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# Pareto principle and sensitivity of soil-footing-superstructure system

## Le principe de pareto et sensivite de system sol-fodation-superstructure

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### ABSTRACT

V. Pareto principle 80/20 (1897), generalized by J.M. Juran (1941), reads “80% of effects are due to 20% of causes, 80% of causes generate 20% of effects”. Site survey data scatter, idealization of soil and soil-footing interaction make conservative decisions inevitable in geotechnical engineering, because the cost of risk is very high. The way to control this conservatism is to evaluate and to rate qualitative regularities of Soil-Footing-Structure System (SFSS) behavior by means of computer simulation of simplified SFSS virtual models. About 10,000 numeric experiments were carried out and 40,000 diagrams of results were obtained for the whole practical range of input data. The results were rated for 84 input-output data pairs (cause-effect) and tabulated (Table 1). Some of the these pairs show that the output result is very sensitive to variations of the input parameter, and such pairs were rated 2, if such influence is negligible, than the pair was rated 0 and may be ignored, the intermediate cases were rated 1. It was shown that the overall Pareto ratio is close to 80/20.

### RÉSUMÉ

Principe de V.Pareto 80/20 (1897), generalisee par J.M.Juran (1941) dit que “80% des effets sont produites par 20% de causes, et 80% des causes produites 20% des effects”. Dispersions des parameters des sols, idealization des interactions de sol-fondation commnandent toujours solutions conservatives, qui sont inevitables en geotechnique, par ce que le valeur de risqué est tres haut. La methode de controle de ce conservatism est de evaluer les regularitees qualitatifs de comportement de System Sol-Fondation-Stucture (SSFS) par simulations en ordinateur de modeles virtuelles de SSFS. Pour le faire, 10,000 essais numeriques etaient faites et 40,000 diagrams graphique etaient obtenus pour toute les valeurs pratiques des donnees de reference. Les resultats etaient evaluees pour 84 paires de donnees-resultats (cause-effet) et etaies entrees en une table (Table 1). Quelques paires montrent que une resultat et tres sensitive au variations d’une donnee, et ces paires etaies evaluees comme 2, l’influence negligible etais evaluee comme 0, l’influence intermedie non-negligible etait evaluee comme 1. Le sommation respectif montrait que le proportion de Pareto est tres proche a 80/20.

Keywords: soil-footing-superstructure, sensitivity, numeric experiements

## 1 INTRODUCTION

Medieval philosopher William Occam (1285-1349) wrote: “What can be done with fewer assumptions is done in vain with more”. In 1897 Italian mathematician Vilfredo Pareto formulated his principle 80/20. Its more up-to-date definition belongs to J. M. Juran (1941): “80% of effects are due to 20% of causes, 80% of causes generate 20% of effects”. The principle challenges conventional logic i.e., “all effects are equally due to all causes”. 80 and 20 are not exact physical values, and ratio 80/20 might vary, still the respective practical rule: “essential factors are few, while trivial factors are many” is very realistic. Such asymmetry is the inherent property of cause-effect links in complex systems. Similar asymmetry pertains to Soil-Footing-Structure System (SFSS) behavior. Virtually, variations of input data (“causes”) generate variations of SFSS analysis output results (“effects”). Virtual SFSS can be sensitive or robust to different input data variations. Underestimation or overestimation of cause-effect links leads either to conservative or to unsafe design. Site survey data features high rate of uncertainty, soil theoretical models are approximate, therefore, conservative solutions prevail in geotechnical engineering, because the value of risk is very high. It is worthwhile to assess the above uncertainty by numeric simulation in terms of SFSS sensitivity/robustness and to identify essential cause-effect links. The paper presents qualitative evaluation of about 10,000 numerical tests of virtual SFSS, represented by a simplified virtual model, consisting of soil base, raft footing, supports (columns) and superstructure, subjected to uniform load in plain strain. Exact analytical solution of the problem was obtained, which makes it possible to avoid “noise” of numerical methods (FEM). The solution was coded in MathCad. There were investigated input-output (cause-effect) pairs. The results were rated in accordance with SFSS sensitivity. Particularly, it was found that bending moments and shear forces in the raft footing

are very sensitive to soil behavior under footing edges and are robust to soil base stiffness. Quantitative ratios of Pareto type were obtained for input-output data pairs.

## 2 VULNERABILITY, ENSITIVITY, OBUSTNESS, DAPTATION AND RISK MANAGEMENT.

These concepts are broadly applied to describe behavior of various systems (power grids, telecommunications, climatology, environment, etc.). They play different roles in specific disciplines. Geotechnical project design is always conservative, because it focuses on SFSS safety and compliance with construction codes rather than optimization. observational method, introduced by K.Terzaghi and further developed by P. Peck and A. Powderham [1], introduces risk management during construction period. New data is obtained at subsequent stages of construction with the project being redesigned and updated on-line to take into account on-line events and data, registered by monitoring. Risk management in anomalous situations with the structure being already in service, requires very labor-intensive or even impossible emergency and/or repair operations. For example, in the city of Santos (Brazil) dozens of high-rise buildings have inadmissible tilts [2]. If any preventive systems to adapt structures during service period had been included in the project design the tilts could have been corrected effectively. Risk management could be facilitated if qualitative features of SFSS behavior – its sensitivity i.e., response of the system to various input data were evaluated [3]. Monitoring data provides a basis for realistic evaluation of SFSS behavior, but the data is obtained after events take place, such data could be really prognostic if complemented by numerical simulation. The prognosis can be done using simplified SFSS numerical models - primitives, which both reflect the essential generic features of real structures behavior

