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The measurement of K_0, the coefficient of earth pressure at rest by the strain path loading technique

La mesure de K_0, le coefficient de pression de la terre au repos par la technique de chargement de chemin de contrainte

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ABSTRACT

An experimental study of the K_0 of a loose reconstituted sand is presented. The experimental technique consists of drained vertical compression of a triaxial specimen using the strain path control, in which a positive control of the strain increments rather than the conventional stress increments is exercised. The method does not require the use of any lateral strain sensors as used in previous studies and is free from any side friction affects, typical of oedometer testing. Possible influence of stress/strain history the soil inevitably experiences during sampling or reconstitution together with the initial state of stress prior to initiating zero lateral strain loading on the measured K_0 are critically reviewed and evaluated experimentally. It is also shown that the requirement of absolute zero lateral strain may not be essential for obtaining credible estimates of K_0 as long as the ratio of radial to axial strain during vertical compression does not exceed about 3 to 5 %.

RÉSUMÉ

Une nouvelle méthode de laboratoire pour estimer K_0 d'un sol est décrite. La méthode se compose de la compression verticale vidangée d'un spécimen à trois axes sous une commande de chemin de contrainte dans laquelle les incréments de la contrainte axiale et volumétrique sont égale maintenue. La méthode n'exige pas l'utilisation d'aucune sonde latérale de contrainte comme utilisée dans des études précédentes et est exempte de tout frottement latéral affecte, typique de l'essai d'oedometer. Influence possible de l'histoire de stress/strain que le sol éprouve inévitablement pendant le prélèvement ou la reconstitution ainsi que l'état initial d'effort avant le chargement latéral nul de contrainte sur le K_0 mesuré sont en critique passées en revue et expérimentalement évaluées. On lui montre également que la condition de la contrainte latérale nulle absolue peut ne pas être essentielle pour obtenir des évaluations croyables de K_0 aussi longtemps que le rapport du radial à la contrainte axiale pendant la compression verticale n'excède pas environ 3 à 5 %.

1 INTRODUCTION

Many soils are deposited under lakes or seas, over areas of large lateral extent. Symmetry dictates that the compression history of such soils under increasing overburden stress σ'_v must have been entirely one-dimensional in the vertical direction, and zero strain in the lateral direction. When these deposits get unloaded, such as by erosion following an episode of deposition, the rebound that occurs would similarly be one-dimensional in the vertical direction. The horizontal effective stress σ'_h in these one-dimensionally compressed soil deposits is often expressed as:

$$\sigma'_h = K_0 \sigma'_v \tag{1}$$

in which K_0 is called the coefficient of earth pressure at rest. In geotechnical engineering problems, particularly those which requires determining ground movements and distribution of stresses and pore pressures induced on loading, a credible estimate of in-situ K_0 is thus of central practical importance.

Both, in-situ and laboratory testing methods have been employed to estimate the magnitude of K_0. All in-situ methods, including the self-boring pressuremeter and lateral stress measuring earth pressure cells, however carefully installed in the ground, result in the soil suffering some lateral strain away from the ideal zero in-situ state, hence influencing the measured K_0.

Instrumented oedometers capable of measuring the hoop stress (e.g. Brooker & Ireland, 1965; Mejia et al., 1988; Mesri & Hayat, 1993), and the triaxial test devices with radial strain sensors have been used for K_0 determination (e.g. Bishop & Henkel, 1962; Daramola, 1980). These techniques when used with undisturbed specimens must commence loading from an initial condition where the soil is no longer in a state of absolute zero lateral strain, as it would ideally be under the undisturbed in-situ condition. The soil would have experienced a small, but, an unknown amount of compressive lateral strain (assuming in-situ $K_0<1$) due to the inevitable release of in-situ shear stresses on undisturbed sampling (Ladd & Lambe, 1963; Skempton & Sowa, 1963). Moreover, after sampling the residual stress state in the soil becomes hydrostatic (since $\sigma_a = \sigma_r = 0$ where σ_a and σ_r are axial and radial stresses, respectively, in the triaxial test). It must be emphasized that the laboratory procedures can, at best, impose the constraint of zero lateral strain only with respect to the specimen configuration that ensues after sampling, and any additional unknown strains associated with sample extrusion, trimming and set up in the apparatus under a small hydrostatic effective stress σ'_0.

