Influence of random soil parameters on seismic vibration of extended structures
Influence des paramètres aléatoires du sol sur les oscillations sismiques des bâtiment oblong

M.L. Kholmyansky
NIIOSP Research Institute, State Research Centre "Civil Engineering", Moscow, Russian Federation

ABSTRACT
The influence of random soil dynamical parameters on seismic vibrations of extended structures is studied. The excitation is modeled by vector random process. Soil dynamic parameters are supposed random variables stochastically independent on random excitations. The problem is solved as linear. General equations are given. A simple specific case is being considered: system with two elastically connected masses, each of them is situated on soil. The numerical results obtained show that accounting for random soil parameters may alter the system response significantly.

RÉSUMÉ
L'influence des paramètres dynamiques aleatoires sur vibrations des ouvrages etendues est analysee. L'excitation est simulees par un vecteur aleatoire de processus. Les paramètres dynamiques du sol sont prises stochastiquement indépendants des excitations aleatoires. Le problème est resolue comme lineaire. Les équations generales sont presentees. En cas simple est analysee: un système avec deux masses liees elastiquement, chacune reste sur sol. Les resultats numeriques obtenuees montrent que le compte tenue des parametres dynamiques de sol peut changer la réponse considerablement.

Keywords : soil dynamics, earthquake engineering, random soil parameters, random processes, extended structures

1 INTRODUCTION

The influence of soil-structure interaction on vibration under seismic excitation was studied in many works. Since the seismic excitation is indeterminate to a high degree it was often described as random processes (Newmark & Rosenblueth 1971). The paper considers seismic excitation as random process with many components.

Randomness of parameters of vibrating system itself must be considered also. Some results are found in the literature.

Discrete system with Rayleigh damping representing machine foundation was supposed having random parameters with given distribution and small coefficients of variation; response correlation function was obtained using modal decomposition (Mironowicz & Śniady 1987).

Numerical results were obtained for spectral characteristics of response for single-degree-of-freedom vibratory systems with stiffness and damping being random variables with uniform distribution under stationary random excitation (Kotulsky & Sobczyk 1987).

Udwadia (1987) considered single-degree-of-freedom oscillator under force and kinematic random excitation with its parameters being random variables. For the stationary random excitation closed form expression for the response power spectral density function was found. Numerical examples showed that the system parameters randomness leads to significant response randomness.

Numerical results were obtained by Chaudhuri & Gupta (2002) for structural response when shear wave velocity and Poisson's ratio of the soil were assumed to be statistically independent variables with Gaussian distributions. Rieck and Houston (2003) provided numerical results for nuclear reactor containment building on soil with random properties under earthquake excitation (stationary random process).

General results were obtained (Kholmyansky 1997; Kholmyansky 2000) within the framework of Bolotin (1984); soil randomness was taken into account and problems concerning vibration of soil masses (Kholmyansky 2003) and structures (Kholmyansky 2007) were solved. It was supposed (Kholmyansky 2003; Kholmyansky 2007) that seismic excitations are equal and in-phase along all the structure or soil mass and may be described with a single random process.

Yet existing methods for soil-structure interaction calculation may account for spatial variability of seismic vibrations leading to different random excitations for bridge piers (Tseng & Penzien 2003), random properties of soil itself are not accounted for.

Some approaches to improve that situation were obtained recently (Kholmyansky 2008); the present paper is devoted to their illustration and development: the problem of vibration of an arbitrary structure interacting with soil with random dynamical parameters under several scalar random excitations (or single vector excitation) is stated and solved.

2 DESCRIPTION OF GENERAL RESULTS FOR SYSTEM WITH RANDOM PARAMETERS UNDER NON-STATIONARY AND STATIONARY RANDOM VECTOR EXCITATION

2.1 *Problem statement*

The periods of alteration of properties of soils and structural materials are some orders of magnitude greater than the periods of natural vibration of structures, so system parameters are assumed to be time invariant (random variables).

Spatial variation of soil dynamic properties may be considering indirectly (by increasing the overall parameters of scatter for soil). Soil variability is of substantial level, so perturbation method will not be used. The consideration is limited to dynamic loads of moderate level leading to linear behaviour.

