Drained and undrained elastic moduli of reconstituted clay
Modules élastiques drainés et non drainés d’une argile reconstituée

T. Kawaguchi
Hakodate National College of Technology, Hakodate, Japan

T. Mitachi
Graduate School of Engineering, Hokkaido University, Sapporo, Japan

S. Shibuya
Faculty of Engineering, Kobe University, Kobe, Japan

ABSTRACT
The behavior of elastic stiffness involved with strains less than 10-5 was examined over a wide stress range when a reconstituted clay is subjected to drained and undrained shear. The tests were carried out by using a fully automated triaxial apparatus equipped with bender elements. The drained/undrained elastic Young’s moduli in the vertical direction, (E_v)_d and (E_v)_u, from stress-strain relationship, together with the elastic shear modulus, G_h, from shear wave velocity measurement using bender elements, were each successfully characterized in terms of the current stress as well as the stress history. Interrelationship among (E_v)_d, (E_v)_u and G_h was also discussed, bearing the stiffness anisotropy at small strains in mind.

RÉSUMÉ
L’évolution des rigidités élastiques mises en jeu pour des déformations inférieures à 10^{-5} m/m, a été étudiée pour une argile reconstituée sous des chargements déviatoires en conditions drainées et non drainées, pour un large domaine de contrainte. Les essais ont été effectués au moyen d’un appareil triaxial entièrement automatisé et muni de capteurs piezoelectriques de type «Bender». Les modules d’Young élastiques verticaux, drainés et non drainés, respectivement (E_v)_d et (E_v)_u, sont issus de la relation contrainte-déformation, tandis que le module de cisaillement (G_h), est obtenu à partir de la mesure des vitesses des ondes de cisaillement émises par les éléments de type «Bender». Chacun de ces modules a pu être caractérisé avec succès par l’intermédiaire de l’état de contrainte courant et de l’histoire de chargement. D’autre part, les relations liant (E_v)_d, (E_v)_u et (G_h) sont discutées et analysées dans un cadre élastique anisotrope en petites déformations.

1 INTRODUCTION
The stress-strain behavior of geomaterials may well be characterized as ‘elastic’ when the induced strains at shearing are less than 10^{-5} (e.g. Tatsuoka et al. 1997). Cross-anisotropy in stiffness is often assumed for the small-strain behavior (e.g. Atkinson, 1975, Bishop & Hight 1977 among others). In sandy soils, such anisotropy resulted in the responses of Young’s modulus, E, and shear modulus, G, both of which depended on the directions of loading and shearing, respectively (e.g., Yamashita & Suzuki, 2001, Yimsiri & Soga, 2002, 2003). In cohesive soils with poor permeability, the measurement of E is technically not straightforward, so the anisotropy may be observed for the G-value from shear wave velocity measurement in the laboratory using bender elements (BEs). For example, the G_v-value when the shear wave propagates in the vertical with the grain motion in the horizontal was smaller than G_h (e.g. Jovičić & Coop, 1998).

Despite that five independent parameters are required to fully define the cross-anisotropy of the stiffness, it is difficult to directly measure all of these in tests on cohesive soils, in particular. In this paper, the behavior of drained and undrained Young’s moduli in the vertical direction, (E_v)_d and (E_v)_u, together with G_h, of a reconstituted clay was measured over wide ranges of stress and the stress history by using triaxial apparatus equipped with BEs. The results were interpreted within the framework of cross-anisotropy with an aid of micromechanics model (Yimsiri & Soga, 2002).