and are as simple as possible for exact mathematical solutions to be applied and translated into fast computer codes, free of numeric “noise”. Such codes shall be fast enough for performing many computer experiments to evaluate SFSS sensitivity. Similar primitives were already applied to analyze SFSS sensitivity [5,6]. In spite of low capacity of the then computers quite unexpected results were obtained [7]. Such analysis can be very effective for optimizing design solutions, for developing normative codes, for interpreting experimental and monitoring data, for training practical intuition and for developing corrective measures.

### 3 CONTACT MODELS (CM) AND SFSS PRIMITIVES

SFSS design analysis is based on application of contact models (CM), defined by a Green function or by differential operator [8]. In FEM terms CM is a super-element, simulating behavior of the ground at the interface with SFSS. CMs have been verified and calibrated by monitoring data and are included in Russian construction codes [10,11]. Boundary elements method is based on CMs.

The simplest Winkler one-parameter CM does not account for lateral ground distribution capacity while two-parametric (C1 and C2) CMs (Pasternak model and elastic layer) are distributive. But introduction of the second parameter results in singularities of contact pressures under footing edges, which are characteristic of Fredholm integral equations of the first type. These singularities give rise to instability of mathematical solutions (Fig. 1, 2). But these singularities do not exist in reality because of disruptions of soil that propagate to certain depth under footing edges, usually described as “plastic zones”, but should be better called “cuts”. The disruptions prevent interaction of the top layer of the ground under the footing with the ground outside the raft, hence, this top layer can be simulated by a Winkler layer, having stiffness C3. Thus, a 3-parametric CM (CCC) is obtained, which does not produce any singularities under footing edges, and its behavior is defined by Fredholm integral equation of the second type, for which contact pressures are limited [12]. The parameters of CCC can be defined as follows:

$$C_1 = \frac{E}{H(1-\nu^2)}; \quad C_2 = \frac{E \cdot t}{H(1-\nu^2)}; \quad C_3 = \frac{E}{H_0(1-\nu^2)}; \\ H_0 = \frac{p - \gamma h}{\pi \gamma} (ctg \phi + \phi - \frac{\pi}{2}) - c \cdot ctg \phi - h, \quad (1)$$

where  $E, \nu, \gamma, c, \phi$  – soil mean deformation modulus and Poisson ratio, soil weight density, cohesion and angle of internal friction,  $H$  – depth of compressible zone;  $H_0$  – depth of soil disruptions (plastic zone),  $p$  – contact pressure under raft edge,  $h$  – depth of the raft bottom from the ground surface. (Note: these parameters are given for homogeneous soil base, for heterogeneous soil they may depend on coordinates).

Non-uniform Winkler CM is often used, because it is easily dovetailed with FEM. Distribution  $C=C(x,y)$  is calculated with the help of an iterative procedure, adjusting  $C(x,y)$  to a known distributive CM (this procedure is known as Schwarz algorithm). For two-parametric CM CC the iterations do not converge, but they are converged after one or two iterations for three-parametric CM CCC. So the latter is best suited for SFSS analysis.

In order to simulate SFSS behavior virtually, a “primitive” in plain strain conditions is used here, consisting of CM CCC, simulating the ground, on top of which sits a structure, consisting of a plate – raft footing, supporting a superstructure, simulated by yet another plate on compressible supports. The load is applied on top of the upper plate (Fig. 3).

Firstly, a solution was obtained for settlements  $S=S(x)$  of a  $2a$  wide plate, having  $D$  bending stiffness, in plain strain conditions. The plate sits on CM CCC. Load  $q=q(x)$  is applied to the

plate. This problem can be expressed as the following system of equations:

$$DS^{IV} = q - p; \quad C_3(S - V) = p; \quad C_1V - C_2V'' = p \quad \text{if } -a \leq x \leq a \\ C_1V - C_2V'' = 0 \quad \text{if } x < -a \text{ or } x > a \quad (1)$$

where  $p=p(x)$  contact pressure distribution,

$V=V(x)$  – settlement function of the lower layer (CM CC).