This paper describes a study of the measurement of K_0 of a loose reconstituted sand. The experimental techniqure utilitzes strain incremental instead of the usual stress incremental technique of loading the specimen in a fully drained triaxial test. The condition of zero lateral strain, from the very start of vertical loading beyond σ'_0, is enforced by exercising a direct and positive control on axial and volumetric strain increments ($d\varepsilon_a$ and $d\varepsilon_v$) that are equal in magnitude (Menzies, 1988; Vaid & Eliadorani, 1998). It automatically guarantees zero raidal strain incrment.

Clearly, the loading rate in these tests must be sufficiently slow so that negligible excess pore pressures develop in the specimen. Such a rate is determined by trial. A number of identical specimens are subjected to successively slower rates of loading. When further reduction in the loading rate does not influence the results (i.e. σ'_r/σ'_a) at any mobilized σ'_a, this is taken as the satisfactory rate for the test. The effects of deliberately allowing small lateral strain increments to occur during loading beyond the σ'_0 state, instead of maintaining them strictly zero, on the measured K_0 are also explored.

2 EXPERIMENTAL PROCEDURE AND TEST MATERIALS

The tests were performed on sand dredged from the Fraser River in British Columbia, Canada. The batch of this sand used in this study has an average particle size (D_{50}) of 0.35 mm and the coefficient of uniformity (C_u) of 1.72. The specific gravity is 2.695, according to ASTM 854, and the maximum and minimum void ratios determined according to ASTM D4253 and D4254 were found to be 0.900 and 0.594 respectively.

Triaxial specimens of sand, nominally 62 mm diameter × 125 mm high, were reconstituted by water pluviation (WP). All test specimens were deposited in the loosest conditions and then densified to target the desired density.

The details of the triaxial apparatus that enables strain path control and experimental procedures are described elsewhere (Vaid & Eliadorani, 1998; Eliadorani, 2000). All test specimens, following their reconstitution by WP, were confined by a vacuum of about 20 kPa. After setting up the triaxial cell in the loading frame, this confining vacuum was replaced by the difference in the applied cell and back pressures. The configuration of the test specimen under this finite hydrostatic stress state σ'_0 was taken as representing zero strain - both axial and radial. An assumption of this kind is necessary as the precise zero reference of lateral strain cannot be known.

Appropriate account was taken of the membrane penentration effects while targeting the volumetric strain increments. Membrane penetration was separately assessed by the method suggested by Vaid and Negussey (1984). A relationship between unit membrane penetration and relative denstiy was established, from where the membrane penetration value specific to the test density was read off.

A detailed series of tests was carried out on Fraser River sand. In the first series, the effective stress state σ'_{ri} sprior to initiating K_0 loading using the $d\varepsilon_r$=0 strain path was selected as hydrostatic, at the set up value σ'_0 of 20 kPa. In other series, this initial hydrostatic stress of 20 kPa was first increased to a target value σ'_{ri} before $d\varepsilon_r$ =0 loading was initiated. This was intended to determine what influence does the level of σ'_{ri}, and hence small radial compressive strains, exert on the measured K_0 value. Tests were also carried out using identical σ'_{ri} = 20 kPa, but the specimen was first loaded conventionally drained to a targeted R = σ'_a/σ'_r, prior to initiating $d\varepsilon_r$ =0 loading. Thus the initial state of stress was now nonhydrostatic instead of the hydrostatic used in earlier series. This phase of pre $d\varepsilon_r$ =0 vertical compression loading will cause some additional compressive axial and compressive or even extensional lateral strain, depending on the selected magnitude of R. Clearly this will result in a further departure from the state of zero lateral strain over and above what is inevitable during sampling or reconstitution of specimen in the laboratory. The effects of this additional departure from a state of ideal zero lateral strain prior to the commencement of $d\varepsilon_r$=0 loading should become apparent in this series of tests.

In addressing to what degree the deliberately induced lateral strain affects the mobilized value of K_0, both ε_a and ε_r refered to are total strains that have occurred beyond the reference sample configuration under σ'_0.