2 ELASTIC DEFORMATION MODULI IN TRIAXIAL TEST
Assuming cross-anisotropy in the stiffness, the vertical (i.e., axial) and horizontal (i.e., radial) strain increments, ∆ε_v and ∆ε_h, induced by the increments of the corresponding effective stresses, ∆σ_v' and ∆σ_h', in axi-symmetrical conditions of deformation, are expressed in Eq.(1), respectively.

\[ \Delta \sigma_v' = \frac{1}{E_v} \Delta \varepsilon_v - \frac{2 \nu_h}{E_h} \Delta \varepsilon_h, \quad \Delta \sigma_h' = \frac{1-v_h}{E_h} \Delta \varepsilon_v - \frac{v_h}{E_v} \Delta \varepsilon_h \]  \tag{1}

where \( \nu_h \) and \( \nu_h \) are Poisson’s ratio. Similarly, the shear strain increments, ∆γ_v and ∆γ_h, are given in Eq.(2).

\[ \Delta \gamma_v = \frac{1}{G_v} \Delta \tau_v, \quad \Delta \gamma_h = \frac{1}{G_h} \Delta \tau_h \]  \tag{2}

Postulating the symmetry of the compliance matrix, together with isotropy of deformation in the horizontal plane, the elastic parameters are correlated as shown in Eqs.(3) and (4);

\[ \nu_v = \frac{\nu_h}{E_v} / E_h \]  \tag{3}

\[ G_h = \frac{E_h}{2(1+\nu_h)} \]  \tag{4}

When the undrained conditions at small strains, i.e., \( \Delta \varepsilon_v + 2 \Delta \varepsilon_h = 0 \), is applied to Eq.(1), the undrained Poisson’s ratio, \( (v_h)_u \), \( (v_h)_u \) and \( (v_h)_u \) are;

\[ (v_h)_u = 0.5, \quad (v_h)_u + (v_h)_u = 1 \]  \tag{5}

(refer to Atkinson, 1975, Bishop & Hight 1977)

Since \( (E_v)_u \), \( (\Delta \sigma_v' - \Delta \sigma_h')/\Delta \varepsilon_v = (\Delta \sigma_v' - \Delta \sigma_h')/\Delta \varepsilon_v \) is obtained by substituting Eqs.(3) and (5) into Eq.(1), (E_v)_u can be directly estimated from the relationship between deviator stress, \( q (=\sigma_v' - \sigma_h) \) and \( \varepsilon_v \) in undrained monotonous or cyclic loading test. Similarly, \( (E_v)_d \) can also be estimated from the similar relationship in drained monotonous or cyclic loading test, since \( \Delta \sigma_v' = \Delta \sigma_v \) and \( \Delta \sigma_h' = \Delta \sigma_h \) in drained conditions.
Assuming that the shear modulus, \( G \), is not affected by drainage conditions, three stiffness parameters, i.e., drained and undrained Young's modulus in the vertical direction, \( (E_v)_\lambda \) and \( (E_h)_\lambda \), and \( G_{vh} \), can be directly measured in triaxial test with shear wave velocity measurement, whereas the other stiffnesses are unknown.

3 TESTS PERFORMED

The powder clay (called NSF clay) with \( IP = 26 \) was mixed thoroughly with distilled water. The initial water content was about twice the liquid limit \( w_l = 55 \% \). The vertical preconsolidation pressure applied was 150 kPa that was maintained constant over 10 days. A fully digitized triaxial apparatus equipped with a direct drive motor was used for the testing. The apparatus is featured by an extremely high resolution of axial displacement pressure applied was 150 kPa that was maintained constant in cyclic loading, fully automated servo-control and data acquisition system, etc (Shibuya et al, 2003). All the triaxial tests performed are summarized in Table 1 (refer also to Kawaguchi et al, 2003).

Note that in tests CD2 and CD5, the \( (E_v)_\lambda \) value was also measured at the start of undrained shear by using the axial strain rate of \( \varepsilon \) at small-strains was measured at the start of shearing by using the axial strain rates of \( \dot{\varepsilon}_v, \dot{\varepsilon}_h \) and \( \dot{\varepsilon}_\lambda \) was determined by means of linear fitting applied to the stress-strain relationship over a range of \( \varepsilon \).

