# INTERNATIONAL SOCIETY FOR SOIL MECHANICS AND GEOTECHNICAL ENGINEERING



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# Reverse behaviour and critical state of sand with small amount of fines

## Comportement renversé et etat critique de sable contenant une faible quantité de particules fines

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ABSTRACT

Some recent publications suggest that loose sand with a small amount of fines manifests the so-called "reverse behaviour". Whilst such unusual behaviour was interpreted in terms of the micro-structural arrangement of fines in the sand matrix, reverse behaviour does not appear to be compatible with the critical state framework. On these premises, this paper presents experimental results that indicate the effect of a small amount of fines is to change considerably the shape and the relative position between the critical state and normal consolidation lines. Additionally, a modified state parameter is defined so that the undrained behaviour of sand with a small amount of fines can be predicted within the critical state framework.

RÉSUMÉ

Des publications récentes suggèrent que le sable lâche contenant une faible quantité de particules fines présente ce que l'on pourrait nommer un "comportement renversé". Tandis qu'un tel comportement a été interprété en terme d'arrangement microstructural des particules fines dans la matrice sablonneuse, le comportement renversé ne semble pas être compatible dans le cadre de l'état critique. Dans ce contexte, l'étude présente les résultats expérimentaux qui indiquent que la présence d'un faible quantité de particules fines modifie considérablement la forme et la position relative entre les courbes d'état critique et de consolidation normale. De plus, un paramètre d'état modifié est défini afin de prédire le comportement du sable humide comportant une faible quantité de particules fines, ceci dans le cadre de l'état critique.

## 1 INTRODUCTION

Significant research progress has been made in the understanding and modeling of liquefaction behaviour of sands. In spite of such achievements, liquefaction behaviour of sand with fines has received less attention, maybe because of early observations by Seed et al. (1983) and Pitman et al. (1994) which suggest the presence of fines in the sand matrix may increase the resistance against liquefaction. More recently Zlatovic and Ishihara (1997), Yamamuro and Lade (1998), Thevanayagam & Mohan (2000) suggested that the previous understanding could have limitations especially when dealing with sand with non-plastic fines content below a certain limit. As further pointed out by Yamamuro and Lade (1998), the behavioural trend of these soils is more liquefiable at lower confining pressure as opposed to the behaviour of clean sand which is less liquefiable with the reduction in the confining pressure. In view of such departure from the "normal behaviour" exhibited by clean sands, the behaviour trend of sand with fines was referred to as "reverse behaviour" (Yamamuro and Lade 1998).

The liquefaction behaviour is often explained within the critical state framework in terms of the state parameter, $\psi$. The state parameter was initially defined by Been and Jefferies (1985) as the difference between the current void ratio and the void ratio at the same mean stress on the critical state line. A positive $\psi$ value at the commencement of undrained shearing is associated with a flow liquefaction response whereas a negative $\psi$ value is associated with a non-flow behaviour. Figure 1 illustrates a Critical State (CS) line and normal consolidation (NC) line of a soil specimen formed at an initial state 'I' at (with void ratio $e_i$ and effective confining stress $p'_i$). Hence, undrained shearing commenced from the NC line at a low effective confining pressure such as at point 'L' is expected to manifest a non-flow response as the state at 'L' is located under

the CS line and $\psi$ has a negative value. By similar reasonings, this soil specimen consolidated to a high effective confining stress such as point 'H' is expected to manifest flow liquefaction as $\psi$ has a positive value. Thus the reverse behaviour does not appear to fit into such a framework.

This paper has two main objectives. First is to show reverse behaviour does occur in the undrained shearing of a relatively "usual" loose sand with a small amount of fines. Second is to explain the reverse behaviour within the critical state framework. In achieving the second objective, a modified state parameter is introduced.
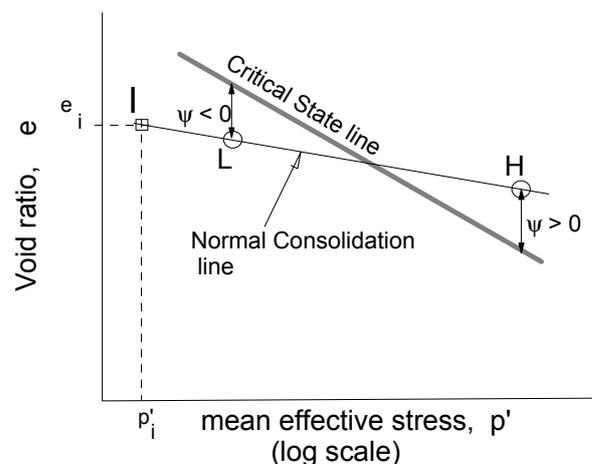


Figure 1. State parameter and liquefaction behaviour

## 2 EXPERIMENTAL STUDY

### 2.1 Material Tested

The soil material chosen for this study consisted of 90% Sydney sand and 10% fines. Sydney sand is a medium quartz sand and has been used in a number of experimental studies. Its index properties given in Lo et al. (1989). The fines were 2/3 well graded silt and 1/3 commercial kaolin.