3 TEST RESULTS

The results of a series of tests, namely, "vertical $d\varepsilon_r$=0 compression commencing from initial hydrostatic stress state" on Fraser River sand at a loose relative density (D_{rc}) of about 20% are illustrated in Fig. 1. Several levels of σ'_{ri} were selected after the specimens were set up under the reference stress state of σ'_0 = σ'_r = σ'_a = 20 kPa. In the σ'_r versus σ'_a plot (Fig. 1a) the data from tests with σ'_{ri}≤50 kPa may be noted to soon align themselves along a straight line passing through the origin. The slope of this line (~0.442) should be the desired K_0-value. Since the magnitude of lateral effective stress ratio K= σ'_a/σ'_r at the commencement of $d\varepsilon_r$ =0 loading is 1.0, it will be expected that some threshold value of σ'_a must be exceeded before this K reduces to approach K_0. In fact, as seen in Fig. 1(b), the value of K reduces rapidly from its initial value 1.0 to within less than about 5% of the eventual K_0 state, by the time σ'_a has exceeded about 400 kPa. The fact that K_0 remains constant over a considerable range of σ'_a in excess of this threshold value may be seen as a confirmation of the typical feature of normally consolidated soils. It appears that there is a tendency for the threshold σ'_a to reduce if σ'_{ri} selected is smaller (Fig. 1b).

It would be of interest to examine to what extent the mobilized σ'_a and σ'_r are affected in tests with σ'_{ri} in excess of the reference 20 kPa. Fig. 1(c) illustrates the variation of $\varepsilon_a/\varepsilon_r$ with σ'_a. It may be noted that as soon as this strain ratio $\varepsilon_r/\varepsilon_a$ falls below about 3%, the approach to the K_0 state appears virtually complete. It should be remembered that in this series of tests with σ'_{ri} = 20 kPa, there is no increase in ε_r, which stays at the pre- $d\varepsilon_r$ =0 loading value (assumed zero). Only ε_a increases, and consequently the ratio $\varepsilon_r/\varepsilon_a$ decreases with increase in σ'_a (or ε_a). The magnitude of $\varepsilon_r/\varepsilon_a$ may be noted to be approximately 2.0 at the start of loading, regardless the level of σ'_{ri} from where $d\varepsilon_r$ =0 loading was initiated. This is a typical characteristic of sands during hydrostatic loading, and is considered a reflection of its inherently anisotropic nature (El-Sohby 1969; Negussey & Vaid 1986). The magnitude of this ratio depends on the relative density of sand. It increases as the density decreases. The results presented in Fig. 1(c) thus show

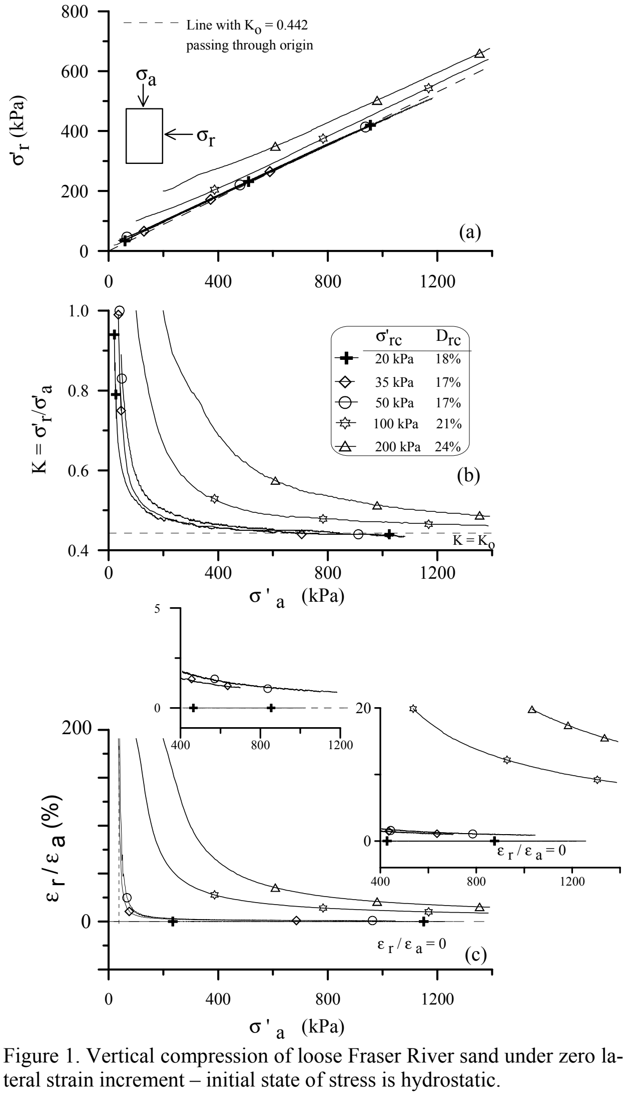

Figure 1. Vertical compression of loose Fraser River sand under zero lateral strain increment – initial state of stress is hydrostatic.