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Input excitations $x_j(t)$ are regarded as random processes with zero mean (either a force or a foundation displacement). The random processes $x_j(t)$ and the system random parameters are assumed to be stochastically independent. Soil-structure system may be considered having multiple responses also (displacements, accelerations, forces etc.).

The system is supposed linear, so general approaches (Bendat & Piersol 1980) for systems with multiple inputs (excitations) and multiple outputs (responses) may be used. The responses y(t) and z(t) may be written

$$y(t)=\sum_{j=1}^{n}\int_{-\infty}^{\infty} g_j(\tau)x_j(t-\tau)\,d\tau\,,\quad z(t)=\sum_{k=1}^{n}\int_{-\infty}^{\infty} h_k(\tau)x_k(t-\tau)\,d\tau \quad (1)$$

where $g_j(t)$, $h_k(t)$ are the weighting functions (for physically realizable system $g_j(t) = h_k(t) = 0$ for $t < 0$), n is the number of inputs and t — the time. It is easy to see that y(t) and z(t) are zero-mean random processes.

2.2 *Non-stationary excitation: time domain*

Cross-correlation function for y(t) and z(t) is obtained easily from independence of system parameters and excitations:

$$K_{yz}(s,t)=\sum_{j=1}^{n}\sum_{k=1}^{n}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{K_{g_j h_k}(\sigma,\tau)}K_{x_j x_k}(s-\sigma,t-\tau)\,d\sigma d\tau \quad (2)$$

where lower indexes of K denote random processes (whose cross-correlation function is being calculated). Further, cross-correlation function for y(t) and $x_l(t)$ is

$$K_{yx_l}(s,t)=\sum_{j=1}^{n}\int_{-\infty}^{\infty}\overline{g_j(\sigma)}K_{x_j x_l}(s-\sigma,t)\,d\sigma \quad (3)$$

where overbar means expected value. For the special case of systems with one input and with z = y Equation 2 was obtained earlier (Pugachov 1962).

2.3 *Stationary excitation: frequency domain*

If all $x_j(t)$ are stationary random processes then y(t) and z(t) are stationary too. Power spectral density functions (PSD) describing vibration energy distribution among the vibration angular frequencies ω are convenient in that case. They are obtained from correlation functions by Fourier transformation (Bendat & Piersol 1980) giving

$$S_{yz}(\omega)=\sum_{j=1}^{n}\sum_{k=1}^{n}T_{g_j h_k}(\omega,-\omega)S_{x_j x_k}(\omega) \quad (4)$$

where S is PSD with lower indexes denoting corresponding random processes and

$$T_{x_j x_k}(\lambda,\mu)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{K_{g_j h_k}(\sigma,\tau)}\varepsilon^{-i(\lambda\sigma+\mu\tau)}d\sigma d\tau \quad (5)$$

Equation 5 may be transformed to use random transfer functions $G_j(\omega)$ and $H_k(\omega)$ directly:

$$T_{x_j x_k}(\lambda,\mu)=\overline{G_j(\lambda)H_k(\mu)} \quad (6)$$

That makes possible to avoid analysis in time domain and use spectral properties only.

PSD for structures on soil with uncertain (random) properties of both soil and structures were calculated by Mochio et al. (1992) using stochastic finite element method and first-order second-moment method; for rather low level of coefficient of variation 0.1 the obtained results are confirmed by Monte Carlo method. The equation obtained by Udwadia (1987) for single-degree-of-freedom system is equivalent to Equation 4.

3 A SOIL-STRUCTURE INTERACTION MODEL FOR MULTIPLE SUPPORT EXCITATION OF TWO-MASS STRUCTURE

As it is well-known waves travelling in soil approach separate foundations of a long-span structure with some phase lag (and sometimes with amplitude difference). This process is shown on Fig.1 with soft soil layer and underlying bedrock. The two supports of the structure are distant, so there is no cross-interaction of their foundations through soil.