### 2.2 *Testing Arrangement*

All tests were performed with a triaxial testing system in which the control of the stress path and data logging was automated. The axial load was applied by a Digital Force Actuator in a deformation controlled mode, but measured with an internal load cell. The axial deformation was recorded by a pair of internal Linear Variable Differential Transformers (LVDTs) mounted directly across the platens. A Digital Pressure Volume Controller (DPVC) was used to control the cell pressure whereas a second DPVC was used to measure the pore water pressure and to control drainage condition.

### 2.3 *Soil Specimen Preparation*

A modified moist tamping method was employed to form the loose soil specimens. A total of 10 layers of predetermined quantities of moist soil were compacted into a prescribed thickness to ensure specimen uniformity. An as-formed soil specimen had 100 mm for both diameter and height. Free ends with enlarged platens were used to minimize end restraint, whereas bedding and membrane penetration errors were reduced to an insignificant value by using the liquid rubber technique developed by Lo et al. (1989).

Saturation of soil specimens was accomplished in multiple steps. Initially the soil specimen was percolated with carbon dioxide followed by a double vacuum flushing under a small constant water head. Finally, back-pressure was applied to ensure a B-value of at least 0.98 was achieved, thus indicating a high degree of saturation. The effective stress was kept at less than or equal to 20 kPa in all saturation stages. At the completion of saturation, a soil specimen was maintained at a state defined by an initial isotropic effective consolidation stress ($p'_i$) of 20 kPa and a void ratio ($e_i$) in the range of 0.830 to 0.850. This state was also used to define replicate specimens.

## 3 TEST RESULTS

Only representative test results are presented in this paper and the notations used to denote the stress and strain are defined below.

mean normal stress: $p' = (\sigma'_1 + 2\sigma'_3)/3$
deviatoric stress: $q = (\sigma'_1 - \sigma'_3)$
volumetric strain: $\varepsilon_v = \varepsilon_1 + 2\varepsilon_3$
deviatoric strain : $\varepsilon_q = \frac{2}{3}(\varepsilon_1 - \varepsilon_3)$

where $\sigma'_1$ and $\sigma'_3$ are the major (axial) and minor (radial) principal effective stresses respectively. Similarly, $\varepsilon_1$ and $\varepsilon_3$ are the major and minor principal strains. The subscript '0' is used to denote the state at the commencement of undrained shearing.

### 3.1 *Drained Behaviour*

Drained tests were conducted on the sand with fines. The test results did not revealed any unusual behaviour although the soil specimens were volumetrically more compressible than the

parent sand. Details of these drained test results can be found in Bobei and Lo (2003).