that as long as the ratio $d\varepsilon_r/d\varepsilon_a$ does not exceed about 3%, a strict adherence to the requirement of zero lateral strain may not be necessary for obtaining credible estimate of K_0. In this series of tests reported, the small lateral strain prior to $d\varepsilon_r=0$ loading was compressive, which occurred during hydrostatic compression from σ'_{ri} =20 kPa to the value of σ'_{ri} prior to initiating $d\varepsilon_r=0$ loading. It is seen that for initial hydrostatic σ'_{ri} values in excess of 50 kPa, $\varepsilon_r/\varepsilon_a$ does not reduce to a low of about 3%, and therefore no approach to a credible K_0 appears likely even at σ'_a =1200 kPa, when the tests were terminated.

3.1 *Vertical $d\varepsilon_r$ =0 compression commencing from initial non-hydrostatic stress state*

This series of tests was carried out using a value of σ'_{ri} =30 kPa but with initial effective stress ratios $R_i > 1$ prior to $d\varepsilon_r$ =0 loading. It was felt that commencing $d\varepsilon_r$ =0 loading from an estimated anisotropic stress state close to the real K_0 stress state in-situ, might prove more reliable for obtaining K_0. The values of $R_i= \sigma'_{ai}/\sigma'_{ri}$ selected in the test series ranged from 1.0 to 3.4.

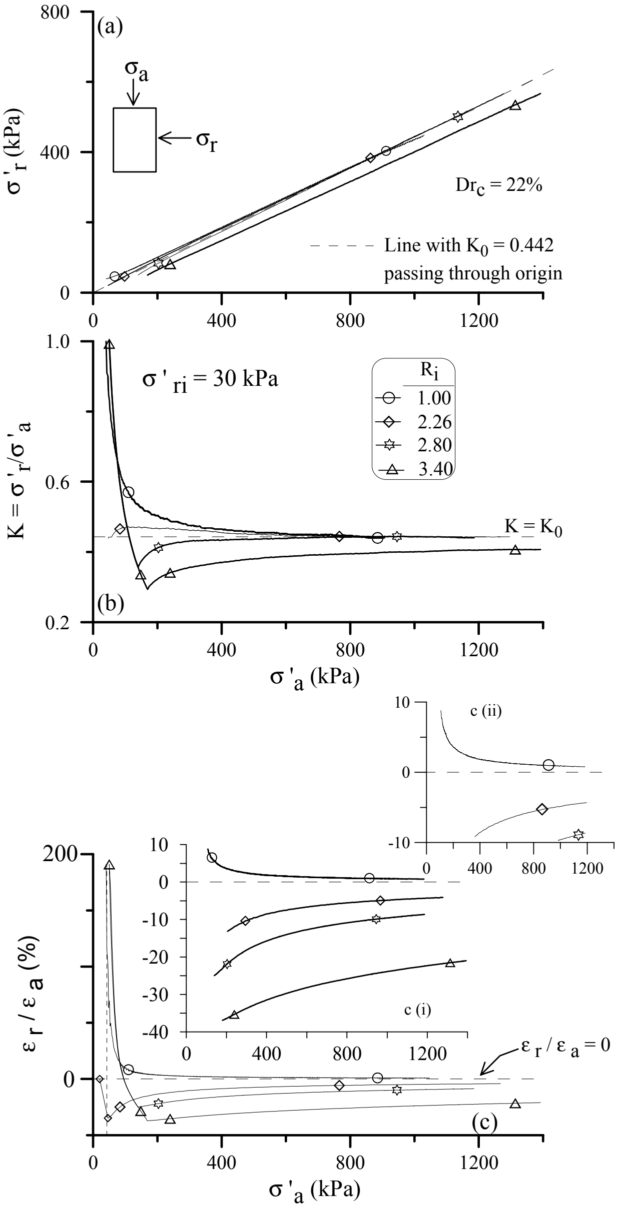

Figure 2. Vertical compression of loose Fraser River sand under zero lateral strain increment – initial state of stress is non-hydrostatic.

