Reliability-based design on the bearing capacity of cement-treated ground considering the spatial variability of shear strength

Dessin fiabilité-basé sur la capacité portée de terre ciment-traitée considérer la variabilité spatiale de résistance cisaillement

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ABSTRACT

Cement-mixing method is gaining popularity as a method for stabilizing soft soils in applications ranging from the improvement of foundation properties to mitigation of liquefaction. However, spatial variability in the shear strength of the cement-treated ground introduces uncertainties in estimating the bearing capacity for design. This paper presents a reliability assessment for the bearing capacity of cement-treated ground based on the results of a probabilistic study in which the shear strength of the cement-treated ground is represented as a random field in Monte Carlo simulations of undrained stability for a surface foundation using numerical limit analyses. A statistical interpretation of the bearing capacity is based on Monte-Carlo simulations using the Random Field Numerical Limit Analyses. The results show how the bearing capacity is related to the coefficient of variation and correlation length scale in the shear strength of cement-treated ground. Based on the result, an overdesign factor, percent defective and resistance factor in LRFD for the bearing capacity of cement-treated ground are proposed to obtain a target reliability index.

RÉSUMÉ

La méthode ciment-mélangeant gagne la popularité se stabiliser des sols doux dans applications qui varient de l'amélioration de propriétés de la fondation à adoucissement de liquéfaction. La variabilité spatiale dans la résistance cisaillement de la terre ciment-traitée introduit des incertitudes dans estimer la capacité portée pour dessin. Ce papier présente une estimation de la fiabilité pour la capacité portée basée sur les résultats d'une étude probabiliste dans laquelle la résistance cisaillement de la terre ciment-traitée est représentée comme un champ aléatoire dans simulations Monte-Carlo qui utilise des analyses de la limite numériques. Une interprétation statistique de la capacité portée est basée sur simulations Monte-Carlo qui utilisent le Champ Aléatoire Limite Numériques Analyses. Les résultats montrent comme la capacité portée est en rapport avec le coefficient de variation et échelle de la longueur de la corrélation dans la résistance cisaillement. Basé sur le résultat, un facteur de l'overdesign, pour cent défectueux et le facteur de la résistance dans LRFD pour porter capacité de terre ciment-traitée est proposé d'obtenir un cible fiabilité index.

Keywords : ground improvement, bearing capacity, reliability, random field, limit analysis, Monte-Carlo simulasions

1 INTRODUCTION

The design for geotechnical structures such as embankment and retaining wall is need to deal with the spatial variability and uncertainty in soil parameters. A reliability-based design can consider variability and uncertainty in input parameters for design quantitatively and evaluate a reliability index and failure probability rationally as the safety of structure. The conventional reliability-based design for geotechnical structure has been applied for evaluating the safeties of embankment and retaining wall and natural slope (Matsuo, 1984). Cement-mixing techniques such as deep-mixing (Terashi & Tanaka, 1981) and pre-mixing (Zen et al., 1992) methods are becoming widely established for stabilizing soft soils in applications ranging from the improvement of foundation properties to mitigation of liquefaction. Although there have been significant advances in the equipment and methods used for cement-mixing, there remains a high degree of spatial variability in the soil parameter such as shear strength. Spatial variability in the shear strength of cement-treated ground introduces uncertainties in estimating the bearing capacity of shallow foundation for design.

This paper presents a reliability assessment for the bearing capacity of a surface strip foundation on spatially random cement-treated ground based on the results of a probabilistic study in which the shear strength of the cement-treated ground is represented as a random field in Monte Carlo simulations of undrained stability for a surface foundation using numerical limit analyses. A statistical interpretation of the bearing capacity is based on Monte-Carlo simulations using the Random Field Numerical Limit Analyses. The results show how the bearing capacity is related to the coefficient of variation and correlation length scale in the shear strength of cement-treated ground. Based on the result, an overdesign factor, percent defective and resistance factor in LRFD for the bearing capacity of cement-treated ground are proposed to obtain a target reliability index.

