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# Treatment of geoseismic data as a non-stationary process

## Traitement des données geoseismic comme processus non stationnaire

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### ABSTRACT

The traditional Fourier-spectra-based tools do not adequately capture the evolutionary and localized features of the natural-systems responses. When these systems are subjected to seismic loads the Fourier analysis may misinterpret the information due to the time-variation of frequency characteristics in non-stationary processes. This study explores the use of the Hilbert-Huang Transform for analyzing earthquake recordings and the associated dynamic-soil behavior. The HHT, integrated by the Empirical Mode Decomposition and the Hilbert Transformation, is an empirical based data-analysis method with an adaptive basis of expansion that can produce physically meaningful representations of data from nonlinear and non-stationary processes. Hilbert-Huang Transform enables engineers to analyze non-stationary oscillation systems and to obtain more detailed intensity descriptions on time-varying frequency diagrams. HHT is used in this work to examine responses of soft-soils deposits in Mexico City. The results indicate that the proposed methodology is able to extract some motion characteristics useful in geoseismic studies which are not properly seen when are employed conventional data processing techniques.

### RÉSUMÉ

Les outils Fourier-spectre-basés traditionnels ne capturent pas en juste proportion les dispositifs évolutifs et localisés des réponses de systèmes naturels. Quand ces systèmes sont soumis aux charges sismiques l'analyse de Fourier peut mal interpréter l'information en raison de la variation temporelle des caractéristiques de fréquence dans des processus non stationnaires. Cette étude explore l'utilisation du Transformation Hilbert-Huang pour analyser des enregistrements de tremblement de terre et le comportement associé de dynamique au sol. Le HHT, intégré par la Décomposition Empirique de Mode et la Transformation de Hilbert, est une méthode basée empirique avec une base adaptative d'expansion qui peut produire des représentations physiquement significatives des données à partir des processus non linéaires et non stationnaires. Le Transformation Hilbert-Huang permet à des ingénieurs d'analyser des systèmes non stationnaires d'oscillation et d'obtenir des descriptions d'intensité plus détaillées sur des diagrammes de fréquence and temps variables. HHT est employé dans ce travail pour examiner des réponses de dépôts de sols meubles à Mexico. Les résultats indiquent que la méthodologie proposée permet extraire certaines caractéristiques de mouvement utiles dans les études géosismiques qui ne sont pas correctement observées lorsque des techniques de traitement de données conventionnelles sont utilisées.

Keywords : Hilbert-Huang Transform, Seismic data analysis, Site amplification, Nonlinear soil behavior

## 1 INTRODUCTION

Spectral analysis using the Fourier Transform has been one of the most important and most widely used tools for studying the vibrations of the soils. Over the past few years, however, researchers have become aware of the limitations of this technique, especially in seismic records that manifest clearly nonstationary characteristics.

The Fourier Transform and its digital analogue, the Fast Fourier Transform FFT carry strong a-priori assumptions about the source data, such as linearity and stationariness. Natural phenomena measurements are essentially nonlinear and nonstationary. The accommodation of this fact in FFT-based analysis often involves using more data samples to assure acceptable convergence and nonalgorithmic procedural steps in the interpretation of FFT results. Wavelet-based analysis may yield some improvement over the FFT because it can handle nonstationary data, but it retains the limitation of requiring the data set to be linear. Wavelet methods may also prove inadequate because, although wavelet is well-suited for analyzing data with gradual frequency changes, its non-locally adaptive approach causes leakage. This leakage can spread frequency energy over a wider range, removing definition from data and giving it an overly smooth appearance.

A recent development, the Hilbert-Huang Transform HHT (Huang et al., 1998), provides a novel approach to the solution

of the nonlinear class of problems previously in the domain of FFT. The HHT allows direct algorithmic analysis of nonlinear and nonstationary data functions by using an engineering and a posteriori data processing method, namely an Empirical Mode Decomposition EMD. Using HHT results in the unconstrained decomposition of the source data function into a finite set of Intrinsic Mode Functions IMFs that can be completely analyzed by the classical Hilbert Transform.

