Proceedings of the 16th International Conference on Soil Mechanics and Geotechnical Engineering

doi:10.3233/978-1-61499-656-9-513

Difference between the values of friction angle Φ derived from the theoretical fracture plane and the reliable one obtained from triaxial tests

Difference entre les valeurs d' angle de friction Φ qui derive de la plane fracturé theorique et les valeurs effectives obtenues des esssais triaxials

I.N. Grammatikopoulos & C.A. Anagnostopoulos
Laboratory of Soil Mechanics and Foundations, Department of Civil Engineering, Aristotle University of Thessaloniki, Thessaloniki, Greece

ABSTRACT
A series of consolidated drained triaxial tests took place in clay-fine sand specimens of different proportions (number of specimens 12) and in clay-fine sand specimens of a constant proportion containing 2 to 12% cement (number of specimens 12). Microfine cement was used with Blaine over 4500 cm^2/g, produced by Titan Co., Greece. Siliceous sand used was uniform with Hazen coefficient 2.45 and its range of particle size was 74 to 840 μm. The basic characteristics of the clay used are: Liquid limit 43.54 %, Plastic limit 25.32%, Water content 61.2%, Activity 0.43, Bulk unit weight 16.68 KN/m^3, Dry unit weight 10.17 KN/m^3.
The angle Θ, defined by the exact failure plane and the horizontal line was measured for each specimen. Continuously, the friction angle Φ_f was calculated by using the equation $\Theta_f = 45 + \Phi_f/2$ and compared with the one which was drawn from Mohr circles. Concluding, the difference between the values of theoretical and reliable friction angle, for clay-fine sand mixtures, is ranging from 1.5 to 6 degrees with the reliable angle always higher. In the case of clay-fine sand specimens containing cement, the difference between the values is fluctuating from 1 to 4.5 degrees.

RÉSUMÉ
Des essais triaxials ont été obtenus à l' argile du sable fin (nombre 12), specimends de differents proportions (nombre de speciments 12). Et à l' argile du sable find des speciments de proportions constantes contenaient 2-12% du ciment (nombre de speciments 12). Du ciment microfin a été utilisé plus de 4500 cm^2/g produit à l' usine TITAN co en Gréce. Le sable du Sylex qui a été utilisé est en uniforme à coefficient Hazen 2.45 et la grandeur de ses grains varie de 74 à 840 μm. Les caracteristiques fondamentals de l' argile sont: limite de liquidité 45.55%, Limite de Plasticité 25.32%, Teneur en eau 61.2%, Activité d' une argile 0.43, Poids specifique apparent 16.68 KN/m^3, Poids specifique sec 10.17 kN/m^3.
L' angle déterminée de lincision du fragment et de la ligne horizontale est mesurée pour chaque speciment. Ensuite, l' angle de friction Φ_f a été calculée par l' equation $\Theta_f = 45 + \Phi_f/2$ et comparée à celle qui résulte des cycles Mohr. En conclusion la difference entre les valeurs de l' angle de friction théorique et réelle pour la mixtoin de l' argile du sable fin varie de 1.5 à 6 degrés avec l' angle réelle toujours plus grande. A l' argile qui continent du ciment aux differents proportions, la difference entre les valeurs varie de 1 à 4.5 degrés.

1 INTRODUCTION

In theory the friction angle Φ_f can be determined straight forward from measurements of the angle Θ_f of the failure plane, which is observed through the colourless cell of triaxial testing machine, by using the equation

$$\Theta_f = 45 + \frac{\Phi_f}{2} \tag{1}$$

Practically, the angle Θ_f of the failure plane is not always equal to $45+\Phi_f/2$ and because the height of specimen should be enough so that the crossing of the failure plane with the bases of the specimen should be avoided, it is more preferable the value of the angle Φ to be calculated from the following equations:

For cohesionless soils: $$sin\Phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} \tag{2}$$

For cohesive soils: $$sin\Phi = \frac{\frac{\sigma_1 + h}{\sigma_3 + h} - 1}{\frac{\sigma_1 + h}{\sigma_3 + h} + 1} \tag{3}$$

where: $h = \frac{c}{tg\Phi}$

So, the determination of Φ, for cohensionless soils, is possible from one triaxial test and for cohesive soils from two triaxial tests (Parry, 1995; Philipponat, 1979).

With the triaxial test the precise determination of Φ and *c* values of a soil is possible and, additionally, the shear strength under different values of water content can be estimated.