The micromechanics model proposed by Yimsiri & Soga (2000) agrees well with Graham & Houlsby’s model for some typical soils (Yimsiri & Soga, 2003). The micromechanics model re-

![Figure 1. Effective stress path of (a) CD & CU, (b) CSD & CSU and (c) CUB series tests](image1)

![Figure 2. Examples for the relationship between \( q \) and \( \varepsilon \), at small strains](image2)
quires some parameters defined at the particulate level, each difficult to be quantified. However, the model can easily express the effects of the ratio of two elastic constants. The details of this model can be found in Yimsiri & Soga (2002, 2003).

Figure 3 shows the variation of \( (E_v)_{NSF} \) and \( G_{NH} \) when calculated according to micromechanics model. In this double-logarithmic plot, the relationship with Eq.(6) is also plotted for comparison. Note also that the \( K_g/K_e \) stands for the ratio of tangential contact stiffness to normal contact stiffness (refer to Yimsiri & Soga, 2002).

According to the results with micromechanics model, the \( (E_v)_{NSF} \) value lies over a narrow range between 0.9 and 1.1 for a wide range of \( E_p/E_e \) from 0.7 to 3.0 (see Fig.3). In addition, the \( G_{NH} \) calculated by micromechanics model agrees well with that by Graham & Housby’s model when \( 0.5 < E_p/E_e < 2 \). Moreover, \( G_{NH} > G_{NH} \) when \( E_p > E_e \) for the results calculated by both the models. Therefore, for some typical soils, it should be stressed that the relationship of \( (E_v)_{NSF} = 3G_{NH} \) (Eq.(7)) is valid except for extreme cases with \( E_p/E_e > 4 \) showing strong anisotropy. Note that the majority of experimental data by wave propagation technique gives rise to \( G_{NH} > G_{NH} \), suggesting that soils are stiffer in the horizontal direction (e.g., Yamashita & Suzuki, 2001, Yimsiri & Soga, 2002, 2003, Jovičić & Coop, 1995).

\[
(E_v)_{NSF} = 3G_{NH} 
\]

A comparison between \( (E_v)_{NSF} \) and \( G_{NH} \) is shown in Fig.4, in which the results of CUB series are examined against the critical mean effective stress \( p' \). Note that the \( G_{NH} \) value coincided closely with \( (E_v)_{NSF} \) irrespective of \( p' \) and the different consolidation paths followed (see tests CUB2 and CUB4). The test results do not indicate directly the effects of the fabric anisotropy, but the effects of stress-induced anisotropy are seemingly insignificant for the behavior of not only \( (E_v)_{NSF} \) but also \( (E_v)_{CSF} \) (see tests CD 1, 4, 6 in Table 1 and Fig.5) in this sample.

5 FORMULATION OF VARIOUS ELASTIC DEFORMATION MODULI

Figure 5 shows the \( \Delta e - \ln p' \) relationship, together with the variations of \( (E_v)_{NSF} \) and \( (E_v)_{CSF} \) of all the tests. It should be mentioned that the \( \Delta e \) means the change of void ratio calculated from the start of consolidation in each test. The \( \Delta e - \ln p' \) relationship of the clay tested is linear, and it is unaffected by the consolidation paths followed, bearing in mind that the \( \Delta e - \ln p' \) relationship of some clays depends slightly on the consolidation path by the effects of dilatancy (e.g., Mitachi and Kitago, 1976) (see Fig.5a).

Similarly, the \( \Delta e - \ln (E_v)_{NSF} \) relationship is linear by showing different slopes at normally consolidated (NC) and over-consolidated (OC) states, and it is unaffected by the consolidation paths followed. This means that the \( (E_v)_{NSF} \) is uniquely related to \( p' \) irrespective of the consolidation paths. Therefore, the \( (E_v)_{NSF} \) has been formulated in Eqn.(8) and (9),

\[
(E_v)_{NSF} = p^{\psi_s'} \cdot \exp \left( \frac{Z_{NSF} - N}{\psi_u} \right) 
\]

\[
(E_v)_{CSF} = p^{\psi_s'} \cdot p' \cdot \exp \left( \frac{Z_{NSF} - N}{\psi_u} \right) 
\]

in which \( \lambda \) and \( \kappa \) are compression and swell indices in \( \Delta e - \ln p' \) relationship, \( \psi_s \) and \( \xi_s \) refer to the slopes at NC and OC states of the \( \Delta e - \ln (E_v)_{NSF} \) relationship and the symbols, \( N \) and \( Z_s \) refer