2 SPATIAL VARIABILITY OF CEMENT-TREATED GROUND

The main factors influencing the shear strength of the cement-treated ground include the types and amounts of cement, physico-chemical proper-ties of the in-situ soil, curing conditions and effectiveness of the mixing process, etc. Because of a number of influential factors on shear strength, the in-situ shear strength of cement-treated ground shows a large degree of spatial variability. For example, the coefficient of variation, *COV*, for unconfined compressive strength, q_u, obtained from core sample cured in the field ranges from 0.14 – 0.99, which is much higher than that expected for the undrained shear strength of natural soils (e.g., Phoon & Kulhawy, 1999). The horizontal and vertical correlation lengths of shear strength in cement-treated ground range 0.15 – 12m, which are much smaller than those for natural soils (Navin & Filz, 2005; Kasama et al., 2006). Figure 1 summarizes the strength ratio (*SR*) of q_u between in-situ specimen obtained from core sample and laboratory

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specimen with a similar cement proportion in the construction project using Pre-mixing method in Japan. Percent defective (*PD*), which is a percentage indicating q_u of in-situ specimen less than that of laboratory specimen, is also shown in Figure 1. An overdesign factor (F_o) as a vertical axis in Figure 1 is a multiple correlation coefficient for the target strength of in-situ specimen for the purpose of decreasing *PD* and guarantee against the spatial variability of shear strength. Namely, the overdesign factor of 2.0 means that the target strength for in-situ specimen is twice of laboratory specimen to satisfy the safety of structure. Without the correction of a target strength (*PD* is 32.8% at F_o =1.0 in Figure 1), however, it is expected to induce the shortage of bearing capacity and local ground failure. Although a target strength is augmented by multiplying F_o with a design strength to increase *SR* and decrease *RD* in current design for cement-treated ground, optimum value of F_o and tolerance of *PD* are not fully understand to guarantee target bearing capacity under the high degree of spatial variability in shear strength by cement-mixing.

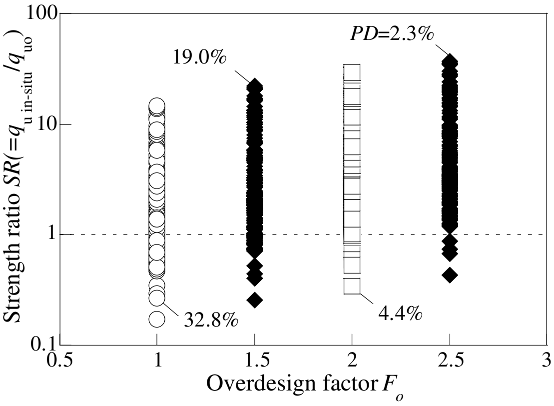


Figure 1. *SR*, F_o and *PD* in Pre-mixing construction project.

Table 1. Input parameters for current study.

Parameter	Selected Values
Mean undrained shear strength μ_c	100kPa
COV_c of undrained shear strength	0.2, 0.4, 0.6, 0.8, 1.0
Ratio of horizontal and vertical correlation length	1.0
Normalized correlation length Θ=θ/B	Random, 0.25, 0.5, 1.0, 2.0, 4.0
Monte Carlo iteration	1000