This study proposes the use of the HHT for analyzing and characterizing the non-linear soil behavior from seismic recordings. The method is applied to earthquake ground motions monitored in the lake zone of Mexico City. Results show that the HHT gives a clear description of the time-frequency energy content, and prove the validity and its great potential for soil and seismic applications.

## 2 HHT BACKGROUND

The HHT was proposed by Huang et al. (1998) and consists of two parts: i) Empirical Mode Decomposition EMD, and ii) Hilbert Spectral Analysis. In the EMD process, signals to be analyzed are adaptively decomposed into a finite number of intrinsic mode functions IMFs. An IMF is described as a function satisfying the following conditions: i) the number of extrema and the number of zero-crossings must either equal or differ at most by one; and ii) at any point, the mean value of the

envelope defined by the local maxima and the envelope defined by the local minima is zero. An IMF defined as above admits well-behaved Hilbert transforms. For an arbitrary function,  $X(t)$ , in linear programming-class (Titchmarsh, 1948), its Hilbert transform,  $Y(t)$ , is defined as

$$Y(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{X(t')}{t-t'} dt' \quad (1)$$

where  $P$  indicates the Cauchy principal value. Consequently an analytical signal  $Z(t)$ , can be produced by

$$Z(t) = X(t) + iY(t) = a(t)e^{i\theta(t)} \quad (2)$$

where

$$a(t) = [X^2(t) + Y^2(t)]^{1/2} \text{ and } \theta(t) = \arctan \frac{Y(t)}{X(t)} \quad (3)$$

are the instantaneous amplitude and phase of  $X(t)$ , respectively. Since the Hilbert transform  $Y(t)$  is defined as the convolution of  $X(t)$  and  $1/t$  by Equation (1), it emphasizes the local properties of  $X(t)$  even though the transform is global.

In Equation (2), the polar coordinate expression further clarifies the local nature of this representation. Using Equation (2), the instantaneous frequency of  $X(t)$  is defined as

$$\varpi(t) = \frac{d\theta(t)}{dt} \quad (4)$$

However, there is still considerable controversy on this definition. A detailed discussion and justification can be found in (Huang et al., 1998). Applying the EMD process, any signal  $X(t)$  can be decomposed into finite IMFs,  $imf_j(t) (j = 1, \dots, n)$ , and a residue  $r(t)$  ( $r$  indicates the signal tendency), where  $n$  is nonnegative integer depending on  $X(t)$ , i.e.

$$X(t) = \sum_{j=1}^n imf_j(t) + r(t) \quad (5)$$

For each  $imf_j$  let  $X_j = imf_j(t) + r(t)$ , its corresponding instantaneous amplitude  $a_j(t)$  and instantaneous frequency  $\varpi_j(t)$ , can be computed with Equations (3) and (4). By Equations (2) and (4)  $imf_j$  can be expressed as the real part RP in the following form

$$imf_j(t) = RP \left[ a_j(t) \exp\left(i \int \varpi_j(t) dt\right) \right] \quad (6)$$

Therefore, by Equations (5) and (6),  $X(t)$  can be expressed as the IMF expansion as follows

$$X(t) = RP \sum a_j(t) \exp\left(i \int \varpi_j(t) dt\right) + r(t) \quad (7)$$

which generalize the following Fourier expansion

$$X(t) = \sum_{j=1}^{\infty} a_j e^{i\varpi_j t} \quad (8)$$

by admitting variable amplitudes and frequencies.

Consequently, its main advantage over Fourier expansion is that it accommodates non-stationary data perfectly. Equation (7) enables us to represent the amplitude and the instantaneous frequency as functions of time in a three-dimensional plot, in which the amplitude is contoured on the time-frequency plane.

The time-frequency distribution of amplitude is designated as the Hilbert amplitude spectrum or simply Hilbert spectrum, denoted by  $H(\varpi, t)$ . Having obtained the Hilbert spectrum, the marginal spectrum is defined as follows

$$h(\varpi) = \int_0^T H(\varpi, t) dt \quad (9)$$

The marginal spectrum offers a measure of total amplitude (or energy) contribution from each frequency value.

### 3 HHT ON EARTHQUAKE DATA

In the following, and due to space limitations, some basic aspects of the HHT-geoseismic implementation will be described.