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Figure 3. Variations of \((E_v)_{NSF} \) and \(G_{NH} \) with \( p' \) according to micromechanics model

Figure 4. Variations of \((E_v)_{NSF} \) and \(G_{NH} \) with \( p' \)

Figure 5. Relationship between \( \Delta e \) and \( p' \), \((E_v)_{NSF}, (E_v)_{CSF} \) for a), b), c)
to $\Delta \sigma$ at $p' = 1$ kPa, $(E_v)_h = 1$ MPa, respectively. $\rho'$ is consolidation stress (refer to Kawaguchi et al, 1999, Shibuya et al, 2002).

Since those trends are very similar for the $\Delta \sigma$ – $\ln (E_v)_h$ relationship, the $(E_v)_h$ is given in Eqs.(10) and (11),

$$E_v = \rho' \exp \left( \frac{Z_d - N}{\psi_d} \right)$$

(10)

$$E_v = \frac{\frac{\lambda}{\rho'} \exp \left( \frac{Z_d - N}{\psi_d} \right)}{\frac{\lambda}{\rho'} \exp \left( \frac{Z_d - N}{\psi_d} \right) + e}$$

(11)

where $\psi_d$ and $\xi_d$ refer to the slopes at NC and OC states of the $e - \ln (E_v)_h$ relationship and the symbol $Z_d$ refers to $\Delta \sigma$ at $(E_v)_h = 1$ MPa. It should be noted that the relations of $\psi_d = \psi$ and $\xi_d = \xi$ seem to be valid for the clay tested (see Fig.5c).

The variations of $(E_v)_h$, $(E_v)_p$, and $3G_v$ with $p'$ of all the tests are shown in Fig.6, in which the results of calculation using Eqs.(8)-(11) are also plotted for comparison. The calculated results with $\psi = \psi$, and $\xi = \xi$ well coincided with the observed data. In addition, the $3G_v$ is also successfully predicted based on Eq.(8). The results strongly suggest that these elastic deformation modules are all uniquely related to void ratio alone. Since the clay exhibits considerably larger volume change as compared to sandy soils, and the volume change in clays is governed by $p'$ rather than $\sigma_v^e$ alone, for example, the stiffness seems as if related to $p'$ (or $e$).

Figure 7 shows the relationship between $\xi$ and $\lambda$, and also between $\xi$ and $\kappa$ of clays showing a variety in the intrinsic properties (data from Li, 2003). As can be seen in Fig.7, the parameters of $\xi$ and $\xi$ are well correlated to $\lambda$ and $\kappa$ respectively. By applying the postulation of $\psi = \psi$ and $\xi = \xi$, the variations of $(E_v)_h$, $(E_v)_p$, and $3G_v$ with $p'$ (or $e$) may be predicted by knowing the parameters of $\lambda$ and $\kappa$ from conventional oedometer or triaxial test.

6 CONCLUSIONS

i) According to the micromechanics model, the $(E_v)_h$/3G_v value is close to unity unless the stiffness anisotropy is extremely strong. The $(E_v)_h$ was approximately equal to 3 G_v for the clay tested.

ii) Like $\Delta e - \ln p'$ relationship, the deformation moduli, $(E_v)_h$, $(E_v)_p$, and $G_v$ all varies linearly against void ratio in the semi-logarithmic plot. Each of these moduli can therefore be formulated with this rule.

iii) The slope of $\Delta e - \ln (E_v)_h$ relationship in compression is more or less the same as that of $\Delta e - \ln (E_v)_p$ relationship. The same is true for the behavior at swelling.

iv) The variations of $(E_v)_h$, $(E_v)_p$, and $G_v$ with $p'$ (or $e$) may be estimated by knowing a set of $\lambda$ and $\kappa$ from conventional oedometer or triaxial test.

REFERENCES