3 RANDOM FIELD NUMERICAL LIMIT ANALYSES

The Numerical Limit Analyses (*NLA*) used in this study were based on 2-D, plane strain linear programming formulations of the Upper Bound (*UB*) and Lower Bound (*LB*) theorems for rigid, perfectly plastic materials presented by Sloan & Kleeman (1995) and Lyamin & Sloan (2002). One of the principal advantages of Numerical Limit Analyses is that the true collapse load is always bracketed by results from the upper and lower bound calculations. For example, Ukritchon et al. (1998) were able to achieve estimates of the collapse for footings under combinations of vertical, horizontal and moment loading to an accuracy ±5% for a wide range of undrained strength profiles in the underlying clay. The effects of inherent spatial variability are represented in the analyses by modeling the cohesive strength of the cemented-treated ground, c_u, as a homogeneous random field. The cohesive strength is assumed to have an underlying log-normal distribution with mean, μ_c, and standard deviation, σ_c, and an normalized correlation length, $\Theta = \theta/B$ (θ: correlation length of shear strength, *B*: footing width). The use of the log-normal distribution is predicated by the fact that c_u is always a positive quantity. Current study uses mid-point Cholesky Decomposition method (e.g. Baecher & Christian, 2003) for representing inherent spatial variability of shear strength in Monte Carlo simulations. Noted that the results of Upper Bound Numerical Limit Analysis (*UBNLA*) are mainly shown in this paper.

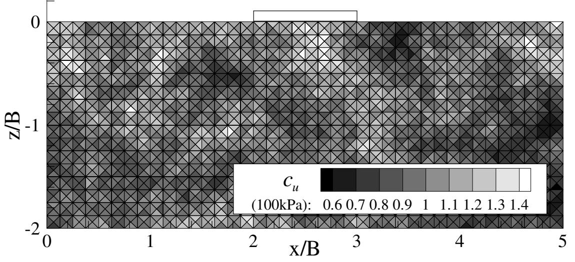
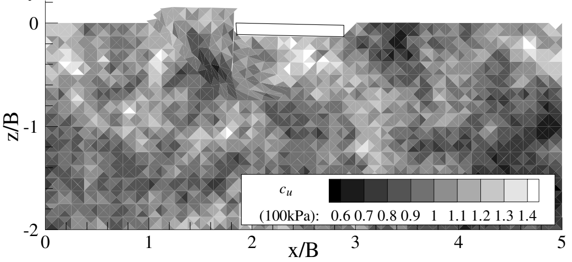
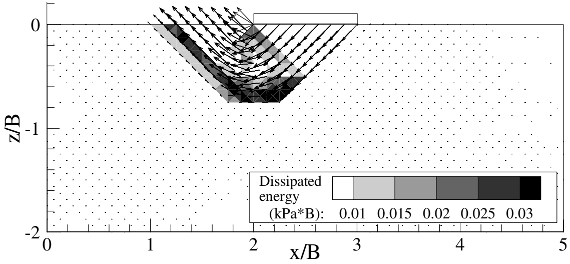


Figure 2. Typical finite element mesh used in *UB* numerical limit analyses (Θ=1.0, COV_c =0.2).



a) Deformed mesh.



b) Vectors of displacement and zone of plastic shear distortion.

Figure 3. Deformed mesh, vectors of displacement and zone of plastic shear distortion (Θ=1.0, COV_c =0.2).

Figure 2 shows typical finite element meshes used in the current *UBNLA* for surface foundations on cement-treated ground. In Figure 2, the spatial distribution of shear strength is shown for one example simulation with input parameters μ_c = 100kPa, COV_c ($=\sigma_c/\mu_c$) = 0.4 and Θ = 1.0. The lighter shaded regions indicate areas of higher shear strength. The model considers a soil layer with depth *z*/*B* = 2.0, where *B* is the width of the surface strip foundation under vertical loading. The soil is underlain by a rigid base, while far-field lateral boundaries of the mesh extend beyond the zone of all potential failure mechanisms. The analyses assume full improvement of soils with cement-mixing around the footing such that the zone of cement-treated ground extends to the boundary. The current simulations also assume that the cement-treated ground have the similar total unit weight. The sliding resistance at the soil-foundation interface is controlled by the shear strength of the cement-treated ground. Based on the literature review of the variability and correlation lengths for cement-treated ground, a parametric study has been performed using the ranges listed in Table 1.

