cases a, b, c and d are 1.74, 2.54, 1.37 and 2.09, respectively. For c_h = 1m^2/year, the corresponding actual times in years are 2.2, 3.2, 1.7 and 2.7. With respect to case a (Hansbo 1981), the increase in time (or time factor) required for 90% consolidation is equal to 46% and 20% for cases b and d, respectively; for case c, the time required for 90% consolidation decreased by 21%.

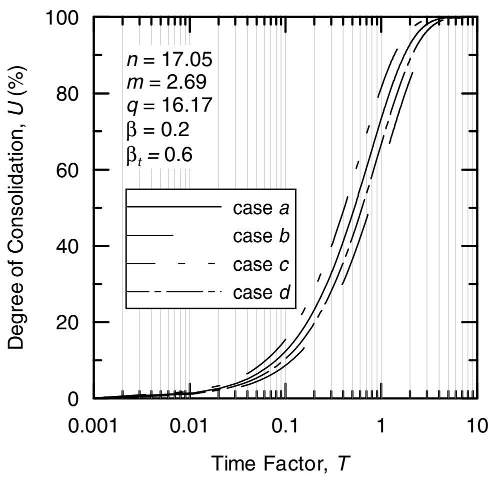

Figure 3. Plots of degree of consolidation versus time factor for different hydraulic conductivity profiles.

It is clear that proper knowledge of the hydraulic conductivity profile in the disturbed zone is needed for accurate design. Neglecting the gradual variation of the hydraulic conductivity in design may lead to error in the estimation of the consolidation rate. Knowledge of the degree of soil disturbance in the immediate vicinity of the drain is of utmost importance for predicting drain performance. This is evident by comparing the curves for cases b, c and d. For cases b and d, k/k_c is around

0.2 near the vicinity of the drain. There is an increase in this ratio (from 0.2 to 0.6) near the drain for case c. Consequently, the difference in response between cases c and b or cases c and d is more than the difference in response observed when cases b and d are compared.

4 EXAMPLE

As the hydraulic conductivity profile has an impact on the rate of consolidation, an example is worked out for some of the hydraulic conductivity profiles reported in the literature (see Figure 1) to illustrate how the analytical solutions can be used in practice. The installation of the PVDs is assumed to have been done with a mandrel of 120 mm × 120 mm ($r_{m,eq}$ = 67.7 mm) cross section. The PVD has a cross section of 100 mm × 4 mm ($r_{d,eq}$ = 33.1 mm), and the clay has a c_h = 10 m^2/year.

Using the data presented in Figure 1a for cases b, c, and e assumptions were made regarding the size of the smear and transition zones and the degree of disturbance. For case b, $r_{sm} = 2r_{m,eq}$ and $r_{tr} = 11r_{m,eq}$, while for case c, $r_{sm} = 4.5r_{m,eq}$ and $r_{tr} = 13r_{m,eq}$. For case e, $r_{sm} = 2r_{m,eq}$, $r_p = 7r_{m,eq}$ and $r_{tr} = 15r_{m,eq}$. The degree of disturbance β near the drain was taken as equal to 0.2 for all these cases. For case c, $\beta_t = 0.75$, and for case e, $\beta_p = 0.9$. A square arrangement of PVD installation was chosen with a center-to-center spacing of 2 m ($r_{c,eq}$ = 1128.4 mm). Based on these input values, μ was found to be equal to 11.00, 7.50, and 10.32 for cases b, c, and e. T can then be calculated for U = 90%: T = 3.17, 2.16, and 2.97 for cases b, c, and e, respectively. With c_h = 10 m^2/year, the calculated actual time required for 90% consolidation is equal to 1.6 years, 1.1 years, and 1.5 years for cases b, c, and e, respectively.

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