### 3.2 *Undrained Behaviour*

The undrained $q$-$\varepsilon_q$ and effective stress path responses of a set of isotropically consolidated soil specimens are presented in Figure 2a and 2b respectively. This set of tests also included an undrained test conducted at $p'_0 = 30$ kPa. This test is not shown in the figure because of the small stress value, but need not be addressed separately as its behaviour pattern fits into the trend of $A_1$ to $A_5$.
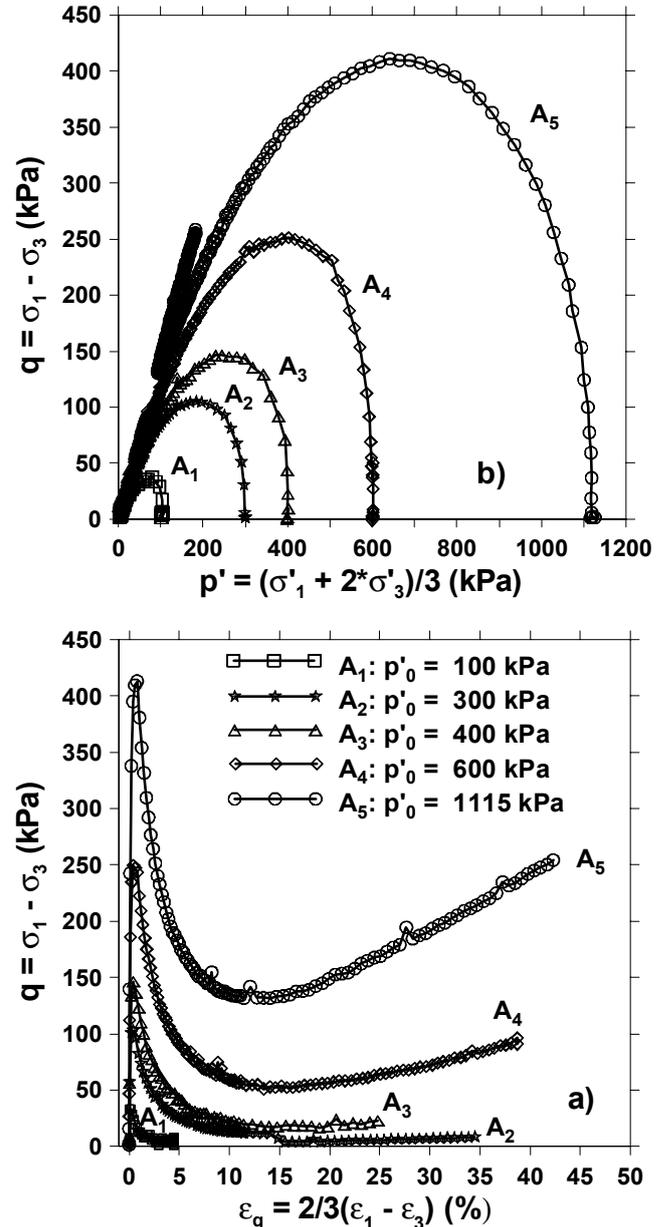


Figure 2. Response of sand with fines in undrained shearing. a) Stress-strain relationships b) Effective stress paths.

The $q$-$\varepsilon_q$ curves all tests initially show a similar response: a sharp increase in q to a peak at $\varepsilon_q \leq 0.5\%$, and then followed by a rapid strain softening. However, whilst the deviator stress in tests $A_1$ to $A_3$ remained at a steady value with further shearing, the deviator stress in tests $A_4$ and $A_5$ increased with shearing after reaching a minimum value termed quasi-steady state by Alarcon and Guzman (1988). As such and as further illustrated by the effective stress paths, the responses changed from limited

flow at higher $p'_0$ values to flow liquefaction at lower $p'_0$ values. This is in accordance with the behaviour pattern of "reverse behaviour" reported by Yamamuro and Lade (1998).

### 3.3 *Critical State and Steady State*

It has been pointed out in Bobei and Lo (2001) that there is no theoretical distinction exists between the Critical State (CS) and Steady State (SS) except for the choice of wording in their definition. However over the years experimental results have lead to controversial debates on the uniqueness of CS and SS in the e-log(p') space. It is pertinent to note that CS is approached in a drained test whereas SS is approached in an undrained test. As the drained and undrained tests have different inherent experimental errors, this may lead to differences between the measured CS and SS. Therefore, the experimental testing techniques adopted for this study were carefully conceived to minimise these testing errors.

In the light of the above and to interpret the "reverse behaviour" in the critical state framework, the uniqueness of CS and SS for sand with fines was studied by conducting a number of drained and undrained tests with the CS and SS points plotted in Figure 3. As shown, the CS and SS points are located on the same curve and consequently no distinction will be made between CS and SS states throughout this paper.
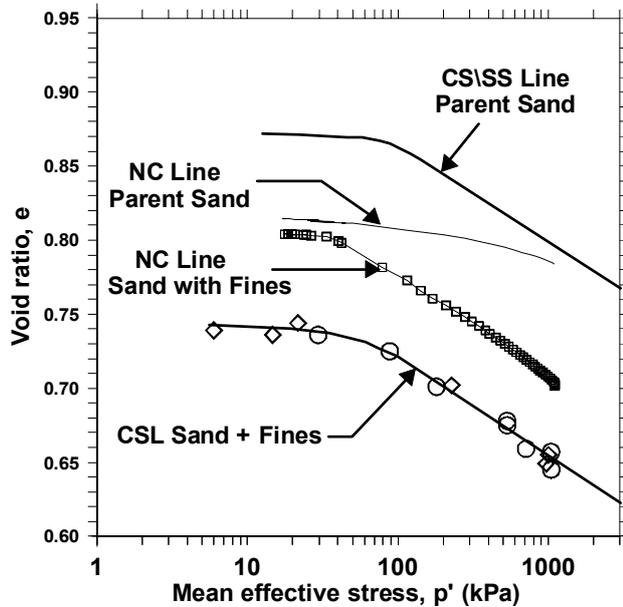


Figure 3. Critical state and normal consolidation lines of parent sand and sand with fines (O for CS and ⌀⌀ for SS).