4 STOCHASTIC BEARING CAPACITY

Figures 3a and 3b show the UB failure mechanisms against vertical loading for the initial UB mesh of Figure 2. Each figure shows the deformed mesh, vectors of the UB velocity field, zone of plastic shear distortion (dark zones within in velocity field). It can be seen that the computed failure mechanisms is no longer symmetrical and find paths of least resistance, passing through weaker regions of the cement-treated ground with active passive rigid body wedges under the foundation. It can be emphasized that it is important for the design of shallow foundation to examine the shear strength distribution around the foundation above the depth $z = B$ because the failure region does not appear below the depth $z = B$.

In order to evaluate the stochastic property of bearing capacity for cement-treated ground with spatial variability in shear strength, the computed bearing capacity factor N_{ci} can be reported for each iteration, i, of the shear strength field. Hence, the mean, μ_{Nc}, and standard deviation, σ_{Nc}, of the bearing capacity factor are recorded through each set of Monte Carlo simulations, as follows:

$$\mu_{Nc} = \frac{1}{n}\sum_{i=1}^{n} N_{ci} \; ; \; \sigma_{Nc} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(N_{ci}-\mu_{Nc})^2} \qquad (1)$$

Figure 4 shows a 25-bin histogram of the bearing capacity factor from one complete series of Monte Carlo simulations with n = 1000, Θ= 1.0 and COV_c = 0.4. It can be characterized that all of the UB simulations satisfy the χ^2 goodness-of-fit tests at a 5% significance level for both normal and log-normal distributions. The role of spatial variability in reducing the expected bearing capacity can be more conveniently seen in Figure 5, which reports the reduced mean bearing capacity ratio $R_{Nc} = \mu_{Nc}/N_{cDet}$, where N_{cDet} is the bearing capacity factor of the analytical Prandtl solution, $N_{cDet} = (2+\pi)$.

Figure 5 is the result of UBNLA. There are large reductions in R_{Nc} as COV_c increases for a given normalized correlation length, Θ, while the reduction rate of R_{Nc} increases with decreasing Θ. It can be characterized that the expected mean bearing capacity of cement-treated ground for typical coefficients of variation (COV_c = 0.4 – 0.8) is 50 – 80% of that expected capacity for an uniform strength ground.

5 RELIABILITY ASSESSMENT ON THE BEARING CAPACITY

In a conventional design for shallow foundation on cement-treated ground, an average undrained shear strength and simplified failure mechanism obtained by assuming uniform strength for ground has been used to estimate the ultimate bearing capacity. In the current calculations, considering the reduce in bearing capacity with spatial variability and increase in mean undrained shear strength by cement-mixing, the probability that the bearing capacity is less than N_{cDet} can be obtained by assuming that N_c can be described by a log-normal distribution. If N_c is log-normally distributed and multiplied by an overdesign factor F_o to increase an average undrained shear strength, the probability that the bearing capacity is less than N_{cDet} is given by:

$$p_r[F_o \times N_c < N_{cDet}] = \Phi\left(\frac{\ln([2+\pi]/F_o) - \mu_{\ln N_c}}{\sigma_{\ln N_c}}\right) \qquad (2)$$

where Φ(..) is the cumulative normal function, $\mu_{\ln Nc}$ and $\sigma_{\ln Nc}$ are mean and standard deviation of lnN_c obtained by following equations using $COV_{Nc} = \mu_{\ln Nc}/\sigma_{\ln Nc}$.