The EMD stage builds on the assumption that any data set consists of different, simple, intrinsic modes of oscillation that need not be sinusoidal, with the nonsinusoidal character of each mode of oscillation derived from the data. At any given time, the recorded data may have many different coexisting modes of oscillation, which may or may not relate to different seismological phases. Figure 1 shows high- and low-frequency components of a seismic signal recorded in Mexico City. For this particular signal, 16 IMFs were produced and they represent the original data vector broken down into frequency components from highest to lowest frequency. If all of the IMFs are added together, the resulting "summation" signal is a near perfect match for the original signal.

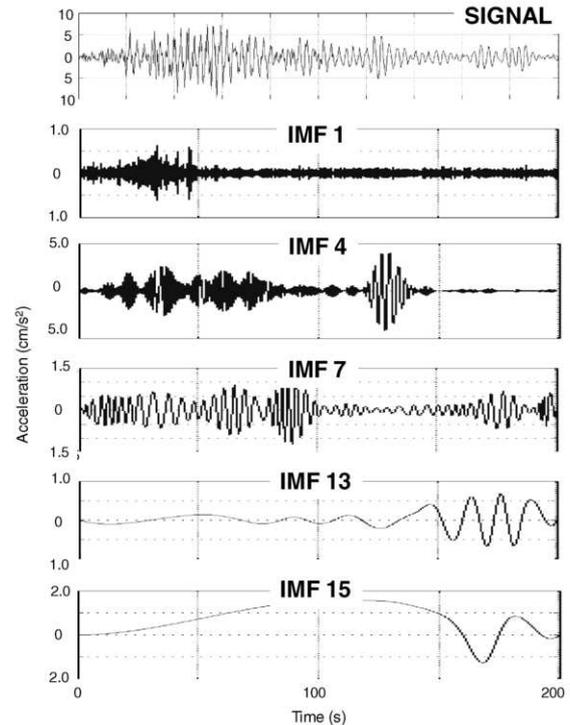


Figure 1. An accelerogram and its IMFs (CDAO site, very soft-clay, Oct 24th 1993 event,  $M_c = 6.5$ ).

All IMFs are well-suited for the Hilbert Transform. The key advantage of using HT, rather than FFT or Wavelets, is that it allows the use of instantaneous frequency to display the data in a "time-frequency-energy" format. This produces a more accurate "real-life" representation of the data without any of the artifacts imposed by the non-locally-adaptive limitations of FFT or Wavelets processing.

As an example, the accelerogram (E-W component) of the September 14th 1995 earthquake recorded in a soft-clay deposit in Mexico City (CDAO site), its corresponding Fourier spectrum and its Hilbert amplitude spectrum HAS are shown in Figure 2. The Fourier spectrum indicates that the frequency content of the waves is spread out with the maximum spectral amplitudes at 0.3 and 0.9 Hz. The HAS illustrates quantitatively the temporal-frequency distribution of vibration characteristics in the ground-motion recording: two sets of ground motion in the time intervals 70–75 and 135–140 s consisting of low-frequency signals between 0.25 to 0.80 Hz.