Bobei (2004) also found that the CS and SS lines for parent sand was essentially identical and hence a single trend line was presented in Figure 3. Additionally, the NC line of a replicate parent sand specimen [viz. $e_i = 0.81$ and $p'_i = 20$ kPa] was also shown. This NC line for parent sand was located under the CS line for the early stage of consolidation; and with increase in confining pressure the NC line eventually intersect the CS line at p' in the range of 1100 to 1200 kPa in a manner similar to that shown in Fig. 1. As expected, the response of parent sand conforms to normal behaviour.

By reference to Figure 3, the presence of fines in the parent sand matrix was found to have the effect of shifting the position of both the CS and NC lines "downwards", but the CS line was shifted more. The shape of CS line of both sand with fines and parent sand are similar, not linear but is curving at lower confining pressures. However, the shape of the NC lines for sand with fines differed from that of the parent sand. The net

effect is that the relative shape and position of the CS and NC lines were altered significantly by the introduction of fines. Furthermore, these two lines do not intersect.

## 4 MODIFIED STATE PARAMETER

Based on the shape and relative position of NC and CS lines illustrated in Figure 3, $\psi > 0$ for all tests $A_1$ to $A_5$. As the NC line remains essentially parallel to the CS line for $p'_0$ in the range of 100 to 1115 kPa, the value of $\psi$ for these five tests are essentially the same. However, as shown in Fig. 2b, the behaviour pattern is reduction in liquefaction tendency with increase in $p_0'$. Thus it appears that $\psi$ has limitation in predicting the undrained behaviour of sand with fines.

In the light of the above, a modified state parameter, $\psi_m$, is proposed which is defined as:

$$\psi_m = \psi \left| \frac{\Delta p'}{p'_0} \right| e_0 \tag{1}$$

where $\Delta p'$ is the horizontal distance between the state at the start of undrained shearing and the CS line as shown in Fig. 4.
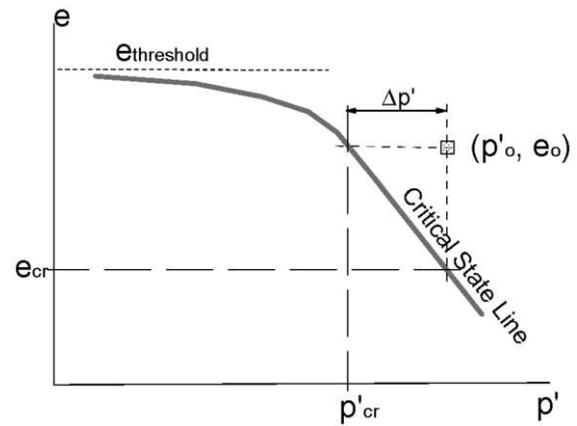


Figure 4. Schematic illustration of the parameters employed in the definition of the modified state parameter.

The inclusion of the factor $|\Delta p'/p'_0|$ in the definition of $\psi_m$ is because it can be related to the State Pressure Index, $I_p = p'_0/p'_{cr}$, as proposed by Wang et al (2002) via the following relationship:

$$\frac{\Delta p'}{p'_0} = \frac{p'_0 - p'_{cr}}{p'_0} = 1 - \frac{1}{I_p} \tag{2}$$

Noting that $I_p$ that has the ability to reflect some of the effects of the curved shape of CS line when assessing the liquefaction behaviour, $\psi_m$ will have a similar property as it assimilates $I_p$ in its definition.

As the factor $|\Delta p'/p'_0|$ has a limiting value of unity, the definition based on equation (3) has a limiting value of $\psi e_0$. In equation 3, the factor $|\Delta p'/p'_0|$ is not defined when $e_0 > e_{thr}$; where $e_{thr}$ is a threshold void ratio as defined by Ishihara (1993). Therefore, for $e_0 > e_{thr}$, the definition of $\psi_m$ is modified to:

$$\psi_m = \psi . e_0 \tag{3}$$

The value of $\psi_m$ given by equations 1 and 3 converge when $e_0 = e_{thr}$, and hence the value calculated by equation 1 will change smoothly to that of equation 3 at the threshold void ratio.