$$\sigma_{\ln N_c} = \sqrt{\ln(1+COV_{N_c}^2)} \qquad (3)$$

$$\mu_{\ln N_c} = \ln \mu_{N_c} - 0.5 \cdot \sigma_{\ln N_c}^2 \qquad (4)$$

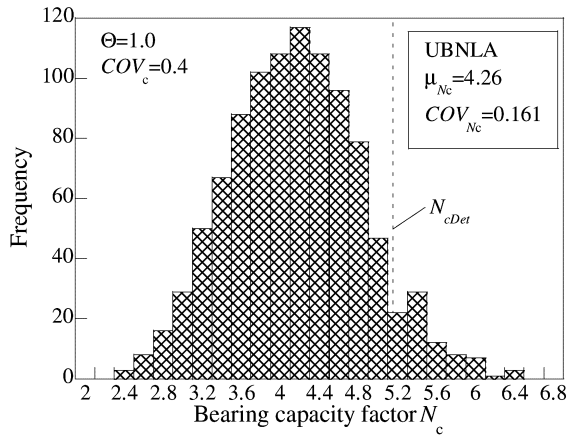


Figure 4. Histogram of bearing capacity factor (Θ=1.0, COV_c=0.4).

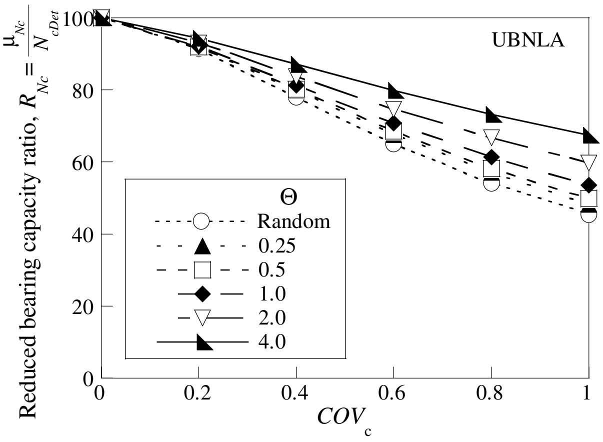


Figure 5. Reduced bearing capacity ratio R_{Nc} and COV_c.

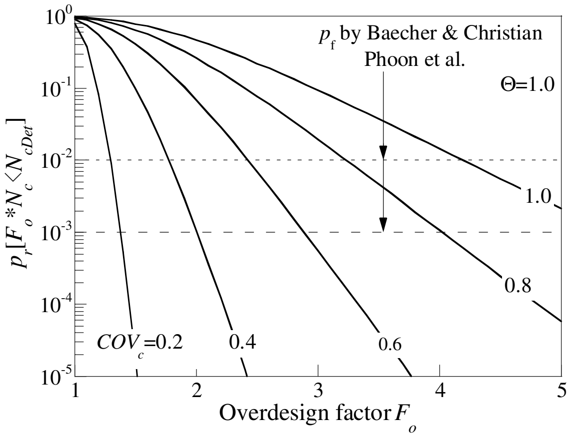


Figure 6. $p_r[F_o \times N_c < N_{cDet}]$ and F_o (Θ=1.0).

Figure 6 summarizes predictions that $p_r[F_o \times N_c < N_{cDet}]$ for Θ = 1.0 as a functions of F_o and COV_c. The target probabilities of failure considered in LRFD codes for shallow foundations are reported in the range, p_f = 10^{-2} – 10^{-3} (Baecher & Christian, 2003; Phoon et al., 2000). It can be seen that $p_r[F_o \times N_c < N_{cDet}]$ decreases sharply with increasing F_o irrespective of COV_c, while $p_r[F_o \times N_c < N_{cDet}]$ increases with increasing COV_c for a given F_o. $p_r[F_o \times N_c < N_{cDet}]$ also depends on values of Θ, however, the maximum probability is obtained at Θ = 1.0 in current analysis.

Figure 7 summarizes an optimum F_o for a given COV_c to satisfy $p_f = 10^{-3}$. It can be seen that optimum F_o increases with the increasing COV_c and Θ. It can be considered that cement-treated ground with a large degree of variability as well as $COV_c \geq 0.6$ and Θ = 1.0 needs a large F_o over 3.0 although F_o = 1.5 – 2.5 is appropriate for small spatial variability of COV_c = 0.2 – 0.4 similar to naturally deposited soils.