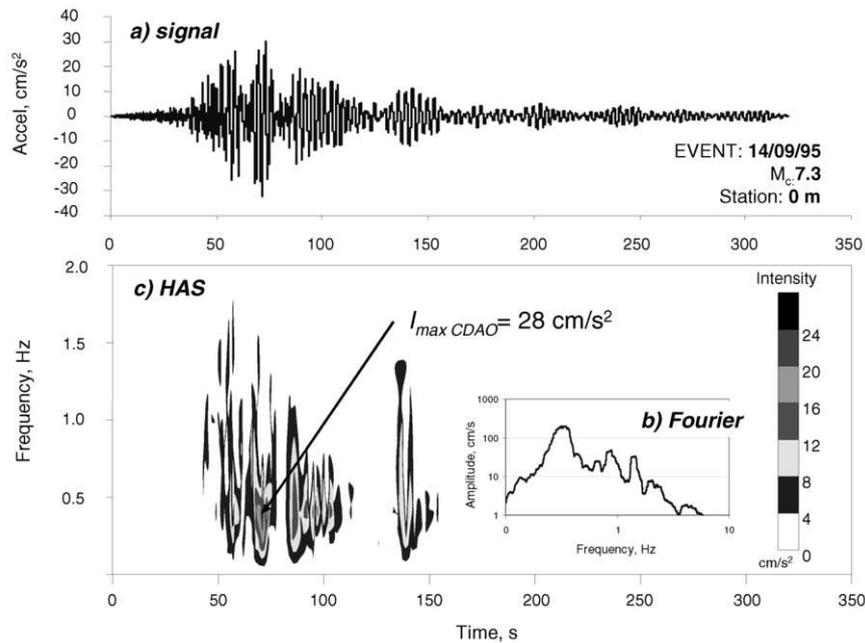


Figure 2 . Temporal-frequency distribution of vibration characteristics using HHT.

Comparison between Figures 2.b and 2.c indicates that the Fourier amplitude spectrum of the original data has a well-defined peak amplitude value ( $\sim 170 \text{ cm/s}^2$ ) at the dominant frequency (0.3 Hz), while the HAS shows the highest intensity  $I_{\max}$  ( $\sim 28 \text{ cm/s}^2$ ) at 0.4 Hz surrounded by an important concentration zone (iso-acceleration curves  $\sim 20 \text{ cm/s}^2$ ) on a wider frequency band (0.2 to 0.5 Hz).

The second motion arrival, with accelerations on the middle of the intensity-scale registered, covers frequencies around 1.0 Hz. Since these amplitudes are quite often used as an index to measure the seismic energy, the distinction between values based on the two different methods can be critical to the seismic design, retrofit guidelines and codes. It is worth further investigation to see which value (Fourier or HHT-based) is more appropriate for structural design, but the differences between frequency-energy content can be critical when site effects are being analyzed. The full understanding of these characteristics is a subject of continuing study, for now, the potential exists for a useful quantitative measure of a motion's input energy to structural and geotechnical systems.

### 3.1 HHT analysis using down hole records

A unique opportunity to detect nonlinear amplification effects is provided by borehole vertical arrays. To illustrate the HHT application when analyzing down-hole records two earthquakes with different magnitudes ( $M_c=7.5$ , strong, and weak,  $M_c=6.5$ ), recorded at CDAO site (0, 12 and 30 m), are presented in Figure 3. The HAS permit to outline the differences between responses and to mark the effects of the soils on the amplification ratios. HHT analysis depicts a deamplification from 12 to 0 m (more energy is dissipated during the strong event) and because of the modification of the frequency content this superior layer can be considered as a low-pass filter. This is not observed in a FFT analysis (not shown here because of the space restrictions). On the other hand, in the frequency range containing most of the radiated energy, it is expected that the attenuation of the strong motion by hysteretic damping be larger than for weak motions. The HAS analysis permits seeing that weak response is due to a collection of acceleration spikes of low frequency from bedrock to the surface while the strong shaking contains a broader intensity-frequency range where higher harmonics are generated increasing strong-motion amplitudes.