The range selected would then surely have straddled the real R corresponding to K_0. The results presented in Fig. 2 show that with R_i = 2.26 the approach to the approximate K_0 state occurred at a threshold σ'_a, which strangely is greater rather than that with R_i=1.0. Possible reason for this increased σ'_a will be explained later. Nevertheless, the K_0 values from the two series of tests are essentially identical. As noted earlier R_i=2.26 was specifically chosen because it represented commencement of $d\varepsilon_r$ =0 loading from an initially approximate K_0 (estimated by $K_0 \approx 1-\sin\varphi'$) rather than the hydrostatic effective stress state, and it was expected that this starting condition might cause the real K_0 state to be reached at σ'_a lower than that observed for the initial state with $R_i=(1/K_i)=1$. It may be argued that such a behavior might have occurred because this specimen had experienced (during loading from the zero strain reference hydrostatic σ'_{ri} =20 kPa, R_i =1.0 to non-hydrostatic σ'_{ri} =30 kPa, R_i =2.26) an extensional lateral strain of about -0.33% prior to the initiation of $d\varepsilon_r$ =0 loading. It is interesting to note that the measured K first increase above the initial selected value of 0.442, but finally did stabilize back to an equilibrium K_0, when the strain ratio $|\varepsilon_r/\varepsilon_a|$ dropped below about 3 to 5%. It, therefore, appears that no benefit was achieved by starting $d\varepsilon_r$=0 loading commencing from an approximately K_0 rather than the K=1 state. In fact any extensional lateral strain history prior to $d\varepsilon_r$ =0 loading, if it did occur, may delay rather than accelerate an ultimate approach to K_0 state with a lower threshold σ'_a.

It is also of interest to point out that the approach to a $|\varepsilon_r/\varepsilon_a|$ level of 3 to 5% for initial R_i=1 occurred from the positive, whereas that for $R_i \geq 2.26$ it did from the negative values. An undisturbed sample after its retrieval from the ground, as pointed out earlier, has already experienced some compressive lateral strain (for the case if K_0 <1). Therefore, the net strain ratio $\varepsilon_r/\varepsilon_a$ would have never ever been extensional if R_i (=K_i) =1. On the other hand an extensional lateral strain history may occur for $d\varepsilon_r$=0 loading tests which commence from some initial states with R_i perceived close to corresponding to estimated K_0, or larger. For in-situ K_0>1, extensional lateral strain on perfect sampling is inevitable. Any negative initial ε_r may be responsible for a slower approach to the final K_0-state.

3.2 *Vertical $d\varepsilon_r \neq 0$ compression commencing from initial non-hydrostatic stress state*

Tests were carried out on three identical specimens of Fraser River sand with initial state σ'_{ri} = 20 to 28 kPa and R_i = 2.28 prior to the strain path vertical loading. One specimen was loaded under $d\varepsilon_r/d\varepsilon_a$ =0 (Test #1), but not the other two. These were allowed to experience finite lateral strains, i.e. $d\varepsilon_r/d\varepsilon_a \neq 0$. During the loading of second specimen (Test #2), no membrane penetration volume corrections were applied to account for increasing σ'_r. Consequently it got subjected to lateral extensional strain increments, whose total magnitude increased with increasing σ'_a. In the third specimen (Test #3), membrane penetration volume correction were applied, which were less than those needed consistent with increases in σ'_r. As a consequence again radial extensional strain increments accumulated as σ'_a increased. These extensional strains, however, were smaller than in Test #2 at any mobilized σ'_a.

The results illustrated in Fig. 3 (a & b) show that the smaller excursions into extensional lateral strains in Test #3 do not appear to influence the value of final K= K_0 arrived at, when compared to what results when no excursion were allowed (Test #1). However, larger excursions into extensional lateral strain region typical of Test #2 caused an underestimate in the measured K_0. Moreover, the smaller lateral extensional strain excursions do not appear to affect the threshold value of σ'_a at which K_0 is practically realized compared to that under the $d\varepsilon_r$=0 loading path. On the other hand somewhat larger excursions into extensional lateral strains region typical of Test #2 tend to delay the approach, even to an underestimated value of K_0.