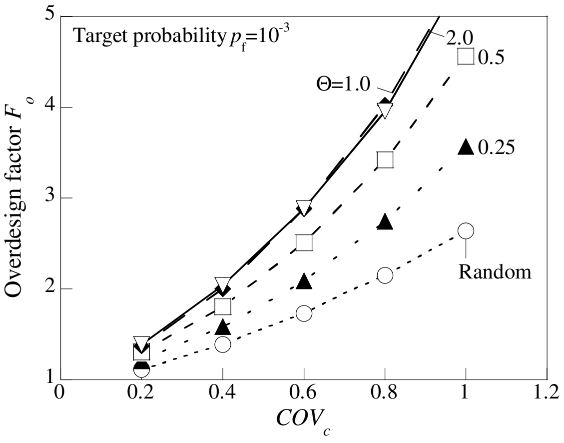


Figure 7. Optimum overdesign factor F_o against COV_c.

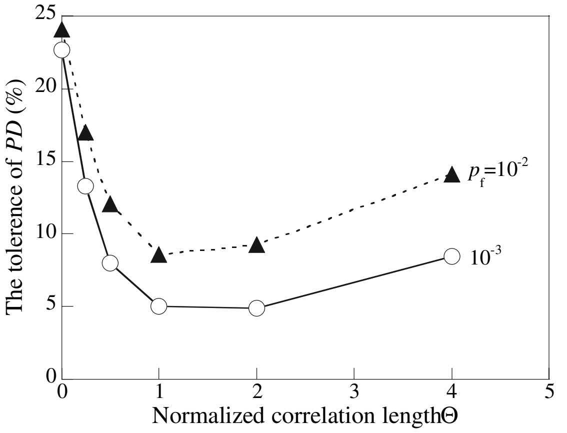


Figure 8. The tolerance of percent defect *PD*.

Figure 8 shows the tolerance of percent defective TPD to obtain p_f = 10^{-2} and 10^{-3} as a function of Θ. Noted that *TPD* is evaluated as minimum *PD* for a given Θ to obtain a target probability in current input parameter. The *TPD* for p_f = 10^{-2} and 10^{-3} decreases sharply for Θ < 1.0 and increases gradually for Θ > 1.0. For a small correlation length less than the width of foundation such a typical cement-treated ground, *TPD* of 5 – 10% is appropriate for a quality management in in-situ strength of cement-treated ground.

In order to extend the numerical results to LRFD code for cement-treated ground, a resistance factor ϕ_R for the bearing capacity of cement-treated ground is calculated assuming that the load component to a strip foundation on the cement-treated ground is modeled by log-normal distribution as follows (JGS, 2006):

$$\phi_R = \frac{1}{\sqrt{1+COV_{N_c}}}\exp(-\alpha\cdot\beta_T\cdot\sigma_{\ln N_c})\frac{R_{N_c}}{N_{cDet}} \qquad (5)$$

where α is a sensitivity factor to represent the ratio between variabilities of load component and resistance, and β_T is a target reliability index. Noted that the nominal resistance in Equation (5) is similar to N_{cDet}. Figure 9 shows a resistance factor ϕ_R for Θ = 1.0 and α = 0.8 against target reliability index β_T. It is seen that the resistance factor ϕ_R decreases with increasing target reliability index β_T and COV_c. For cement-treated ground with $COV_c \geq 0.6$, ϕ_R less than 0.5 is estimated for β_T = 2.5 – 3.5.