Taking into account all the above conceptions, the experimental work presented in this paper, studies the difference between the values of the angle Φ obtained by plotting the failure envelope in the Mohr diagram with the values of the angle Φ_f derived from the laboratory observations of the angle Θ_f of the failure plane and the performing of the equation (1).

2 BASIC CONCEPTIONS

The component dominating the soil deformations is that which consists of the mutual slides of the grains. These slides are activated under a stress state and inhibit the shear fracture, so that under a critical combination of shear and normal stresses is not only attributed to the friction forces developed between the grains but also by the mutual interlocking of the grains. Particles interlocking causes a strength enhancement during shearing, in order for which to be overcome, the particles must be self-displaced resulting in the increase of the soil volume.

The contribution of the cohesion to the in situ strength is low compared with that of the friction. Strength is developed between the contact surfaces of the grains and directly related to the effective stresses. In soil materials, fracture is studied by

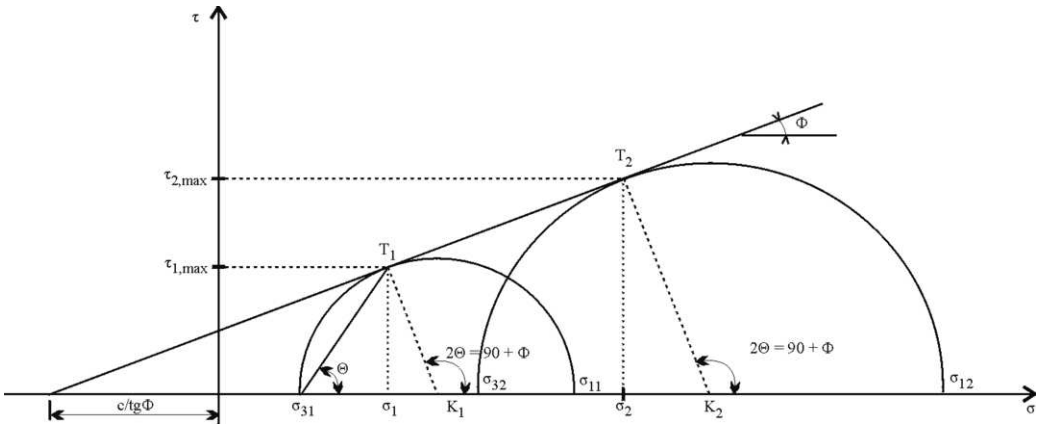

Figure 1. Failure envelope, angle Θ of the failure plane and determination of the friction angle Φ and cohesion.

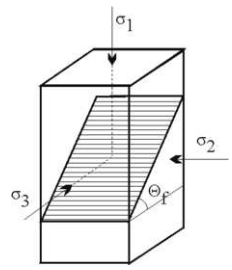

Figure 2. Stress state and angle Θ_f of the failure plane.

using Mohr's theory of the angle of internal friction (Fig. 1). According to this theory, failure occurs on a level at which shear stress appeared to have a critical value depends on the value of normal stress acting on this level.

Thus, $\tau = f(\sigma)$ and assuming linear failure envelope, the critical shear strength of the soil is expressed by Coulomb equation, $\tau = \sigma \cdot tg\Phi + c$. Failure occurs when the value of shear stress reaches the value of shear strength of the material. That means that fracture is expected on a plane at which the difference between the existing shear stress and the shear strength is minimized. Thus,

$$\frac{\partial(\tau_{max} - \tau)}{\partial\theta_f} = \Phi \Rightarrow \Theta_f = 45 + \frac{\Phi_f}{2} \qquad (4)$$

where Θ_f is the angle of the failure plane with horizontal (Fig. 2).

3 MATERIALS

3.1 *Sand*

Siliceous fine sand used in the testing programme was collected from Axios river near Thessaloniki. Its grain size distribution ranging from 74 to 840 μm, It had dry unit weight of γ_d=14.85KN/m^3, saturated unit weight of γ_{SAT}=19.35KN/m^3, porosity of n=0.45 and Hazen coefficient 2.45.

3.2 *Clay*

Clay used was taken from excavations at a depth of 10 to 15m. The basic characteristics of the in situ clay are listed in Table 1. It is defined as inorganic clay of medium plasticity.

3.3 *Cement*

Cement used was Microfine Portland cement with specific gravity of γ=3,15g/cm^3, a specific area (Blaine) of s=4500cm^2/g and characteristic compressive strength of 45 Mpa at 28 days.