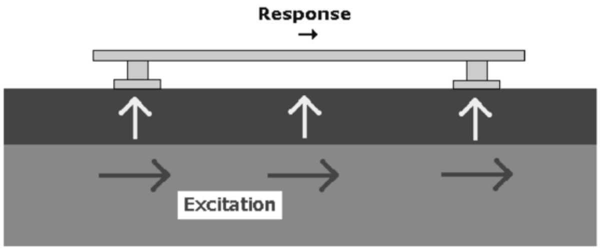

Figure 1. Seismic excitation travelling trough rocks and soft soils to an extended structure

To analyse such a problem a model of soil-structure system with two degrees of freedom on two separately moving supports is applied (see Figure 2). Spring and damping constants for the soil bases are k_j and b_j

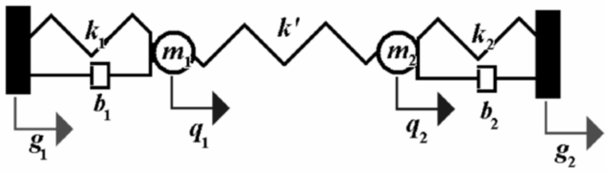

Figure 2. Two-mass structure with two supports on soil

The inputs x_1 and x_2 are free-field displacements g_1 and g_2; the output (response) is the force

$$z=f=k'(q_2-q_1) \quad (7)$$

The resulting transfer functions are

$$H_j(\omega)=(-1)^j\frac{1}{D(\omega)}\frac{p_j(\omega)}{s_j(\omega)}\,, \quad (8)$$

where

$$p_j(\omega)=k_j+i\omega b_j\,,\ D(\omega)=1/k'+1/s_1+1/s_2\,,\ s_j(\omega)=p_j-\omega^2 m_j \quad (9)$$

From now on we shall omit g denoting excitation (input) and second y in indexes. For example, Equation 4 becomes

$$S_y(\omega)=T_{11}(\omega)S_1(\omega)+T_{22}(\omega)S_2(\omega)+2\,\mathrm{Re}\left(T_{12}(\omega)S_{12}(\omega)\right) \quad (10)$$

4 PROBABILISTIC PROBLEM STATEMENT

4.1 *Random excitations*

Input stationary excitations are chosen to have well-known Kanai-Tajimi spectrum (Kramer 1996) with predominant ground frequency ω_g and damping ratio ζ_g:

$$S_1(\omega) = S_2(\omega) = S_0 \frac{1 + 4\zeta_g^{\,2} \left(\omega/\omega_g\right)^2}{\left[1 - \left(\omega/\omega_g\right)^2\right]^2 + 4\zeta_g^{\,2} \left(\omega/\omega_g\right)^2} \tag{11}$$

The cross-spectral density $S_{12}(\omega)$ may vary.

Consider first the symmetric system ($m_1 = m_2$, $k_1 = k_2$, $b_1 = b_2$). It is easy to obtain that $T_{11}(\omega) = T_{22}(\omega) = -T_{12}(\omega)$. Specify three cross-spectral densities.

(a) Excitations are identical: $g_1(t) = g_2(t)$, $S_{12}(\omega) = S_1(\omega)$. In that case the mass displacements are identical and response (force) spectral density is zero,

$$S_y(\omega) = 0 \tag{12}$$

(b) Excitations are non-correlated: $S_{12}(\omega) = 0$, whence

$$S_y(\omega) = 2T_{11}(\omega)S_1(\omega) \tag{13}$$

(c) Excitations have deterministic time lag: $g_2(t) = g_1(t-\tau)$. The excitation cross-spectral density $S_{12}(\omega) = S_1(\omega)\exp(-i\omega\tau)$ and

$$S_y(\omega) = 2T_{11}(\omega)S_1(\omega)[1-\cos(\omega\tau)] \tag{14}$$

Cases (a) and (b) may be considered as limiting special cases of case (c).

From now on only the excitations with deterministic time lag will be considered. This assumption is one of the simplest and corresponds to deterministic bedrock properties and thin soft soil layer.

4.2 *Soil random properties*

Soft soil layer properties determine stiffness and damping of the soil bases. The simplest supposition is made that its random soil dynamical properties are not varying in space. Soil elastic modulus is therefore a random variable; the density is taken a deterministic constant since its random variability is significantly less. Each soil base is described by stiffness and damping — random variables determined by random soil elasticity modulus.