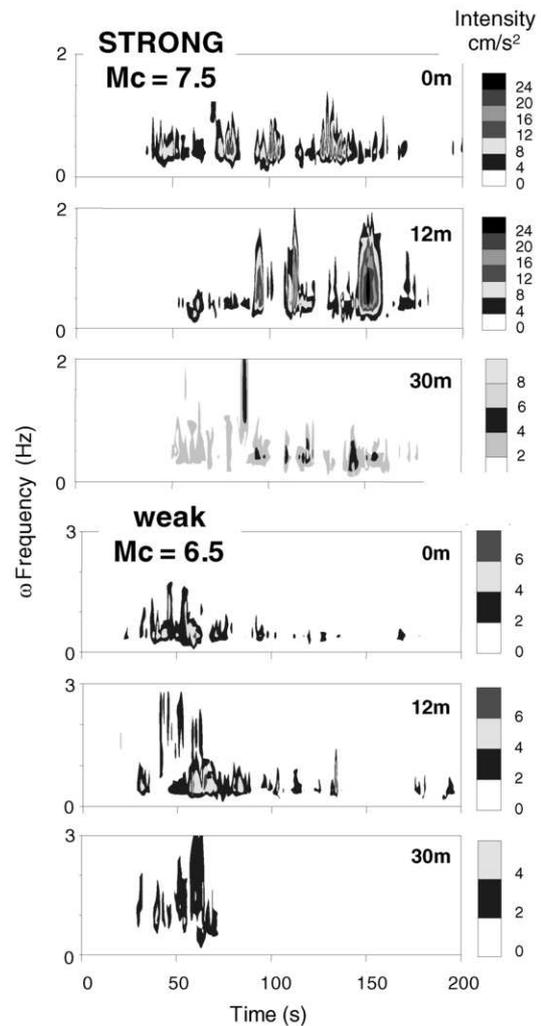


Figure 3. Soil amplification behavior, an HHT analysis.

### 3.2 HHT Nonstationary data processing

Spectral representations of acceleration signals recorded in two lake-bed stations (SCT and CDAO, very soft clay deposits) during the Sept 19th 1985 event ( $M_w = 8.1$ ) are presented in Figure 4. As can be seen in the figure, if FFT is used to compute the soil-site amplification and to evaluate the soil-site potential nonlinearity, an ambiguous indication of nonlinear clay-behavior could be driven by the analysis tool rather than the actual encrypted in the recorded motions. The mixed frequency content of the recordings, containing low and high frequencies, and the correct energy distribution is truthfully reflected in the HAS.

For non-stationary data recorded from a nonlinear systems such as large-amplitude earthquake ground motion, this results show that the Hilbert spectrum can truthfully represent the non-linearity-related non-stationary data in comparison with Fourier amplitude spectrum. An HHT-based approach for characterizing non-linear site effects can quantify soil non-linearity in terms of frequency downshift in the low-frequency range and amplification downshift in intermediate-frequency range than the Fourier-based factor.

## 4 CONCLUSIONS

HHT is found suited for analyzing nonstationary dynamic and earthquake motion recordings, which is better than some conventional Fourier data processing techniques. The decomposed components in EMD of HHT, namely, the IMF components, may contain observable, physical information inherent to the original data. The Hilbert spectra in HSA of HHT show the temporal-frequency energy distribution for dynamic and earthquake motion recordings precisely and clearly.

It is important to point out that nonlinear processes need special treatment. One of the typical characteristics of the seismic waves transmission through the Earth is its intra-wave frequency modulation, which indicates the instantaneous frequency changes within one oscillation cycle. In the past, when the analysis was based on the linear Fourier technique, this variation could not be depicted, except by resorting to harmonics that they may have mathematical meaning, but not a physical one. In this short paper we have addressed the possibility of characterizing natural systems (soil deposits) by the time-frequency variations of system signals (accelerograms that represent the systems dynamics). The objective of describing data from engineering perspectives (time-frequency-intensity domain) for finding specific and indicative behavior patterns reducing the error mechanisms is addressed applying the HHT to the seismic information. Our analysis is of a preliminary nature and many issues have to be investigated rigorously but the HHT seems to have much potential for this approach.

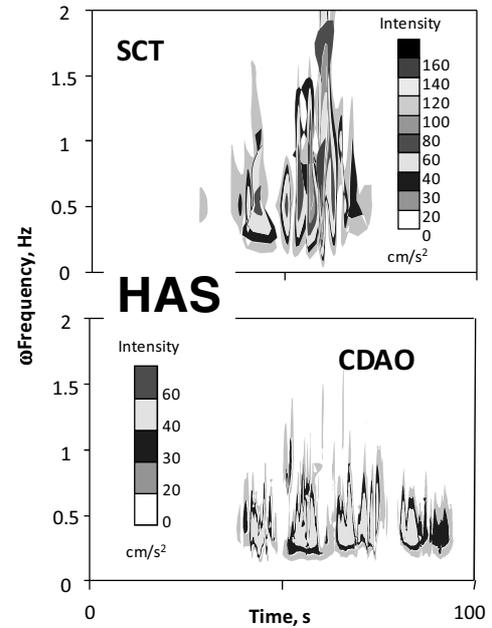


Figure 4. Two lake-bed sites behavior, HHT and Fourier analyses.

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