4 LABORATORY PROCEDURE

In order to investigate the failure plane and consequently the friction angle determined by the combinations of the in situ measurements and Mohr-Coulomb criterion, the following sets of grain materials were prepared and tested.

The first set included reconstituted specimens of clay-fine and in different proportions. Cylindrical specimens, 33mm in diameter by 75mm high, were utilized for the experiments.

Before testing, specimens were subjected under a preconsolidation pressure of 50KN/m^2. Free water in samples was removed by drying in an oven maintained at 105°C for 24 hours.

The second set included also reconstituted specimens of the same size as the above referred having a constant proportion of 70% clay and 30% fine sand and mixed with cement in the range of 2%, 4%, 6%, 8%, 10% and 12%, by total weight of solid material. They were prepared and treated similarly as the first set.

Consolidated - drained triaxial tests were carried out on both set specimens representing the actual fracture state in terms of effective stresses. The axial strain rate was 1.25x10^{-1}%/min. Confining pressures were 0,1 and 0,2Mpa. 12 specimens for

Table 1. Properties of the in–situ soft clay.

Properties	Characteristics	Properties	Characteristics
Liquid Limit LL (%)	43.54	Grain size distribution • Clay (%) • Silt (%) • Sand (%)	 42 48 10
Plastic Limit PL (%)	25.32		
Plasticity Index PI (%)	18.22		
Water Content (%)	61.2		
Activity	0.43	Initial Void Ratio (e_o)	1.653
Bulk Unit Weight (KN/M^3)	16.68	pH (Soil:water = 1:5)	8
Dry Unit Weight (KN/M^3)	10.17	X-Ray Diffraction Analysis	Montmorillonite, Kaolinite, illite, Chlorite, Quartz, Calcite.
Specific Gravity	2.7		
Compression Index (C_c)	0.311		
Swelling Index (Ca_s)	0.093		
Shear Strength (Kpa)	18.6		

Table 2. Comparison between the values of theoretical angle φ and reliable angle φ_f, for clay-fine sand specimens.

Soil composition	Angle Φ obtained from Mohr-Coulomb circles	Angle $\Theta_f = 45 + \frac{\Phi_f}{2}$ of the failure plane	Angle Φ_f derived from equation $\Phi_f = 2\Theta_f - 90$
95% clay + 5% sand	12.5	52	14
90% clay + 10% sand	14	54	18
85% clay + 15% sand	16.5	54	18
80% clay + 20% sand	18	56.5	23
75% clay + 25% sand	20.5	57.5	25
70% clay + 30% sand	22	59	28

Table 3. Comparison between the values of theoretical angle φ and reliable angle φ_f, for clay-fine sand specimens containing 2% to 12% cement.

Cement content (% by weight of solid material)	Angle Φ obtained from Mohr-Coulomb circles	Angle Θ_f of the failure plane	Angle φ_f derived from equation $\Phi_f = 2\Theta_f - 90$
2%	27.5	61	32
4%	29	61.5	33
6%	34	62.5	35
8%	35	63.5	37
10%	36	64	38
12%	37	65	40

each set were tested in triaxial tests and the friction angle Φ was determined by plotting the failure envelope in the Mohr diagram.

After recording the in situ angle Θ of the failure plane with the horizontal, for each specimen, the Φ_f was calculated.

The differences between the friction angle Φ determined by Mohr diagram and the Φ_f derived from the laboratory observations of specimen failure plane are shown in Table 2 and 3.

5 CONCLUSIONS

For clay-fine sand mixtures, the values of friction angle become higher as the sand content increases. This is explained from the fact that as the amount of sand grains increases, the number of grain to grain contact also increases (better packing), resulting in the enhancement of frictional resistance. Also, as the sand content increases, the difference between the values of theoretical and reliable friction angle increases ranging from 1.5 to 6 degrees (Cuidi, 1975; Papacharisis et al., 1999) .

The addition of cement causes a pronounced enhancement in friction angle, especially when added in amounts ranging from 6% to 12% by weight of solid material.

The difference between the values of theoretical and reliable friction angle decreases for soil specimens containing cement, fluctuating from 1 to 4.5 degrees.

These differences for both sets, can be expressed by a non linear regression equation that best relates the corrected angle Φ_f to the Coulomb's angle and the Hazen coeficient υ as follows:

$$\Phi_f = \Phi_{Coulomb} + 3\log\upsilon \quad (5)$$

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