Soil base stiffness is proportional to soil elasticity modulus, while the damping is proportional to the square root of the modulus (Richart et al. 1970; SNiP 1988):

$$k_j = E \times \text{const}\,,\ \ b_j = \sqrt{E} \times \text{const} \tag{15}$$

Since that damping ratios are deterministic.

Elasticity modulus distribution is supposed uniform for simplicity. The coefficient of variation is 0.29 (Kholmyansky 2000) as it follows from the experimental data obtained by Barkan et al. (1974).

5 STUDYING THE INFLUENCE OF RANDOM SOIL PARAMETERS ON STRUCTURAL FORCE

PSD functions corresponding to the system with random parameters (solid line) and deterministic system (dotted line) are shown in Figure 3.

Here k_j have mean values 1 and 1.5; $k' = 0.1$, $m_1 = 1$ and $m_2 = 2$ are deterministic. Damping ratio for each partial system is 0.1 (determined under supposition of no interaction between masses); ground damping ratio is 0.1. Predominant ground frequency is twice the system partial frequency for the first mass. Excitations time lags are 0, 2, 7 and 20.

Fig. 3 shows significant effect of soil randomness. It is seen that the soil stiffness randomness lowers the resonant peak, but increases to some extent the response in its neighbourhood.

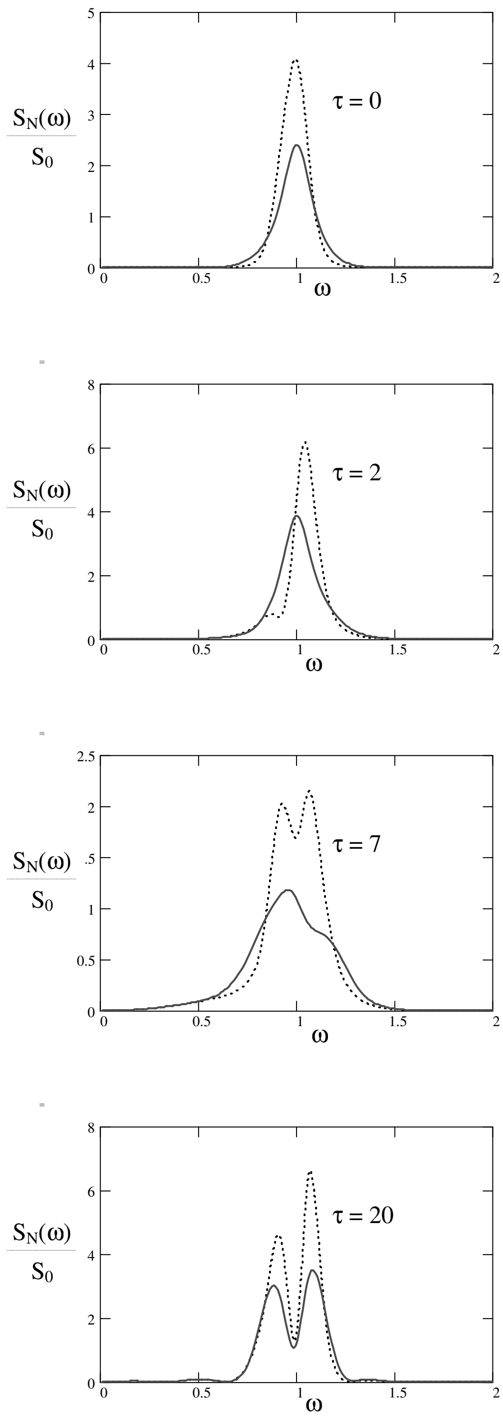

Figure 3. PSD functions for soil-structure system responses with deterministic parameters (dotted line) and with random parameters (solid line) for different time lags

4 CONCLUSIONS

A calculation method of seismic vibration for extended structures interacting with soil with account for soil parameter randomness was developed.

The main results are obtained for the stationary random excitation with multiple components (vector excitation). Different deterministic and stochastic interdependences between the components were considered.

Soil randomness was accounted for in a simple way making it possible to obtain numerical results easily. Possible significant influence of soil random properties on earthquake response was shown.

The results obtained seem promising and showing the need for further research. The main tasks for future investigations are problem statement examining and refining along with proper input data obtaining.

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