By excluding  $S$  in (1), we obtain the following 6th order ordinary differential equation:

$$D \frac{C_2}{C_3} V^{VI} - D(\frac{C_1}{C_2} + 1) V^{IV} + C_2 V'' - C_1 V = -q \quad (2)$$

$$\text{with } V(\pm a \pm 0) = V(\pm a \mp 0); \quad V'(\pm a \pm 0) = V'(\pm a \mp 0); \\ S''(\pm a) = 0; \quad S'''(\pm a) = 0$$

We find the respective Green function  $G=G(x,f)$ , which is the solution for a unit point force  $q=\delta(x,f)$  applied to the plate at point  $f$ , where  $\delta$  is Dirac delta-function.  $G$  can be found from continuity conditions of functions  $S, V, S', V', S'', V'', S'''$  and discontinuity condition

$$S'''(f-0) - S'''(f+0) = \frac{1}{D} D \text{ at point } f.$$

The solution of the problem (2) has been coded in MathCAD. The computer code is very fast to perform many numerical simulations of the virtual SFSS behavior (~10,000 numerical tests with ~40,000 graphs), which were necessary to obtain general qualitative conclusions of SFSS behavior.

### 4 SFSS SYSTEM NUMERICAL SIMULATION RESULTS

The following input parameters have been varied:  $E, \nu, \phi, c, H, H_0$ , raft width and thickness, compressibility of supports and their spacing, distance from the extreme support to the raft edge (cantilever), upper structure stiffness, staged growth of the structure during erection period.

An example of SFSS analysis is given on Fig. 1, where profiles of bending moments in the raft are given for the following cases:

- No upper structure and uniformly distributed load,
- Upper structure of finite stiffness,
- Absolutely stiff upper structure.

The graphs show that absence of upper structure results in considerable change of the bending moments profile. But presence of even relatively moderate-stiffness upper structure changes the profile considerably, and the raft behaves practically as that with infinitely stiff upper structure. In the absence of the upper structure and with short cantilever the raft arches up, with long cantilever the raft sags down.

The results of numeric simulation help assess sensitivity of SFSS raft to variations of a large spectrum of input parameters. Here are the most important results:

- plastic zones under footing edges and stiffness of the upper structure make SFSS robust to variations of input parameters within their practical range, and even raft behavior does not depend on the soil base model;

- effect of structure staged growth during erection on raft behavior is only noticeable if it is taken into account that the cut (plastic zone) depth increases with the structure growth.

The obtained results are partially presented in Table 1, in which sensitivity of SFSS is rated: 0 means that influence of input factor in line  $i$  on output result in column  $j$  is low, and such influence may be neglected; 1 means that influence  $ij$  is essential and shall be taken into account, 2 means that the influence is very high. The main purpose of Table 1 is to show that SFSS is robust to many input parameters i.e., the values of these parameters may be very approximate.

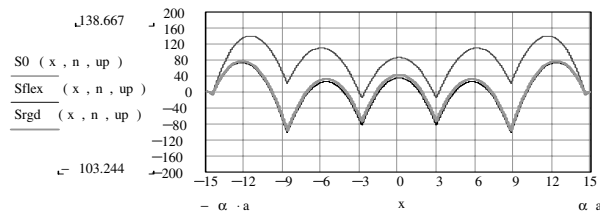


Fig. 1. Comparison of bending moments profiles in raft with no upper structure (S0), with upper structure of finite stiffness (Sflex) and infinite stiffness (Srgd)

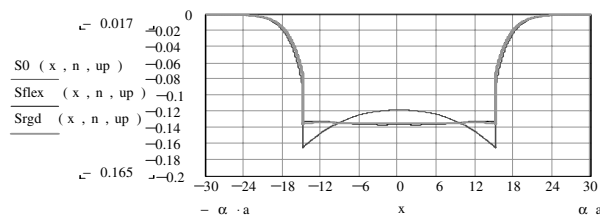


Fig. 2. Comparison of settlements profiles. The notations are the same as on Fig. 1. Coincidence of profiles Sflex and Srgd and diversion of profile S0 (arching up).

Table 1 Sensitivity rating SFSS

Variations of input data	Variations of output data					
	Mean settlements	Deflections	Tilts	Bending moments		Shear forces
				-	+	
Soil deformation modulus $E$	1	1	1	0	0	0
Soil strength parameters $c, \phi$	1	1	1	1	1	1
Footing depth $h$	1	1	1	1	1	1
Soil base distribution capacity ( $C_2$ ) in real range	0	0	0	0	0	0
Soil base heterogeneity	0	1	1	1	1	1
Cantilever	0	1	1	2	2	2
Stiffness of supports	0	0	0	0	0	0
Footing bending stiffness $D$	0	1	1	0	1	1
Relative stiffness of superstructure строения						
$Ds/D < 5$	0	1	1	1	1	1
$5 \leq Ds/D < 20$	0	1	1	0	0	0
$Ds/D \geq 20$	0	0	0	0	0	0
Structure growth during erection, linear deformations of soil base	0	0	0	0	0	0
Structure growth during erection, non-linear deformations of soil base	1	0	0	1	1	0
Nearby construction activity	0	0	1	1	1	2
Pareto ratio	86/14	72/28	68/32	75/25	68/32	68/32

The last line of Table 1 shows Pareto-Jordan ratios, characterizing the influence of input data (causes) on output data (effects). These percentage ratios of the ratings sum in a column to the maximum sum of ratings equal to 28. The greater is the value of such ratio the greater is the number of negligible factors and the less the result (effect) is dependent on the errors of these input data combinations. The ratios in the last line are close to ratio 80/20.

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