The factor $e_0$ improves prediction of liquefaction tendency, particularly for specimens at high void ratios.
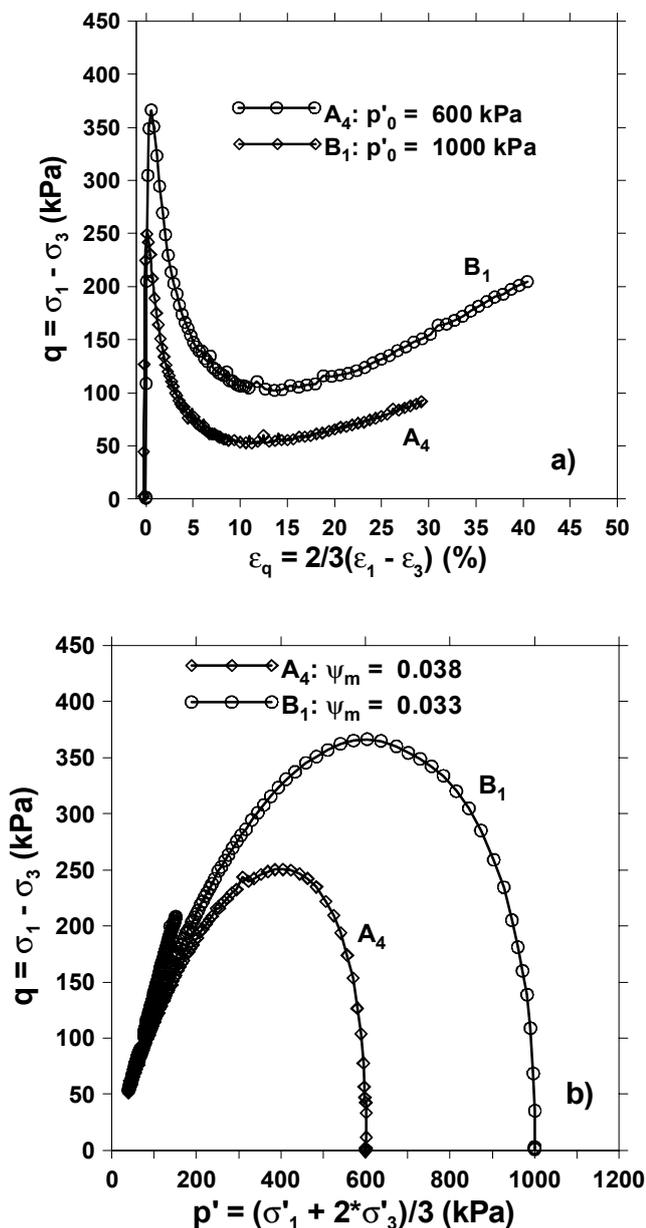
Figure 5. Similar undrained behaviour in shearing commenced at 600 and 1000 kPa for $\psi_m = 0.035$. a) Stress-strain relationships b) Effective stress paths.

## 5  ADDITIONAL VALIDATIONS

Anisotropically consolidated undrained tests and additional isotropically consolidated undrained tests with different $e_i$ were also conducted to verify the applicability of the modified parameter. Furthermore, the modified state parameter was also found to be applicable to the parent sand.  However, paper length prevents the presentation of all supporting test data.

To further validate the applicability of the modified state parameter, the response of two undrained tests labeled $A_4$ and $B_1$ with similar $\psi_m$ values of 0.038 and 0.033 respectively were compared in Figure 5a and 5b. Both undrained tests manifested limited flow.  Besides the similarity in the $q$-$\varepsilon_q$ responses, the similarity is also maintained in the stress paths.  Additionally after the decrease of $q$ to quasi-steady state, the $q$-$\varepsilon_q$ curves show similar slopes with further increase in $q$.

## 6  CONCLUSIONS

The paper presented both experimental results and theoretical developments on the behaviour of sand with a small amount of both plastic and non-plastic fines.  The main findings can be summarized as follows:

1. The undrained response of soil specimens manifested "reverse behaviour" previously reported by Yamamuro and Lade (1998).
2. The reverse behaviour could be explained in the critical state framework because of significant changes in the shape of the NC line and its relative position with respect to the CS line.
3. The original state parameter, $\psi$, in predicting the observed behaviour trend was found to have limitations.  A new state parameter termed modified state parameter, $\psi_m$, was proposed.
4. Similarity of undrained tests commenced at similar $\psi_m$ values, but different confining pressures and void ratios, further validated the suitability of $\psi_m$ to predict the undrained response.

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