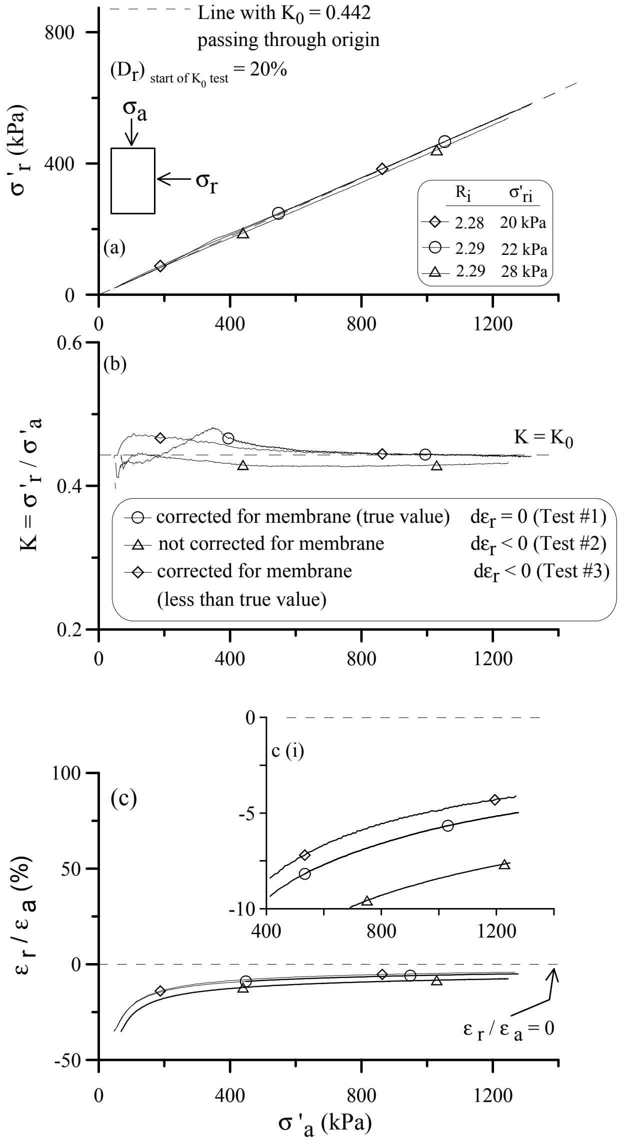

Figure 3. Vertical compression of loose Fraser River sand under extensional lateral strain increment – initial state of stress is non-hydrostatic.

An examination of Fig. 3(c) further shows that the strain ratio $\varepsilon_r/\varepsilon_a$ too does not drop to the small value of about 3 to 5% even after σ'_a in excess of about 1200 kPa, when excursions into larger extensional lateral strains are allowed. Both specimens #2 and #3 experienced the same extensional lateral strains prior to strain path loading. With further increase in σ'_a (and hence ε_a) on vertical loading, additional extensional strains got added. But for specimen #3, because of their smaller incremental values, the ratio $\varepsilon_r/\varepsilon_a$ did decrease to about 3 to 5%. This ratio, for Test #2 on the other hand never got an opportunity to fall below this threshold minimum value. Excursions into larger extensional lateral strains during the strain path axial compression thus appear much more damaging in retrieving the true K_0 of the soil.

4 CONCLUSIONS

An experimental study of the K_0 of a loose reconstituted sand has been carried out. Triaxial specimens were used which were loaded fully drained using the strain path instead of the commonly used stress path control. An important advantage of the method is that it dispenses with the use of any lateral strain sensing device or a continual check on the quality of accumulated axial and volumetric strains.

Based on the tests carried out on reconstituted specimens of Fraser River sand, the following conclusions can be arrived at:

- Credible estimate of K_0 were obtained by vertical compression of specimens initially hydrostatically consolidated using the $d\varepsilon_r=0$ strain path control.
- K_0 state arrived at by initiating loading from initially hydrostatic effective stress state was essentially identical to that using an initially non-hydrostatically state close to the estimated K_0 value.
- The K_0 state was reached sooner, i.e. at a lower mobilized σ'_a if the initial hydrostatic stress prior to strain path loading was lesser.
- The common belief that a strict condition of zero lateral strain must be adhered to for credible estimates of K_0 was found not to be necessary. As long as the ratio $\varepsilon_r/\varepsilon_a$ stayed below about 3 to 5%, no difference in K_0 was noted compared to when it was held strictly zero.
- Deliberate excursions of the lateral strain into the extensional region were found responsible for failure to retrieve K_0 using the proposed method.

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