6 CONCLUSIONS

This paper has presented a reliability assessment for estimating the bearing capacity for cement-treated ground using numerical limit analyses with random field theory and Monte Carlo simulation. The main conclusions are as follows:

1) The bearing capacity factor of cement-treated ground considering the spatial variability of shear strength can be characterized by both normal and log-normal distribution functions with 5% significance level.
2) The expected mean bearing capacity for a typical coefficient of variation (COV_c = 0.4 – 0.8) is 50 – 80% of that expected capacity for an uniform strength ground.
3) Cement-treated ground for $COV_c \geq 0.6$ and normalized correlation length Θ = 1.0 needs a large overdesign factor F_o over 3.0 although F_o = 1.5 – 2.5 is appropriate for COV_c = 0.2 – 0.4. The tolerance of percent defective of 5 – 10% is suggested for a quality management in in-situ strength of cement-treated ground.
4) The resistance factor ϕ_R in LRFD code for the bearing capacity of cement-treated ground is proposed to obtain a target reliability index.

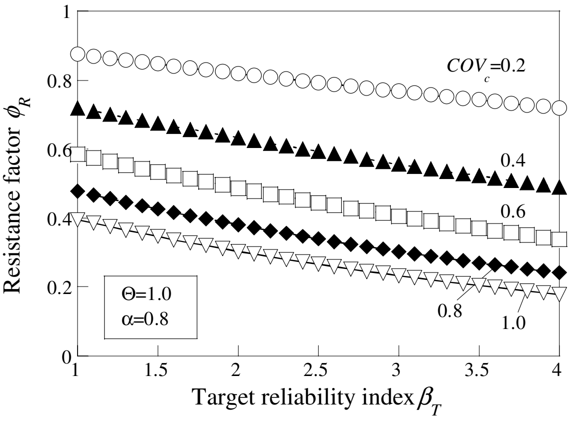


Figure 9. The tolerance of percent defect *PD*.

REFERENCES

Baecher, G.B., and Christian, J.T. 2003. Reliability and statistics in geotechnical engineering, John Wiley & Sons, Ltd., NY.

The Japanese Geotechnical Society. 2006. Principles for foundation designs grounded on a performance-based design concept, JGS 4001-2004.

Kasama, K., Zen, K., and Whittle, A.J. 2006. Effects of spatial variability of cement-treated soils on undrained bearing capacity, Proceedings of Numerical Modeling of Construction Processes in Geotechnical Engineering for Urban Environment, pp. 305-313.

Lyamin, A.V., and Sloan, S.W. 2002. Lower bound limit analysis using non-linear programming, International Journal for Numerical Methods in Engineering, Vol. 55, No. 5, pp. 573-611.

Matsuo, M. 1984. Geotechnical Engineering -Concept and Practice of Reliability-based Design-, Gihodo Shuppan, Ltd. (in Japanese).

Navin, M.P., and Filz, G.M. 2005. Statistical analysis of strength data from ground improved with DMM columns, Proceedings of Deep Mixing '05, Vol. 1.1, pp. 145-154.

Phoon, K.K., and Kulhawy, F.H. 1999. Characterization of geotechnical variability, Canadian Geotechnical Journal, Vol. 36, pp. 612-624.

Phoon, K.K., Kulhawy, F.H., and Grigoriu, M.D. 2000. Reliability-based design for transmission line structure foundations, Computers & Geotechnics, Vol. 26, pp. 169-185.

Sloan, S.W., and Kleeman, P.W. 1995. Upper bound limit analysis using discontinuous velocity fields, Computer Methods in Applied Mechanics and Engineering, Vol. 127, pp. 293-314.

Terashi, M., and Tanaka, H. 1981. Ground improved by deep mixing method, Proceedings of the 10th ICSMFE, Stockholm, Vol. 3, pp. 777-780.

Ukritchon, B., Whittle, A.J., and Sloan, S.W. 1998. Undrained limit analyses for combined loading of strip footing on clay, Journal Geotechnical Engineering, ASCE, Vol. 124, No. 3, pp. 265-276.

Zen, K., Yamazaki, H., Yoshizawa, H., and Mori, K. 1992. Development of premixing method against liquefaction, Proceedings of the 9th Asian Regional Conference SMFE, Vol. 1, pp. 461-464.